DYNAMIC MODEL OF A TWO-LINK ROBOT MANIPULATOR WITH FUZZY UNCERTAIN PARAMETERS

Fabian Andres Lara-Molina, fabianmolina@utfpr.edu.br[1]
Valder Steffen Jr, vsteffen@mecanica.ufu.br[2]

[1] Federal Technological University of Paraná, Campus Cornélio Procópio, Av. Alberto Carazzi, 1640, Cornélio Procópio, PR, 86300-000, Brazil.
[2]Federal University of Uberlândia (UFU) - School of Mechanical Engineering - Campus Santa Mônica, 38400-982 Uberlândia – MG – Brazil

Abstract: *This paper aims at studying the effect of uncertain parameters on a two-link planar manipulator using a fuzzy logic approach. The uncertain parameters are modeled as fuzzy variables and the dynamic simulation of the robot is performed using fuzzy dynamic analysis. Two case studies are considered to analyze the dynamic behavior of the robot manipulator: a) uncertain payload and b) uncertain payload and friction force simultaneously. Numerical simulations illustrate the proposed methodology so that the effect of fuzzy uncertain parameters on the performance of the robot manipulator is analyzed.*
keywords: *Uncertainty Analysis, Fuzzy Variables, Robot Manipulator*

1. INTRODUCTION

Robot manipulators are unavoidably subjected to uncertainties. The main sources of uncertainties include manufacturing and assembling tolerances of the mechanical parts and control errors. Furthermore, in several applications, the manipulators operate with different values of payload to perform a specific task, (e.g. pick and place robots).

Nevertheless, the robot manipulators must be able to execute diverse tasks with high accuracy and repeatability, which requires high reliability (e.g. robots used in surgical applications). Therefore it is necessary to analyze properly the effects of uncertain parameters on the dynamic response in order to observe the behavior of the robot manipulators with uncertain parameters.

Several methodologies have been used to analyze the uncertainties in robot manipulators. The stochastic approach has been widely applied to study the effects of uncertain parameters on the behavior of robot manipulators. In this approach, the uncertain parameters are modeled by means of random variables. In agreement with this approach, effect of tolerances associated with the various manipulator parameters on the reliability was studied Rao and Bhatti (2001); Kim *et al.* (2010); Pandey and Zhang (2012) and the tracking control for a two-link planar rigid manipulator was considered by Cui *et al.* (2013).

Interval analysis has been applied to study the uncertainties of robot manipulators. In this approach the uncertainty is defined by a given interval. Melet (2009) introduced the proprieties of the robot that are sensitive to uncertain parameters and the interval analysis method is used to manage these uncertainties aiming at ensuring the reliability of the robot . The joint tolerances of the manipulator were modeled by using interval analysis Wu and Rao (2007).

Recently, an approach based on fuzzy logic has been used to analyze the uncertain parameters in mechanical structures Moens and Hanss (2011); Lara-Molina *et al.* (2014a). This approach is suitable when the stochastic process that governs the uncertainty is unknown, thus uncertain parameters are modeled by means of fuzzy variables. The fuzzy approach is an extension of interval analysis where a membership function indicates a range of possible uncertain values. The fuzzy analysis requires the solution of the interval problems corresponding to the uncertainty model.

According with the previous discussion, it is necessary to develop a straightforward methodology to evaluate the dynamic behavior of robot manipulators with fuzzy uncertain parameters, i.e., to analyze how the robot dynamics is affected by an uncertain payload when numerical simulation of the robot Lara-Molina *et al.* (2012b). Furthermore, it is necessary to evaluate the effect of uncertain parameters on the dynamic performance Lara-Molina *et al.* (2012a, 2014b).

In this work the dynamics of a planar two-link rigid manipulator with fuzzy uncertain parameters is analyzed. The simulation of the robot is performed by means of fuzzy dynamic analysis. In accordance with this, the uncertain parameters are modeled by means of fuzzy variables.

This paper is organized in three sections. Section 1 introduces the robot manipulator model. In section 2, the methodology to analyze the fuzzy uncertainties is presented. The numerical results are presented in section 3. Finally, the conclusions and further work are formulated.

2. ROBOT MANIPULATOR MODELING

In this work, a two-link planar rigid robot manipulator is considered (see Fig.1). In order to establish a mathematical model to study the dynamic behavior of the robot, the dynamic model of the manipulator was obtained by means of the Lagrange-Euler dynamics (Lewis *et al.*, 2003). In the dynamic model of Eq. (1) the dynamics of the links and frictions of the joints were considered.

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\dot{\theta}) \tag{1}$$

where, $M(\theta)$ is the mass matrix; $V(\theta,\dot{\theta})$, $G(\theta)$ and $F(\dot{\theta})$ are the Coriolis/centrifugal, gravity and friction terms vectors, respectively; τ is the torque in the actuators; θ, $\dot{\theta}$, $\ddot{\theta}$ are the joint positions, velocities and accelerations. The gravity acts along the $-y$ axis direction (see Fig.1).

The friction term $F(\dot{\theta})$ considers both the viscous F_v and dynamic F_d friction forces, thus:

$$F(\dot{\theta}) = F_v\dot{\theta} + F_d(\dot{\theta}) \tag{2}$$

where $F_v = diag\{v_i\}$, $F_d(\dot{\theta}) = K_d sgn(\dot{\theta})$, with $K_d = diag\{k_i\}$, for $i = 1, 2$.

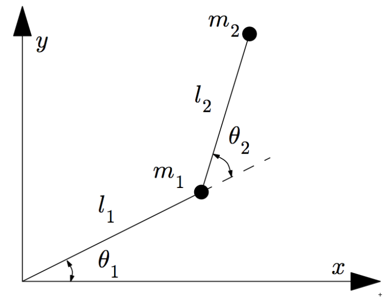


Figura 1: **Two-link planar rigid robot manipulator.**

3. ANALYSES OF DYNAMIC SYSTEMS WITH FUZZY PARAMETERS

In several cases, some parameters of the systems cannot be accurately estimated due to small variations around their nominal values. In these cases, these parameters can be modeled by means of fuzzy variables. The fuzzy set theory was initially formulated by Zadeh (1965) to represent vague or ambiguous information. Thereby, it is possible to represent inaccurate or uncertain parameters using fuzzy variables, specially when the stochastic process which models the uncertain parameter is unknown.

Moens and Hanss (2011) presented a review of the literature focused on the non-probalistic approaches to analyze parameter uncertainty. The two main approaches, presented in this work, to model the uncertainties are interval quantities and fuzzy variables. These two approaches require the solution of interval problems. The methodology to analyze the fuzzy uncertainties in this woks is based on the α-level technique presented by Möller and Beer (2004) . The basic concepts of the fuzzy variables are introduced below.

3.1 Fuzzy Variables

Let $\mathbf{X}$ be an universal classical set of objects whose generic elements are denoted by x. The subset A (where, $A \in \mathbf{X}$) is defined by the classical membership function $\mu_A : \mathbf{X} \rightarrow \{0, 1\}$ (see Fig. 2(a)). Furthermore, a fuzzy set $\tilde{A}$ is defined by means of the membership function $\mu_A : \mathbf{X} \rightarrow [0, 1]$, where $[0, 1]$ is a continuous interval. The membership function indicates the degree of compatibility of the element x to $\tilde{A}$. The closer the value of $\mu_A(x)$ is to "1", the more x belongs to $\tilde{A}$.

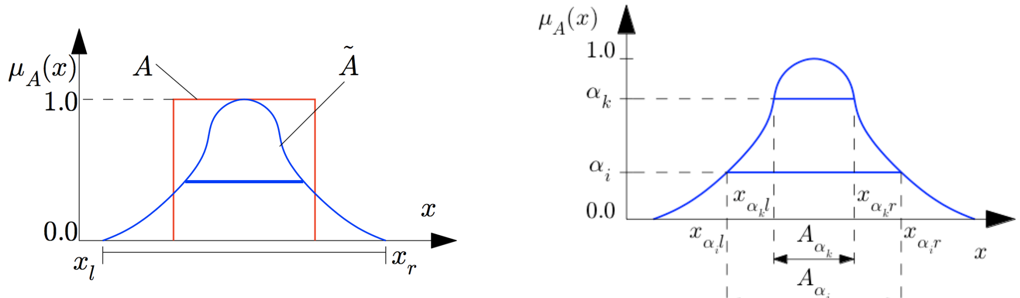
Thus, the fuzzy set is completely defined by:

$$\tilde{A} = \{(x, \mu_A(x))\,|x \in \mathbf{X}\}, \text{where, } 0 \le \mu_{\mathbf{A}} \le 1 \tag{3}$$

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets which are denominated α-levels. These subsets, which correspond to real and continuous intervals, are defined by A_{α_k} (see Fig. 2(b)), thus:

$$A_{\alpha_k} = \{x \in \mathbf{X}, \mu_{\mathbf{A}}(x) \ge \alpha_k\} \tag{4}$$

The α-level subsets of $\tilde{A}$ have the propriety:



(a) Fuzzy set.

(b) α-levels.

Figura 2: Fuzzy sets and α-level representation.

$$\underline{A}_{\alpha_k} \subseteq \underline{A}_{\alpha_i} \forall \alpha_i, \alpha_k \in (0, 1] \quad \text{with} \quad \alpha_i \leq \alpha_k \tag{5}$$

If the fuzzy set is convex (in the unidimensional case), each α-level subset A_{α_k} corresponds to the interval $[x_{\alpha_k l}, x_{\alpha_k r}]$ where:

$$\begin{aligned} x_{\alpha_k l} &= \min[x \in \mathbf{X} | \mu_A(x) \geq \alpha_k] \\ x_{\alpha_k r} &= \max[x \in \mathbf{X} | \mu_A(x) \geq \alpha_k] \end{aligned} \tag{6}$$

3.2 Dynamic Model of Robot Manipulator with Fuzzy Parameters

The dynamic model of the robot described in the Eq. (1) can be extended to a fuzzy model by considering the parameters of the model as a set of fuzzy variables $\tilde{x}$. Thus, the fuzzy dynamic model of the robot is presented in Eq. (7).

$$\tilde{\tau} = \tilde{M}(\theta)\ddot{\theta} + \tilde{V}(\theta, \dot{\theta}) + G(\theta) + \tilde{F}(\dot{\theta}) \tag{7}$$

where $\tilde{M}(\theta)$, $\tilde{V}(\theta, \dot{\theta})$ and $\tilde{F}(\dot{\theta})$ contain fuzzy parameters. Thus, by considering the inputs of the model as the set of fuzzy variables $\tilde{x}$, the dynamic response of this system corresponds to the resulting fuzzy functions $\tilde{z}$. These fuzzy functions result from the mapping process $\tilde{x} \rightarrow \tilde{z}$.

3.3 Fuzzy Dynamic Analysis

The fuzzy dynamic analysis is an appropriated method to map a fuzzy input vector $\underline{\tilde{x}}$ onto the output $\underline{\tilde{z}}$ of a numerical model using the deterministic model of the Eq. (1). The fuzzy dynamic analysis is composed of two stages showed in the Fig. 3.

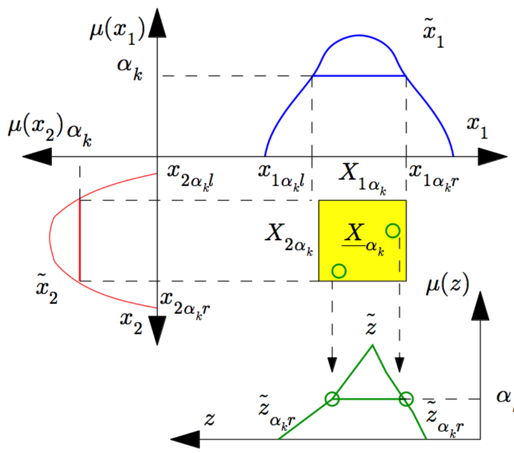


Figura 3: Sub-space $\underline{X}_{\alpha_k}$ and resulting fuzzy variable $\tilde{z}$.

In the first stage, for computational purposes, the input vector which corresponds to the fuzzy parameters is discretized by means of α-level representation of Eq. (4) and Fig. 2(b). Thus each variable of fuzzy parameters vector $\underline{\tilde{x}} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ is considered as an interval $X_{i\alpha_k} = [x_{i\alpha_k l}, x_{i\alpha_k r}]$, where $\alpha_k \in (0, 1]$. Consequently, the sub-space $\underline{X}_{\alpha_k}$ is defined, where $\underline{X}_{\alpha_k} = (X_{1\alpha_k}, \ldots, X_{n\alpha_k})$, thus $\underline{X}_{\alpha_k} \in \mathbb{R}^n$.

The second stage is related to solve an optimization problem. This optimization problem consists in finding the maximum and minimum value of the output using the mapping model $M : \underline{z} = \mathbf{f}(\underline{x})$, where $\mathbf{f}(\underline{x})$ is the deterministic model, thus:

$$z_{\alpha_k r} = \max_{\underline{x} \in \underline{X}_{\alpha_k}} \mathbf{f}(\underline{x}) \qquad z_{\alpha_k l} = \min_{\underline{x} \in \underline{X}_{\alpha_k}} \mathbf{f}(\underline{x}) \tag{8}$$

$z_{\alpha_k r}$ and $z_{\alpha_k l}$ correspond to the upper and lower bound of the interval $z_{\alpha_k} = [z_{\alpha_k r}, z_{\alpha_k l}]$ in the α-level α_k. The set of discretized intervals $[z_{\alpha_k r}, z_{\alpha_k l}]$ for $\alpha_k \in (0, 1]$ compose the whole fuzzy resulting variable $\tilde{z}$.

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems, two on all α-level of interest for each considered time step. Each upper and lower bound of the system analysis at a certain time is obtained with the aid of Differential Evolution algorithm Price *et al.* (2005) since evolutionary strategies have been used to solve optimization problems in robotics with success Lara-Molina *et al.* (2011). The the transient response value at an evaluated time-step is obtained finding the maximum and minimum output by solving the optimization problem in which the objective function is the deterministic model of the system. The inputs of the objective function are the uncertain parameters described previously by means of fuzzy variables.

4. RESULTS AND DISCUSSION

The fuzzy dynamic analysis is applied to study the effects of uncertain parameters on the dynamic performance of the robot manipulator (see Fig.4). The direct dynamic model of the robot was used in the simulation using a code implemented in MATLAB/SIMULINK®.

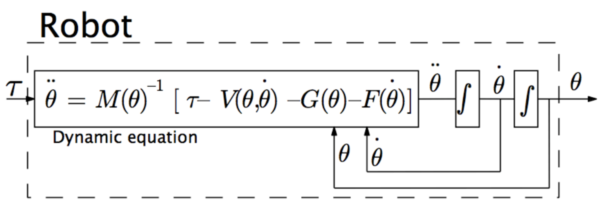


Figura 4: Direct dynamic model of the robot.

The nominal parameters for the numerical simulation are presented in Tab 1.

Tabela 1: **Nominal parameters of the two-Link robot manipulator.**

Parameters	Value	Parameters	Value
$m_1[kg]$	0.25	$v_1[Ns/rad]$	1
$m_2[kg]$	0.25	$v_2[Ns/rad]$	1
$l_1[m]$	0.25	$k_1[N]$	1
$l_2[m]$	0.25	$k_2[N]$	1
k_p	2236	k_v	67

An imposed torque is applied to the joints (see Fig. 5). The joint space position was obtained using the direct dynamic model of the robot of Fig. 4.

The uncertain parameters were considered using fuzzy triangular numbers, which is the simplest notation to describe a fuzzy variable.

The parameters used in the Differential Evolution Algorithm to solve the optimization problem in the fuzzy analysis are described as follows: population size is 10 per uncertain variable, 100 generations, crossover probability rate is $0,8$, perturbation rate is $0,8$ and strategy for mutation mechanism is DE/RAND/1/BIN. These parameters were derived of previous contributions (Price *et al.*, 2005). The objective functions for the optimization are the joint space positions θ (see Eq. (1)).

Two case studies are considered to analyze the dynamic behavior of the robot manipulator. In the first case, the robot dynamics is analyzed with an uncertain payload. In second case, the robot dynamics is analyzed with uncertain friction force and payload.

4.1 Payload Uncertainty

In this case, the effect of an uncertain payload on the dynamic response of the robot manipulator is studied. Specifically, the effect of the uncertain parameter on the joint space position of the robot is analyzed (the imposed torque of Fig. 5 is applied). The uncertain payload is modeled by means of the triangular fuzzy number $\tilde{m}_2 = (0,225/0,250/0,275)Kg$, i.e, the parameter m_2 can vary $\pm\%10$ with respect to its nominal value.

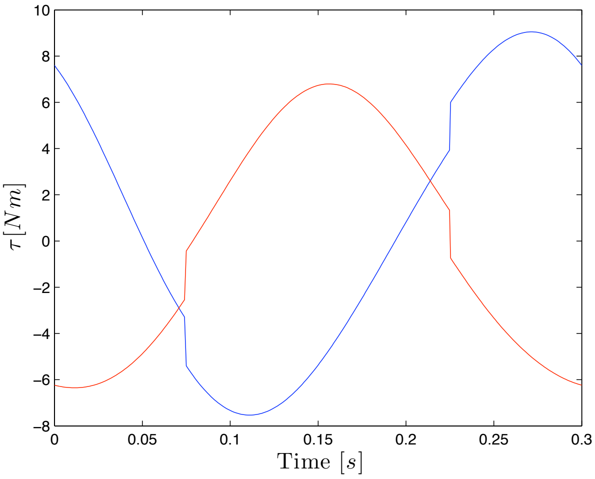


Figura 5: Imposed torque in the joints.

The effect of the uncertain payload on the time domain response of the robot is obtained by using the fuzzy dynamic analysis introduced in Sec. 3.3 The resulting uncertain joint space position, evaluated for $\alpha = 0$, $\alpha = 0,5$ and $\alpha = 1$, is shown in Fig. 6. The joint space position of the robot with the nominal payload corresponds to $\alpha = 1$; in this case, a sinusoidal joint space trajectory is produced by the imposed joint torque. The maximum uncertainty of $\tilde{m}_2$ is considered for $\alpha = 0$. In this case (when $\alpha = 0$), the joint space position is described by an envelope which contains uncertain output. The effect of the uncertain payload on the joint space position is preponderant with time increasing due to the joint space acceleration is proportional to the variation of payload parameter. The 10% of payload uncertainty produce an uncertainty of $2,27\%$ and $0,71\%$ in the first and second joint position respectively at the specific time 0,2 s. This results indicate that a small uncertainty in the nominal parameter of the model may change significantly the dynamic behavior of the robot manipulator.

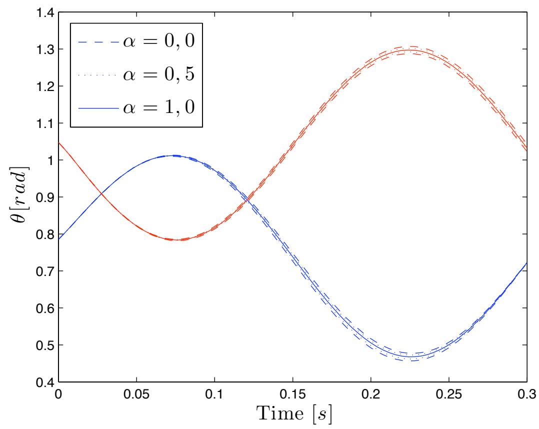


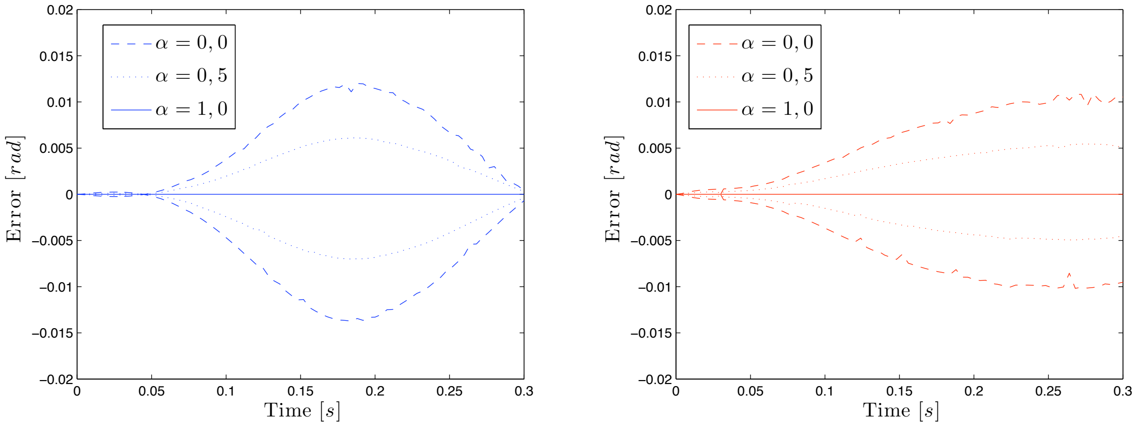
Figura 6: Joint position with uncertain payload.

Moreover, the error between the joint space of the robot manipulator with nominal and uncertain payload is showed in Fig.7. The results indicate that the amplitude of the error is proportional to the uncertain payload; therefore the effect of this uncertainty is not negligible in the model of the robot.

4.2 Uncertain Friction Force

In this case, the effect of an uncertain frictions forces on the joint space position are studied. The imposed torque of Fig. 5 is applied and the uncertain parameters are modeled by means of the triangular fuzzy number, thus: $\tilde{v}_{1,2} = (0,95/1/1,05)Ns/rad$ and $\tilde{k}_{1,2} = (0,95/1/1,05)N$.

The resulting uncertain joint space position, evaluated for $\alpha = 0$, $\alpha = 0,5$ and $\alpha = 1$, is shown in Fig. 8. The joint space position of the robot with the nominal parameters corresponds to $\alpha = 1$; in this case, a sinusoidal joint space trajectory is produced by the imposed joint torque. The maximum uncertainty is considered for $\alpha = 0$. The uncertain frictions modifies value of the joint space position around its nominal value. The 5% of uncertainty in friction produces an uncertainty of $1,31\%$ and $2,13\%$ in the first and second joint respectively at the specific time 0,212 s. This results indicates that small uncertainties in friction parameter modify the dynamic response of the robot manipulator.



(a) Joint Error θ_1.

(b) Joint Error θ_2.

Figura 7: Joint Errors.

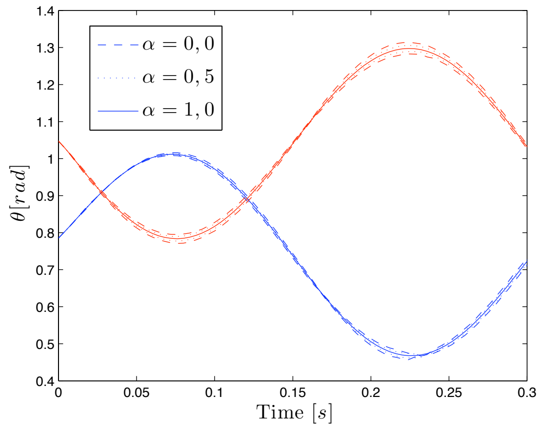
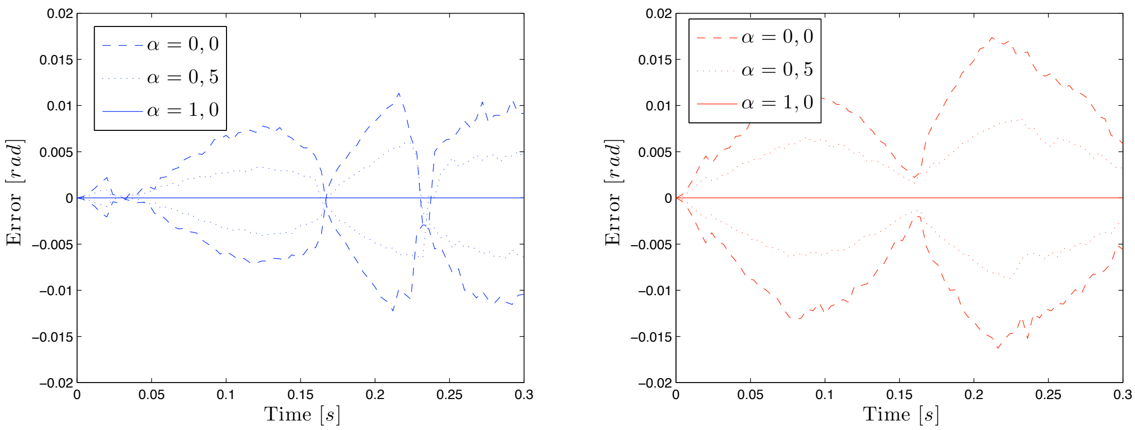


Figura 8: Joint position with uncertain friction.

Furthermore, Fig.7 shows the error between the joint position with nominal parameters for $\alpha = 1$ and time-response with the uncertain frictions. The error indicates that uncertainty friction parameters have a considerable influence on the model of the robot manipulator.



(a) Joint Error θ_1.

(b) Joint Error θ_2.

Figura 9: Joint Errors.

5. CONCLUSIONS

In this paper the effect of uncertain parameters on the dynamic behavior of a two degree planar rigid robot manipulator was studied. Specifically, the direct dynamic model was analyzed with uncertain payload and friction parameters.

The fuzzy dynamical analysis demonstrated to be a straightforward method to quantify the effect of uncertain parameters on the dynamic response of the robot manipulator. Moreover, the uncertain parameters were properly characterized by means of fuzzy variables. Nevertheless, the fuzzy dynamic analysis requires a high computational effort.

The simulation results indicate that small uncertainties in the parameters of the numerical model of the robot manipulator may affect significantly the dynamic behavior of the system. Therefore, uncertain parameters must be taken into account in numerical simulation to obtain numerical models with higher reliability. Accordingly, the study of robot manipulator dynamics should be extended by the inclusion of the uncertain analysis, e.g. involving the simulation of new control techniques, numerical simulation to study kinematic, and dynamic proprieties and design procedure.

Further work will be related to the study of the dynamic performance of flexible robot manipulators in several application with uncertain parameters.

6. ACKNOWLEDGEMENTS

The authors express their acknowledgements to the National Institute of Science and Technology of Smart Structures in Engineering (INCT-EIE), jointly funded by CNPq and FAPEMIG.

7. REFERENCES

Cui, M.Y., Wu, Z.J. and Xie, X.J., 2013. “Stochastic modeling and tracking control for two-link planar rigid robot manipulator”. *International Journal of Innovative Computing, Information and Control*, Vol. 9, No. 4, pp. 1769–1780.

Kim, J., Song, W.J. and Kang, B.S., 2010. “Stochastic approach to kinematic reliability of open-loop mechanism with dimensional tolerance”. *Applied Mathematical Modelling*, Vol. 34, pp. 1225–1237.

Lara-Molina, F.A., Rosario, J.M., Dumur, D. and Wenger, P., 2012a. “Generalized predictive control of parallel robots”. In K. Kozłowski, ed., *Robot Motion and Control 2011*, Springer London, Vol. 422 of *Lecture Notes in Control and Information Sciences*, pp. 159–169.

Lara-Molina, F.A., Rosário, J.M. and Dumur, D., 2011. “Multi-objective optimization of stewart-gough manipulator using global indices”. In *Advanced Intelligent Mechatronics (AIM), 2011 IEEE/ASME International Conference on*. pp. 79–85.

Lara-Molina, F.A., Koroishi, E.H. and Steffen Jr, V., 2014a. “Análise estrutural considerando incertezas paramétricas fuzzy”. In F.S. Lobato, V. Steffen Jr and A.J. da Silva Neto, eds., *Técnicas de Inteligência Computacional com Aplicações em Problemas Inversos de Engenharia*, Omnipax.

Lara-Molina, F.A., Rosário, J.M., Dumur, D. and Wenger, P., 2014b. “Robust generalized predictive control of the orthoglide robot”. *Industrial Robot: An International Journal*, Vol. 41, No. 3, pp. 275 – 285.

Lara-Molina, F.A., Rosário, J.M., Dumur, D. and Wenger, P., 2012b. “Application of predictive control techniques within parallel robot”. *Sba: Controle & Automação Sociedade Brasileira de Automatica*, Vol. 23, pp. 530 – 540.

Lewis, F.L., Dawson, D.M. and Abdallah, C.T., 2003. *Robot Manipulator Control: Theory and Practice*. CRC Press.

Merlet, J.P., 2009. “Interval analysis and reliability in robotics”. *Int. J. Reliability and Safety*, Vol. 3, No. 1/2/3, pp. 104–130.

Moens, D. and Hanss, M., 2011. “Non-probabilistic finite element analysis for parametric uncertainty treatment inapplied mechanics: Recent advances”. *Finite Elementsin Analysis and Design*, Vol. 47, No. 1, p. 2011.

Möller, B. and Beer, M., 2004. *Fuzzy Randomness, Uncertainty in Civil Engineering and Computational Mechanics*. Springer-Verlag.

Pandey, M.D. and Zhang, X., 2012. “System reliability analysis of the robotic manipulator with random joint clearances”. *Mechanism and Machine Theory*, Vol. 58, pp. 137–152.

Price, K.V., Storn, R.M. and Lampinen, J.A., 2005. *Differential evolution a practical approach to global optimization*. Springer-Verlag.

Rao, S. and Bhatti, P., 2001. “Probabilistic approach to manipulator kinematics and dynamics”. *Reliability Engineering and System Safety*, Vol. 72, pp. 47–58.

Wu, W. and Rao, S., 2007. “Uncertainty analysis and allocation of joint tolerances in robot manipulators based on interval analysis”. *Reliability Engineering and System Safety*, Vol. 92, pp. 54–64.

Zadeh, L., 1965. “Fuzzy sets”. *Information and Control*, Vol. 8, pp. 338–353.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.