CHARACTERISTIC PARAMETERS IDENTIFICATION OF MEMS RESONATORS USIGN GRAY BOX MODELING

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Abstract. *The growing demand of MEMS (Micro Electro Mechanical Systems) requires total quality of each device produced. This quality is guaranteed by tests, which raise the cost of the final price. An alternative to minimize these costs is to compare the characteristic parameters defined in designing with those obtained in manufacturing. This work proposes the gray box modeling technique to obtain the characteristic parameters of two microstructures: Double Bridge and Hinge. The technique consists of five steps from System Identification. The first one is to collect data via an experimental platform. Next the model order is selected. Then the discrete model parameters are estimated through an estimator, in this case the least squares. Finally the model is validated by PE (Percentage Error) and MPE (Mean Percentage Error). The characteristic parameters are encountered by an ARX (Auto Regressive with Exogenous inputs) model. These parameters obtained from the gray box model show an error percentage lower than 1% compared to defined parameters. The results indicate that is possible to find the model parameters by gray box modeling and utilize it as criterion for selecting good microstructures.*

Keywords: *ARX model, discretization, system identification, Double Bridge, Hinge*

1. INTRODUCTION

Micro-Electro-Mechanical Systems, or MEMS, is technology that can be defined as miniaturized mechanical and electro-mechanical elements that ate made using the techniques of microfabrication. MEMS emerge as one of the most promising technologies to fulfill the needs of modern society. Automotive applications and electronic products are examples of areas where MEMS sensors have become a mainstream.

The MEMS inertial sensors had the most attention during the early waves of the MEMS technology adoption, during the 1990s and 2000s (Wisniowski, 2013). MEMS accelerometers have been widely used since the 90s in airbags vehicle like crash sensors. Since then, many others devices have benefited from the use of motion sensors. The MEMS inertial sensors, like accelerometers and gyroscopes, are am ong the most prominent sensors used in mobile devices such as smartphones and tablets, according to Figure 1. (Yole Développment, 2012b).

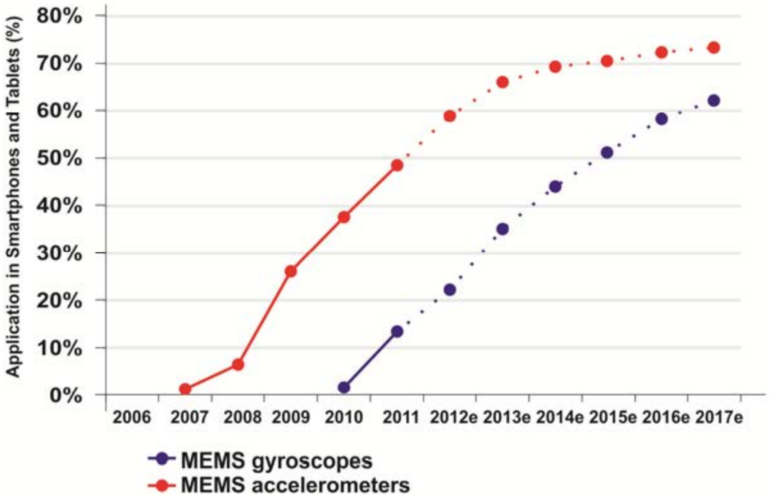


Figure 1. Diffusion of MEMS inertial sensors in smartphones and tablets.

The global market for smartphones and tablets is increasing each year, moving millions of dollars over the years and with predictive to increase, as shown in Figure 2 (Yole Développement, 2012a).

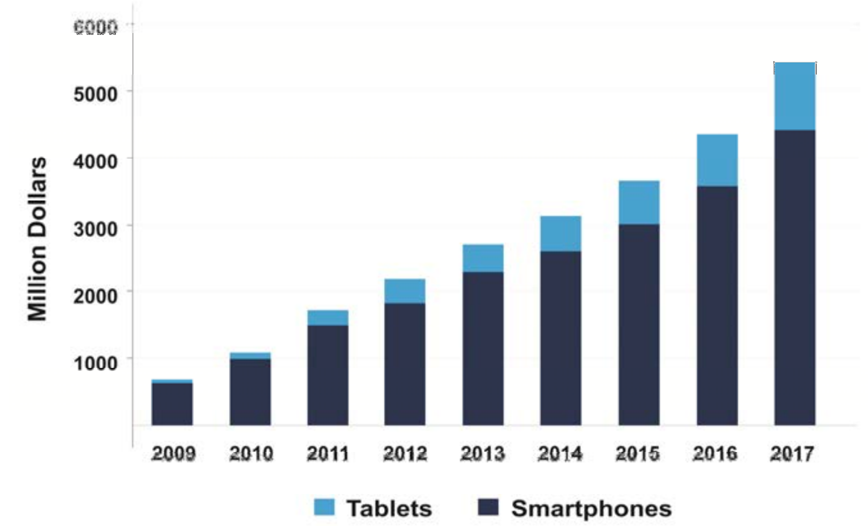


Figure 2. Global MEMS Market for Smartphones and Tablets (in Million Dollars)

The basic operation of MEMS sensors is linked to the physical knowledge of the system, defined by the structures characteristic parameters: mass (M), damping (D) and elastic constant (K). These parameters are influenced by the device geometric shape, the material properties used in its manufacture and the environment which they are inserted to operate (Song et al, 2010).

Run tests of MEMS inertial sensors is a challenging task because their characterization requires an extensive range of physical stimuli (MEMS Investor Journal, 2011). Performing these tests and device calibration makes the production cost high.

As knowledge of the micro scale phenomena are beyond the limits of classical physics, it is necessary an alternative to mathematical modeling of these structures. The system identification is presented as an ideal alternative to mathematically model the MEMS devices. The technique requires few or no prior knowledge of the system, however the input and output data are fundamental in obtaining the model.

The main objective of this paper is to obtain the ARX (Auto Regressive with eXogenous inputs) mathematical model and then find the structure characteristic parameters used in the manufacture of MEMS sensors.

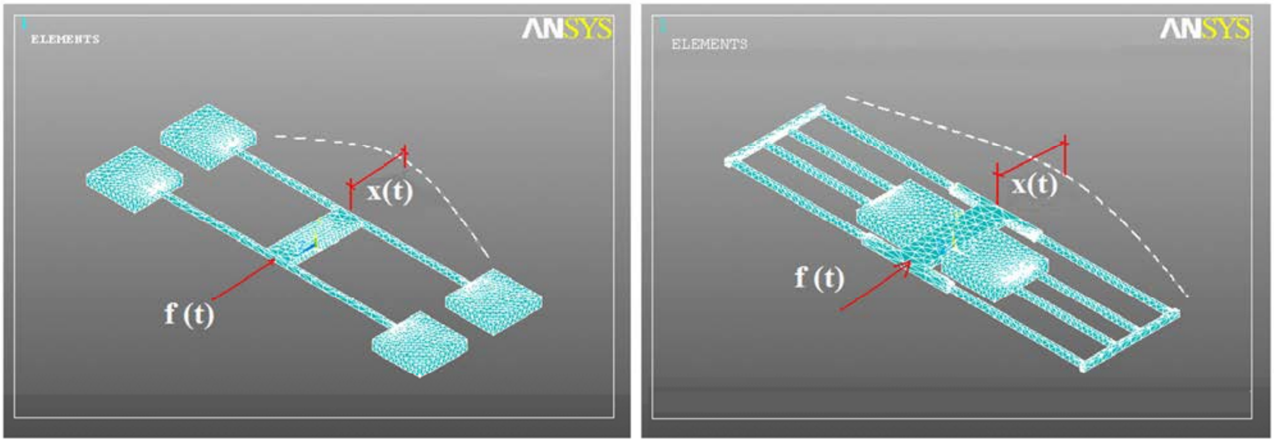
2. MATHEMATICAL MODELING: SYSTEM IDENTIFICATION

It was applied the five step of system identification proposed by Aguirre (2004) in this work. In the identification process, the mathematical model is obtained based on the data collected from the system and can reproduce the dynamics characteristics and statistics of the original system (Correa, 1997).

2.1 Data processing

At this stage the data are collected to be analyzed. The data are obtained from an experimental platform developed on finite element based software, ANSYS ®.

Figure 3 shows the geometries of the developed platforms.



(a) (b)

Figure 3. MEMS structures: (a) Double Bridge (b) Hinge

2.2 Choosing the mathematical representation

There are many ways to represent the same mathematical model, in other words, there are several ways to write equations which describe the behavior of the system (Aguirre, 2004). For the considered problem, was chosen as mathematical representation the ARX model, because the system is considered linear whose input and ouput does not suffer disturbance to be filtered. The format of ARX model is shown by equation (1).

$$A(L)x(n) = B(L)y(n) + e(n) \tag{1}$$

where $x(n)$ is the output, $y(n)$ is the input, $e(n)$ is the error between the real data and estimated data, $A(L)$ and $B(L)$ are polynomial lag operators of order p, for output, and r, for input, according to equations (2) and (3),

$$A(L) = 1 + a_1 L^{-1} + a_2 L^{-2} + ... + a_p L^{-p} \tag{2}$$

$$B(L) = b_1 L^{-1} + b_2 L^{-2} + ... + b_r L^{-r} \tag{3}$$

Expanding the polynomial in equation (1) and applying the lag operator, the ARX model can be written as follows

$$x(n) = -a_1 x(n-1) - a_2 x(n-2) - ... - a_p x(n-p) + b_1 y(n-1) + b_2 y(n-2) + ... + b_r y(n-r) + e(n) \tag{4}$$

It is noted in equation (4) that ARX model describes the present system output as a function of past values of output and input. This type of model is parsimonious because it contains a small number of parameters and the predictions obtained are quite accurate (Morettin and Toloi, 2006).

2.3 Determination of the model structure

The gray box modeling theory combines the advantages of white box and black box modeling. Therefore to determine the ARX(p, r) model order, is necessary to develop the system from white box model that represents the structure behavior (Tang, 1990):

$$M\frac{d^2}{dt^2}x(t) + D\frac{d}{dt}x(t) + Kx(t) = y(n) \tag{5}$$

Since the studied models are discrete and the white box model is continuous, it must turn the equation (5) to discrete time domain. To obtain a discrete representation of any signal, it can be used the definition of Z transform. This implies that has to be known the representation in time at which applies the transform or that is known the Laplace transform (Soares, 1996). The transfer function obtained by Laplace transform of the equation (5) is defined by:

$$\frac{X(s)}{Y(s)} = \frac{1}{Ms^2 + Ds + K} \tag{6}$$

Among the discretization methods, a group is highlighted: open loop methods. The choice is jutstified by the fact of not having interest in system control. Methods in open loop, the process used in the discretization is to replace the term s in the function, for a new term in z (Soares, 1996).

For this work it was studied three methods: Forward Difference, Backward Difference and Tustin. Among the methods, was chosen as the Backward Difference discretization results in a stable system while the Forward Difference discretization results in an unstable system. Also, in comparison with Tustin discretization has a lower number of regressors, which means fewer parameters to calculate.

Then applies the Backward Difference discretization defined by (7)

$$s = \frac{1 - z^{-1}}{T} \tag{7}$$

thus obtains the transfer function in the z domains is defined by:

$$\frac{X[z]}{Y[z]} = \frac{\theta_3}{\theta_2 z^{-2} \theta_1 z^{-1} + 1} \tag{8}$$

where

$$\begin{cases} \theta_1 = -\dfrac{(DT + 2M)}{(KT^2 + DT + M)} \\ \theta_2 = \dfrac{M}{(KT^2 + DT + M)} \\ \theta_3 = \dfrac{T^2}{(KT^2 + DT + M)} \end{cases} \tag{9}$$

This manipulation of variables in the plans s and z, can find a vector of regressors corresponding to a discretization process (Bedendo, 2012). Developing the equation (8) similarly to equation (4) gives:

$$X[n] = -\theta_1 X[n-1] - \theta_2 X[n-2] + \theta_3 F[n] \tag{10}$$

Based on equation (10) it is observed that the structure of model should be ARX (2,1)

2.4 Parameters Estimation

The estimation of the model parameters is performed by the method of least squares. The choice is justified by its ease of implementation and efficiency in estimation of linear systems (Reimbold et al, 2012). Thus, starts from the ARX(2,1) model by writing in matrix form:

$$x(n) = \begin{bmatrix} -x(n-1) & -x(n-2) & f(n-1) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + e(n) \tag{11}$$

Therefore, the system is written according to equation (12)

$$x(n) = \varphi^T \theta + e(n) \tag{12}$$

where φ is the row vector of regressors, $e(n)$ is the model error and θ is the vector containing the parameters to be estimated by the method, which will be obtained by equation (13)

$$\theta = \left(\varphi^T \varphi\right)^{-1} \varphi^T X \tag{13}$$

2.5 Model Validation

The model validation is performed by comparison of the dynamics of the estimated model and the real model. However, only visually comparing the dynamics does not show that there are model errors. So, to validate the effectiveness of the model in a quantitative form, it was used the percentage error indicator to check the percentage error of the estimated data in relation to the real data. The error percentage is defined by equation (14):

$$PE = \left(\frac{x(n) - \hat{x}(n)}{x(n)}\right) \times 100 \tag{14}$$

whereas $x(n)$ is the real values and $\hat{x}(n)$ is the estimated value.

If the PE is negative then the estimated value is smaller than the real value. If the error is positive the estimated value is higher compared to the real value.

It also calculated the Mean Percentage Error (MPE) by the sum of the percentage erros divided by the amount of data analyzed, calculating the average of Percentage Errors. If positive errors offset the negative errors, the result of MPE should be approximately zero. MPE is defined by:

$$MPE = \sum_{i=1}^{n} \left(\frac{x(n) - \hat{x}(n)}{x(n)}\right) \times \frac{100}{n} \tag{15}$$

whereas $x(n)$ is the real values, $\hat{x}(n)$ is the estimated value and n is the amount of analyzed data.

3. RESULTS

It was obtained the following parameters for the ARX model represented by equation (13):

Table 1. ARX Model obtained parameters .

Parameter	Double Bridge	Hinge
θ_1	-1.1130	-1.1979
θ_2	0.9739	0.9680
θ_3	0.7372	1.3342

It is made a comparison between the real performance and the estimated model, according to figures 4 and 5

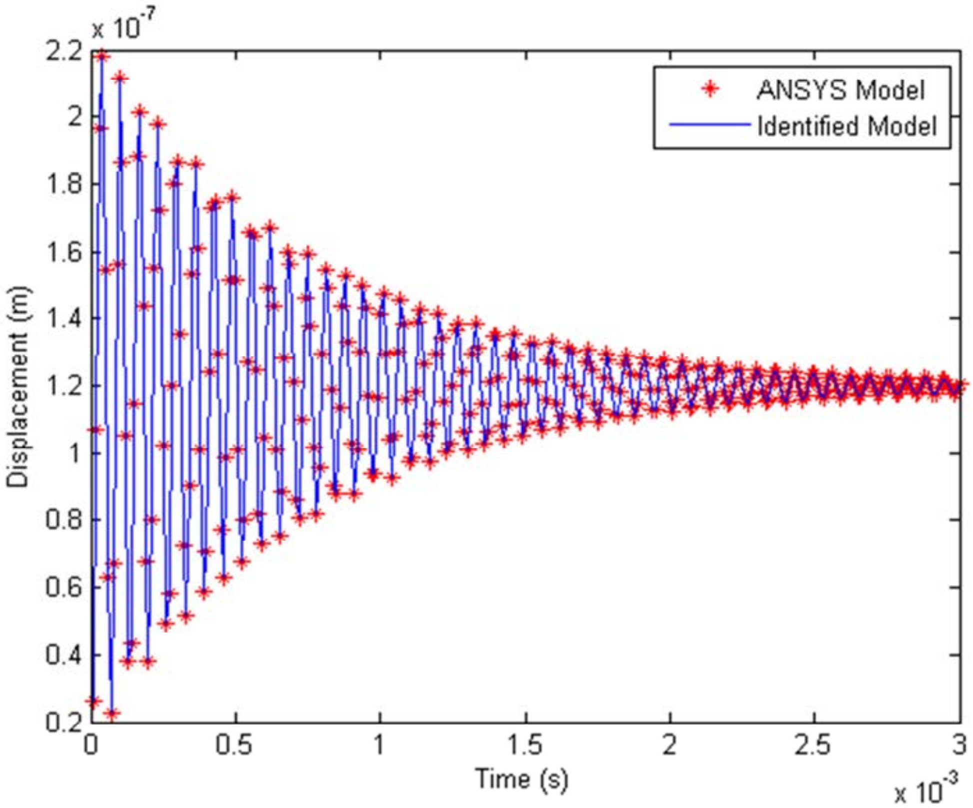


Figure 4. Performance of the Double Bridge structure

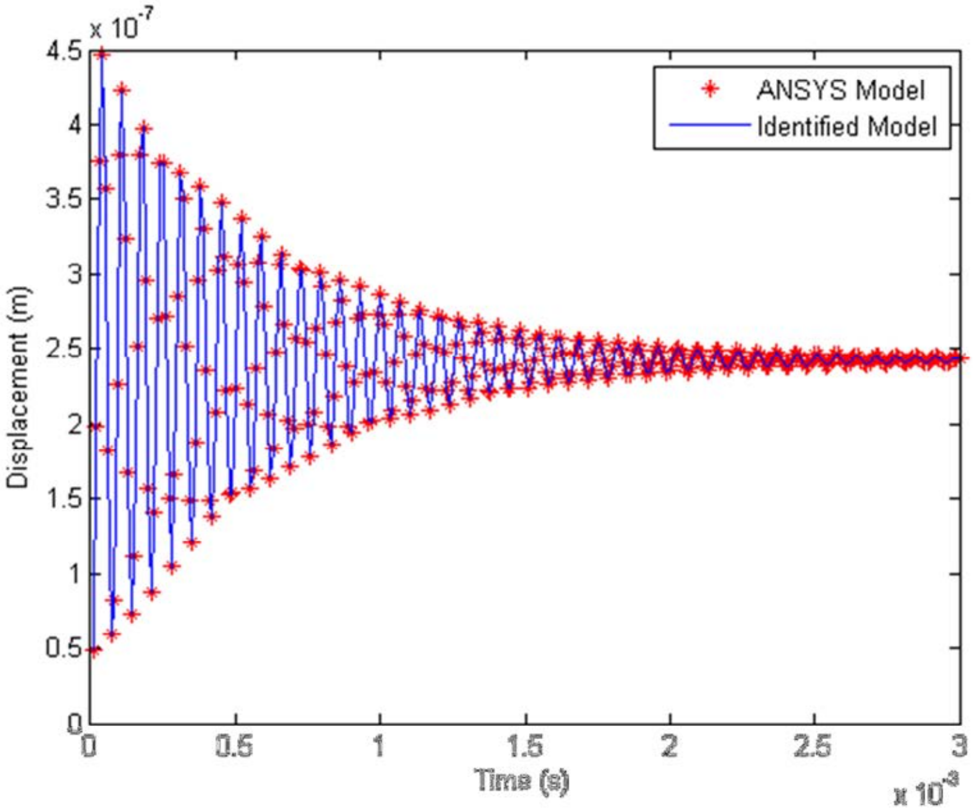
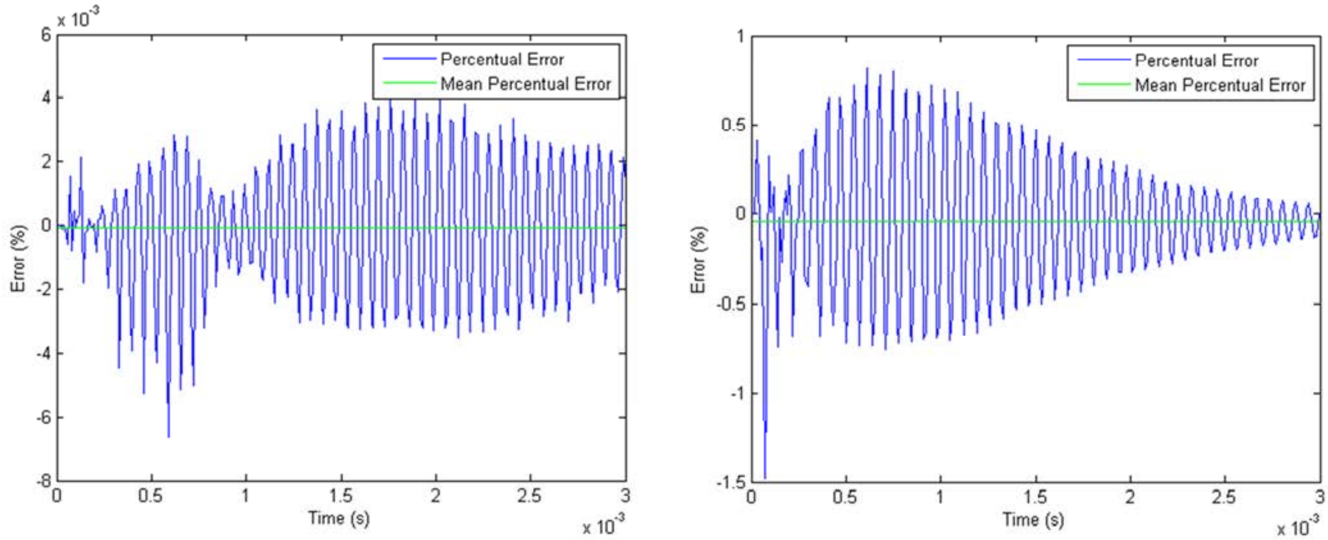


Figure 5. Performance of the Hinge structure

There is the presence of two distinct operating regimes: transient state and steady state. For both structures, the input signal in the step form generates an oscillating movement of displacement at the beginning, characterizing the transient state. As the oscillation is reduced, the structure begins to operate in a stable around a value characterizing the steady state. For both structures, the estimated models feature a dynamic visually compatible with the real dynamic. This fact can be evaluated quantitatively by Percentage Error between the platform data and estimated model dynamic.



(a) (b)

Figure 6. Percentage Error (a) Double Bridge Structure (b) Hinge.

According to Figure 6 it is observed that both structures error rate is extremely low. For both structures the maximum error was achieved in transient state and decreasing and tends to zero. The largest error found in the Double Bridge structure was -0.00662% while the Hinge structure was -1.482%. The MPE for both structures showed a value nearly zero, while for the Double Bridge structure was -0.000804% and Hinge structure was -0.042%.

Making the relationship between the vector of estimated parameters to the equation (10) parameters gives:

Table 1. characteristic parameters .

Characteristic Parameter	Double Bridge	Hinge
M	5.2361e-011	5.8945e-011
D	1.9620e-008	2.0642e-007
K	5.8800e-001	5.8800e-001

4. CONCLUSION

The gray box modeling technique used in this work is shown as an effective alternative for MEMS sensors modeling. As the gray box is a non-invasive technique, this type of modeling eliminates the possibility collapsing, electrical or mechanical, in the devices. Considering the accuracy achieved in the results and practical application, the model can be used in the manufacturing sector to verify the performance of MEMS based on the comparison of the signals. It can also be used in the design sector, since it is possible to obtain the characteristic parameters of the structures. Thus the results obtained in this work are satisfactory. As future works it is desired to apply this technique with seasonal (SARX) and nonlinear (NARX) models.

5. ACKNOWLEDGEMENTS

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