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THREE-DIMENSIONAL SURFACE RECONSTRUCTION USING NURBS

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Abstract. *In this paper will be presented a new methodology for 3D surface reconstruction from unorganized cloud of points.In the first step of the proposed methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. From the new set of points the contour of the object cross-section is drawn using NURBS. The analysis of consecutive cross-sections enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation, as in a conical object. From all defined points the surface can be meshed and then, using commercial software the object rendering can be made for its visualization. From simulations results it can be shown that the proposed method is very effective for clustering unorganized point clouds for meshing surfaces. Examples are shown in order to explain the methodology.*

Keywords*: Surface reconstruction, Unorganized 3D points, 3D shape recovery, NURBS, 3D reconstruction*

1. INTRODUCTION

Due to the increasing interest to use the computer aided design and manufacturer's concept (CAD/CAM) in several areas like as medical science, biomedical engineering, geographical data processing, paleontology and reverse engineering, the digitalization and reconstruction techniques of objects having complex shape and/or free-form have been presented an important development.

The velocity and precision of these techniques are due to the improvement in the areas of physics, electrical, engineering, the lasers' development, CCD cameras, and the high velocity acquisition cards. In the acquisition data process, thousands or millions of data must be acquired. These acquired data consist in a cloud of points in general, not organized, with gaps and noise, acquiring efficient and trustful algorithms capable to obtain computational models from those samples (Curless, 1997).

A surface reconstruction always consists of getting a computational model that resembles more faithfully to the real object and this must be done from a cloud of points obtained from systems based on objects measurements or whose systems that not use measurements. Systems based on measurements uses techniques based on images or on active 3D sensors. Systems not based on measurements are commercial computer animation software that allow the generation of 3D models starting from simple elements like polygonal boxes. In all case the reconstruction method must to identify the correct geometry, the topology and the object shape from a finite ensemble of points (Remondino, 2003).

Several works have proposed techniques to overcome the difficulty involved in the problem of surface reconstruction from unorganized points (Yu, 1999), (Mangiameli et al., 1996), (Fritzke, 1993a, 1993b, 1996), (Ivrissimtzis et al., 2003), (Kawahara and Saito, 1996), (Hoppe et al., 1992), (Amenta et al., 1998).

There is a vast literature on geometry extraction from point clouds. It includes local polynomial fits, global smooth approximations, Voronoi/Delaunay-based methods, and level set methods, like in (Alexa et al., 2003), (Amenta and Kill, 2004), (Edelsbrunner and Mucke, 1994), (Gopi et al., 2000), (Bertalmo et al., 2001), (Xie et al. 2004). Earlier heuristic approaches gave no guarantees, but new powerful methods do give guarantees of faithfulness to the underlying continuous shape in the limit of infinitely dense data, under somewhat restrictive assumptions on object smoothness and the evenness of the sampling (Dey and Sun, 2005a,b), (Kamberov and Kamberova, 2007).

A lot of methods have been developed in order to create a regular and continuous mesh representation from the cloud of points (Mencl, 2001). From the meshed surface others techniques can be used to the pos-processing (textured, shading and rendering) and to visualization of the 3D model (Patias, 2001).

The points digitalization obtained although the acquisition system and image analysis techniques like computed tomography (CT), magnetic resonance imaging (MRI) and ultrasound imaging, have allowed to build computational models that reduce the costs and the time of product creation. Splines and NURBS (Non-Uniform Rational B-Splines) have been useful to model the surfaces, enabling an excellent description of the object surface (Remondino, 2003).

Although the various methods to reconstruct a surface from a cloud of points, in general they have been applied to a specific area or have data restrictions. Problems like noise, shadow (failure of reading due to hidden details), sinuous edges and contours restrict each method making difficult to the development of a generic and efficient method.

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In this paper will be presented a new methodology for 3D surface reconstruction from unorganized cloud of points. In the first step of the methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. From the new set of points every new point will be ordered by an proposed ordering process to define the contour of the object where, by using NURBS the object cross-section is defined. The analysis of consecutive cross-sections enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation, as in a conical object. From all defined points the surface can be meshed and then, using commercial software the object rendering can be made for its visualization. Examples are presented in order to explain the methodology.

2. METHODOLOGY

In a 3D reconstruction process, the results can be an approximated surface or an interpolated surface depending of the used method and the desired objective. The methods are chosen by considering the data processing time and the quality of the final graphics reconstruction (Curless, 1997).

The surfaces are constructed, in general, by using polygon (triangulation, for example) or functions like NURBS.

The reconstruction by using slices (cross section of the object), associated to NURBS method, is a well used method because reproduce with accuracy the object surface, and the application of the contour parameters are more simple (Mencl, 2001).

2.1. NURBS: Non-Uniform Rational Basis Splines.

The NURBS is a standard industrial tool for the representation and design of simple and complex geometry. The NURBS can interpolate the control points reproducing the object shape with accuracy and allows change its shape too (Xie and McDonnell, 2004).

The interest by using the NURBS is due to the following characteristics: the common mathematical model used by the method to represent standard analytical forms and free-form; they offer good flexibility for the complex shape design; the data processing time is small; and they are generalization of the Bezier and B-splines curves and surfaces.

The NURBS surfaces are a curve set of the same class and is the generalization of a tensor-product B-spline surface. It is defined over the parametric variables *u* and *v* as:

$$S(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} P_{i,j} \cdot w_{i,j} \cdot B_{i,k}(u) \cdot B_{j,l}(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{i,j} \cdot B_{i,k}(u) \cdot B_{j,l}(v)} \quad (1)$$

Where $B_{i,k}$ and $B_{j,l}$ are B-splines basis functions of degree *k-1* and *l-1*, respectively. The NURBS surface has *mxn* control points $P_{i,j}$ and weights $w_{i,j}$ that define the influence of the control points on the curve. The number of knots is $(m+k)+(n+l)$.

The number of the control points *m* and *n* are defined by the surface order on each parametric direction *u* and *v*, respectively. Each control point has its own weight that is positive, in general.

The B-splines basis functions are knot's functions t_i in an increasing sequence. Then, the B-splines basis functions can be defined as:

$$B_{i,1}(u) = \begin{cases} 1 \; if \; t_i \le u \le t_{i+1} \\ 0 \; otherwise \end{cases} \quad (2)$$

With

$$B_{i,k}(u) = \frac{u - t_i}{t_{i+k-1} - t_i} \cdot B_{i,k-1}(u) + \frac{t_{i+k} - u}{t_{i+k} - t_{i+1}} \cdot B_{i+1,k-1}(u) \quad (3)$$

The knots are represented by a number list that is commonly called as knot vector. The knot vector need to be an equal or increasing number sequence and the repetition of the each knot cannot be higher than the surface degree on the respective direction. More details about NURBS curves can be found in Piegl and Tiller (1996).

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2.2. The reduction method

In order to simplify the reconstruction method and to reduce the data processing time, we have been proposed a method to reduce the number of the acquired points. This method substitutes the points closed in a defined region by one at its center of mass (CM), considering a point having a unitary mass. As the center of mass is calculated in a defined region, the distance of two consecutives center of mass is defined by the precision of the required reconstruction surface, which is defined by the selected dimensions of the region.

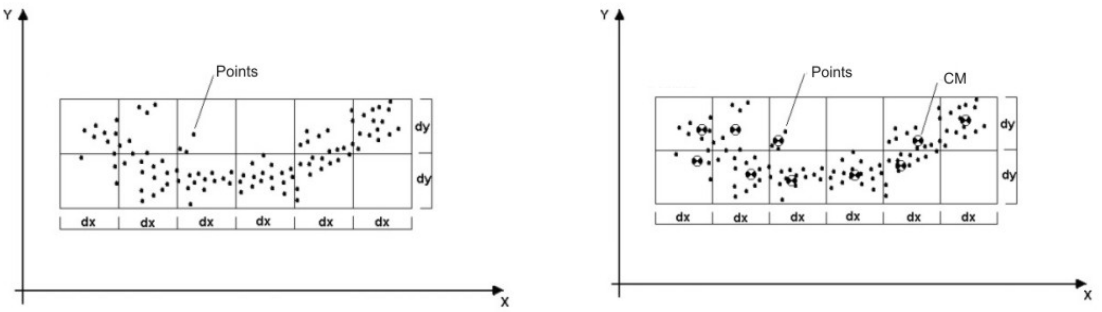
From simulations the better geometry of the region is a parallelepiped which is defined by dimensions "*dx*", "*dy*" and "*dz"*. As the geometry is simple, it is easy to identify its inner points and elaborate the algorithm leading to a small data processing time. Furthermore, it is easy change de values of the dimensions in order to obtain the better set of points that gives a good surface reproduction. Nevertheless, an indiscriminate changing on the region dimensions can give unprofitable results as shown in section 3.

Once both the region dimensions and the number of points in the region are known the center of mass of each region can be obtained from its coordinates x_{CM} , y_{CM} and z_{CM} as

$$x_{CM} = \frac{\sum_{i=1}^{n} x_i}{n} \quad ; \quad y_{CM} = \frac{\sum_{i=1}^{n} y_i}{n} \quad ; \quad z_{CM} = \frac{\sum_{i=1}^{n} z_i}{n} \tag{4}$$

Where n is the number of points in the region and x_i, y_i and z_i are the coordinates of point i.

In Figure 1a is represented a set of a planar acquired points in which regions are defined by *dx* and *dy*, and in Fig. 1b its correspondent center of mass (CM).



(a) (b)

Figure 1. Method for reducing the acquired points. a) The acquired points and the regions; b) Regions and its center of mass.

The final surface quality is function of the chosen resolution (the dimensions of the region), principally for conical surfaces. If the resolution is big, details of the surface will be not represented and the vertices could be truncated. If the resolution is very small, the contour of the cross-section will be irregular and waved.

Numerical simulations have been shown that the resolution can be based on the width of point clouds and the local radii of the section. As result, for radii bigger than ten times the width of cloud points a resolution as two times the width of cloud points can be used. For smaller radii we have been used a resolution equal of the width of cloud points.

As the methodology uses the cross-section of the object to mesh the surface, one coordinate (for example the z coordinate) can be obtained from a common coordinate using the least square method for example.

It is important to note that the width of cloud of points is function of the reading equipment and the used resolution defines the number of CM for defining the contour of the object.

2.3. Approach to select the CM to define the contour of the object

From the set CM previously obtained the contour of the cross-section of the object can be defined. For that an approach to define the sequential set of CM has been proposed. It is important to note that to apply NURBS the sequential set of points must be defined. The selected CM to form the contour of the cross-section of the object must represent the major probability of that CM being a point of the object surface.

From numerical simulations two parameters have been chosen: the number of acquired points in the region (more the number of points in a region bigger is the probability of its CM be a point of the object surface), and the distance between CM in a local macro-region as described following.

The first step consists in choose a CM to be the initial reference CM. Then a local macro-region is defined by all neighbor regions from the reference CM as shown in Fig. 2, where the CM of region named as 1 is the reference CM.

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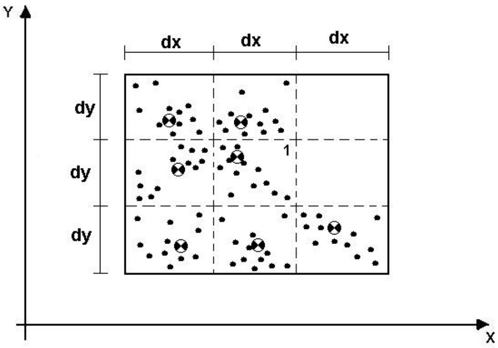

Figure 2. A local macro-region.

Once the local macro-region had been defined, the two parameters, the number of acquired points of each region and the distance between the reference CM and the others one, can be computed. Then, the bigger value of the product of these two parameters will define the next CM belonging to the contour, and that will be the next reference CM. The others regions of the analyzed local macro-region are eliminated for the next step. Thus, the process repeats till close the contour. At the end of the process the CM that will define the contour of the object cross-section in a connectivity order is defined yet.

2.4. Algorithm to mesh the surface

After the two previous approaches, the points (CM) which defines the cross-section of the object are known. Thus it is possible to draw a curve for each slice by computing the control points and knots necessary to represent the object surface by using NURBS. In Figure 3 is shown a set of acquired points, the CM used as control points and segments connecting them, and the obtained NURBS curve.

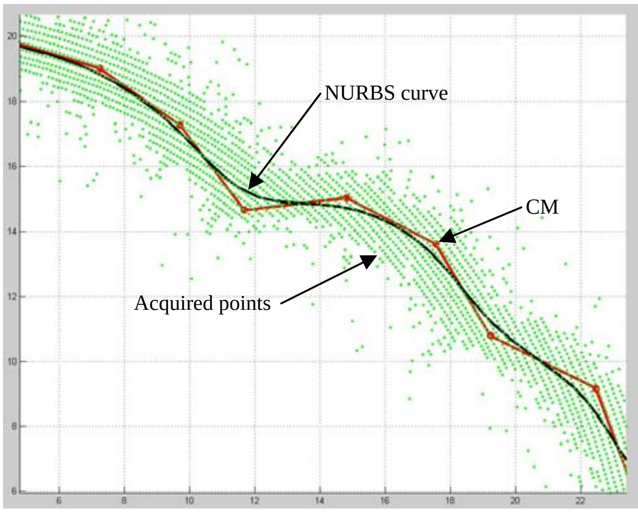

Figure 3. Example of a NURBS curve obtained from initial CM.

On the meshing a tri-dimensional surface from curves representing slices of the object it is necessary to construct transversal curves intersecting the previous ones, from corresponding points that defines the object shape. As each transversal curve must intersect only one point of each slice it is necessary that all cross-section curves have the same number of points.

However, the number of points at each cross-section is not the same, principally in a conical surface. For a conical surface, as near is the vertices less is the necessary number of points to define it. To overpass this problem an approach had been proposed to connect the control points of consecutive cross-section.

The first step of this approach consists in defining a transversal line to be used as reference to numbering the CM. In this work we have been chosen a line which pass by the axis $x=0$ as shown in Fig. 4, and then, at each cross-section, the first CM is the nearest of this reference line, no matter if the CM is before or after the reference line, as shown in Fig. 4. Then, the knots and control points are numbered from these first CM. The CM numbering has been adopted counterclockwise, but can be done another way. From the numbered CM a first curve of the cross-section can be draw and then a novel set of knots and control points can be defined.

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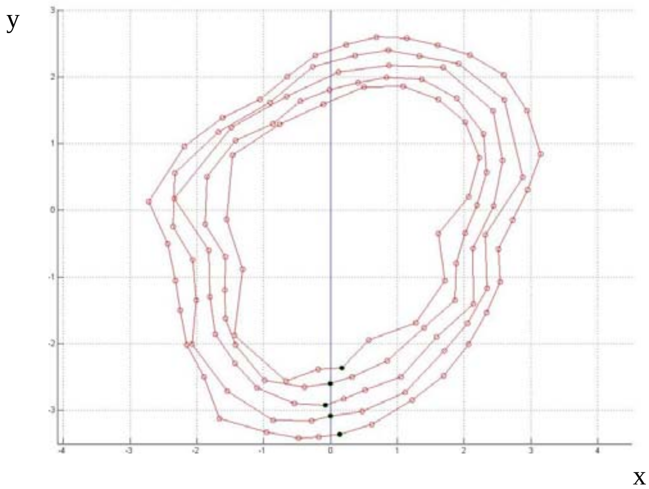

Figure 4. Selecting process for numbering the CM in order to draw the NURBS curve.

To each cross-section the novel set of knots are equidistant and are in same quantity of the CM defining the cross-section. From this set of knots the novel set of control points (CP) can be obtained that will be use to mesh the surface.

When the consecutive cross-section is reduced, a smaller number of points it is necessary to define it. From numerical simulations we have been defined a procedure to connect two control points of consecutive cross-section. This procedure has been defined in view of simplifying the meshing without compromise the surface shape. The procedure is as follows.

a) Compare the number of control points from two consecutive cross-sections. The curve which has a bigger number of control points (the bigger curve) should connect the difference in pairs of control points with the same quantity of control point of the smaller curve. For example: the curve 1 has 15 CP (the bigger curve); the curve 2 has 10 CP (the smaller curve); the difference of CP is 5; then, 5 pairs of CP of curve 1 should be connected to 5 CP of curve 2.
b) To select which CP should be connected, the distance between CP of the bigger curve are computed. The CP whose the distance are smaller are chosen to be connected.
c) The connecting sequence is the same numbering sequence of CP. This is justified because the CP whose are closed in the bigger curve should be closed in the smaller one. It is important to note that two consecutive cross-section must be closed for well represent the surface shape.

In order to obtain a regular mesh with a reduced double connections, we have been used the following algorithm.

a) Select a curve that will be used as a base curve. At the start of the surface reconstruction, the first curve is the first cross-section of the object.
b) If the next curve is till 25% smaller than the length of the base curve, it is imposed to the smaller curve the same number of CP of the base curve, which is bigger. For that the knot vector of the base curve is repeated to the smaller curve, recalculating a new set of knot points for the smaller curve.
c) The procedure is repeated for the subsequent curves up to find a curve where its length is smaller then 25% of the base curve. In this case, the precedent procedure for connecting the CP in pairs is applied.
d) The smaller curve becomes the new base curve and the procedure is repeated till the last curve.

The described procedure can be used for smaller or bigger curve. In the Figure 5 is represented the connecting procedure in pairs.

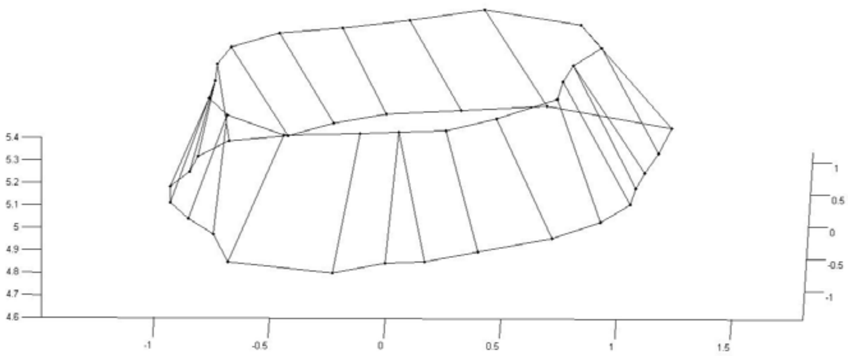

Figure 5. Procedure for connecting control points in pairs.

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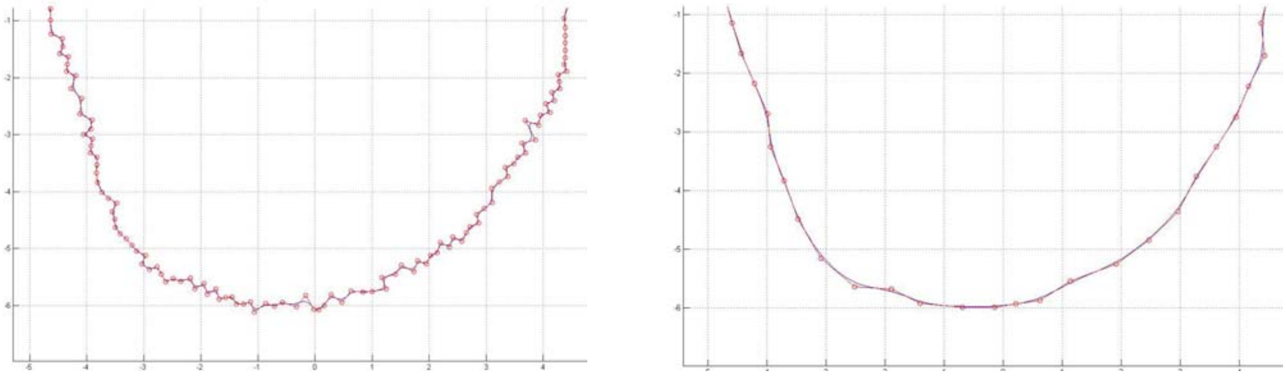
When it is necessary obtain a softy surface, one can create intermediary cross-sections between the acquired cross-sections that is based on the knots and control points of then. More details can be seen in Piegl and Tiller (1996).

3. NUMERICAL SIMULATIONS AND RESULTS

In order to show the behavior of the curve when the resolution of the proposed method is not adequate, as described in section 2.2, an example is presented in Fig. 6. The represented section has a medium diameter of 100 mm and the width of the acquired points is 2 mm. In Figure 6a is represented the curve where the used resolution is 0.8 mm, i.e., less then the recommended on section 2.2. One can see that the curve is not uniform and presents an irregular contour. In Figure 6b the correct resolution is used, i.e., 4 mm. One can see that the curve is regular.



(a) (b)

Figure 6. Methodology for reduction the acquired points related to the resolution process.
a) Curve using an inadequate resolution; b) Curve using the correct resolution.

3.1. Applying to a cylindrical surface

In order to verify the validity of the proposed method we have been used a set of unorganized points and badly acquired resulting in many empty regions (without points) as represented in Fig. 7. We have been token five slices to represent the object. To apply the proposed method the CM of each cross-section have been organized in a same plane by using the least square method.

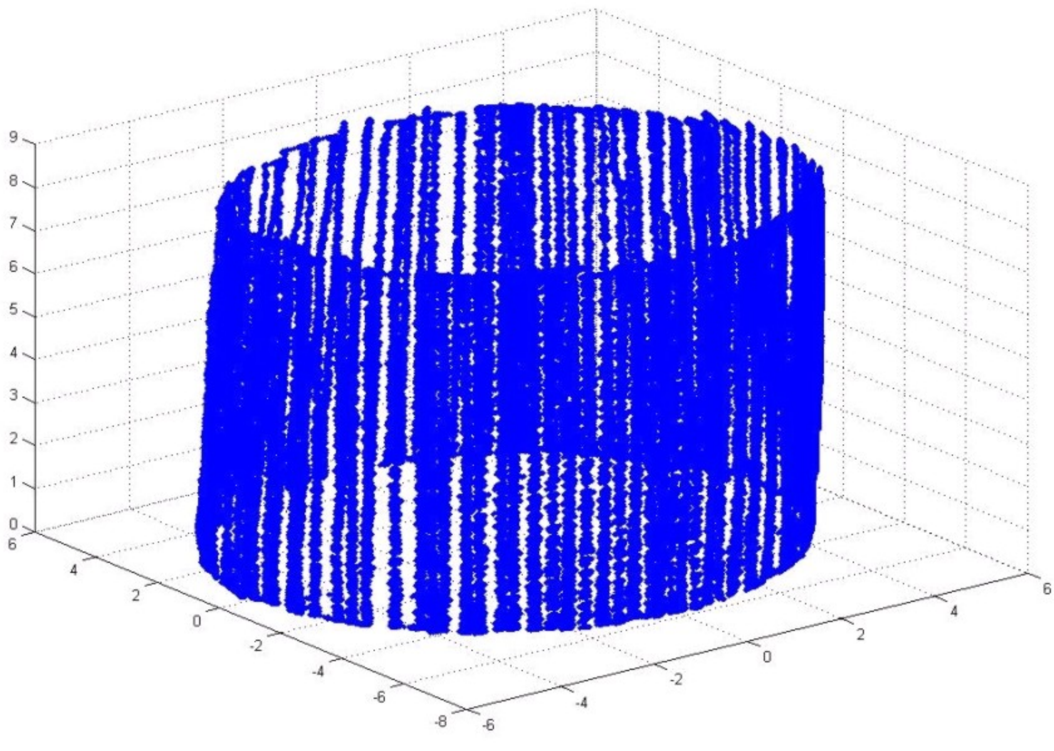


Figure 7. Acquired points representing a cylindrical surface.

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For meshing the cylindrical surface we have been used the presented procedures. As the cross-sections have approximately the same diameter, it was not necessary change the knot points. By using commercial software for rendering the surface, after the mesh had been concluded, we obtained the surface as represented in Fig. 8.

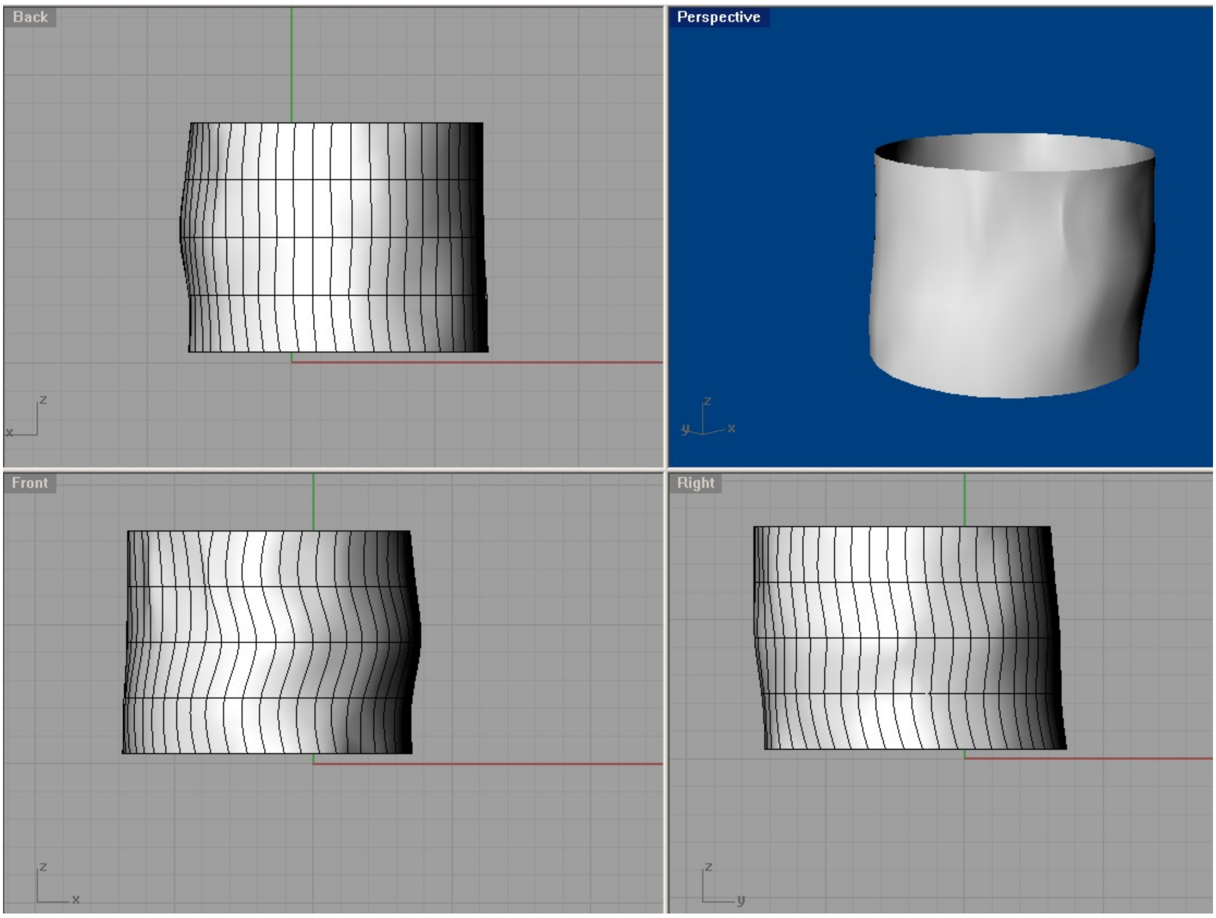
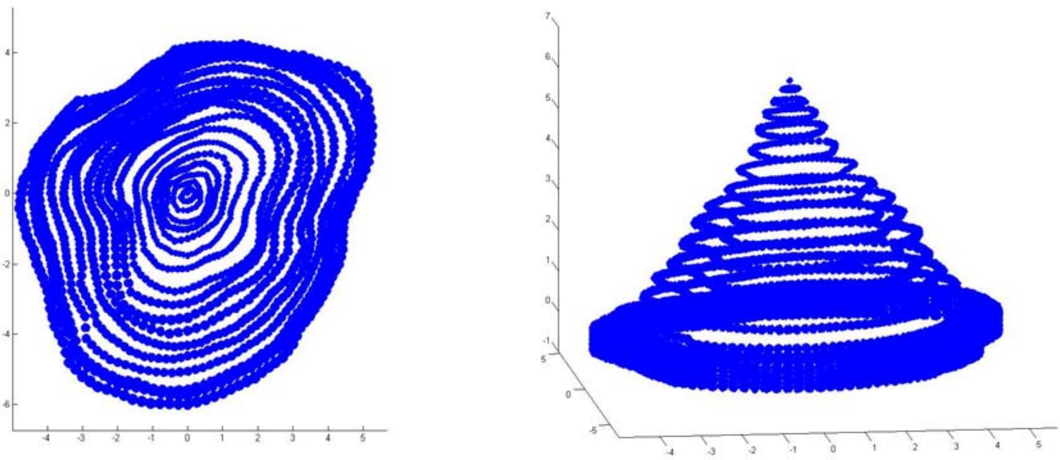


Figure 8. Rendered surface from five slices by using the proposed methodology.

3.2. Applying to a conical surface

The next example we have been used a conical surface where the points are represented in Fig. 9. As the previous example, the points are unorganized and not regular.



(a) (b)

Figure 9. Cloud of points representing a conical surface.
a) Top view; b) Tri-dimensional view.

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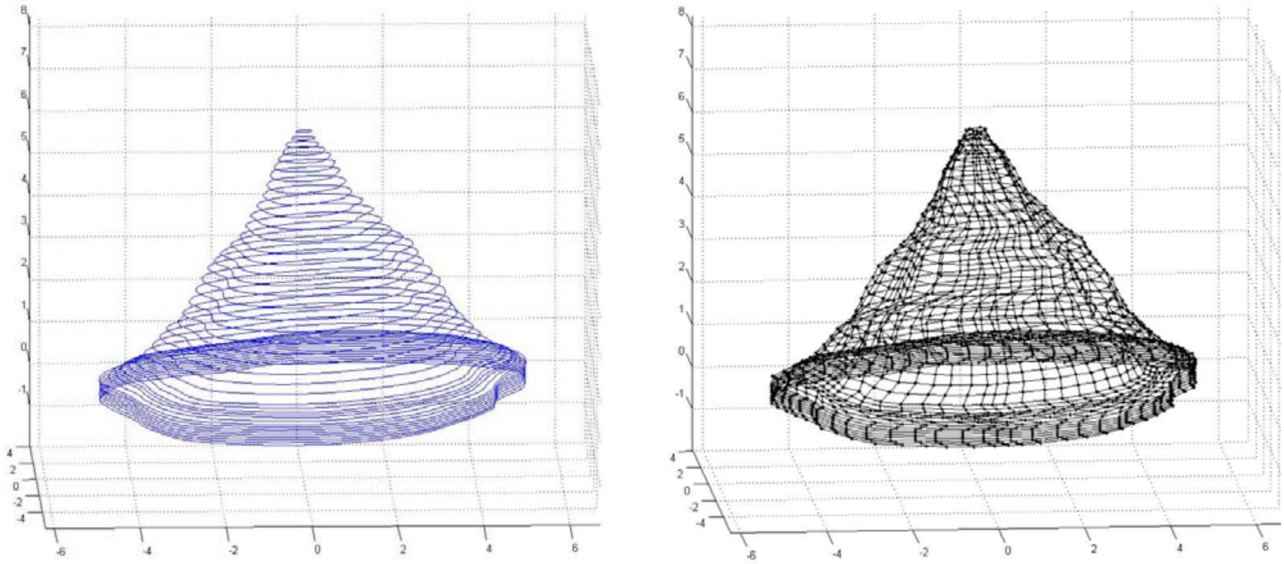
By applying the described methodology we had been obtained the NURBS curves as represented in Fig. 10a and the mesh as in Fig. 10b.



(a) (b)

Figure 10. Applying the proposed methodology to a conical surface. a) The obtained NURBS curves; b) The obtained mesh.

Again, by using commercial software for rendering the surface, after the mesh had been concluded, we obtained the surface as represented in Fig. 11.

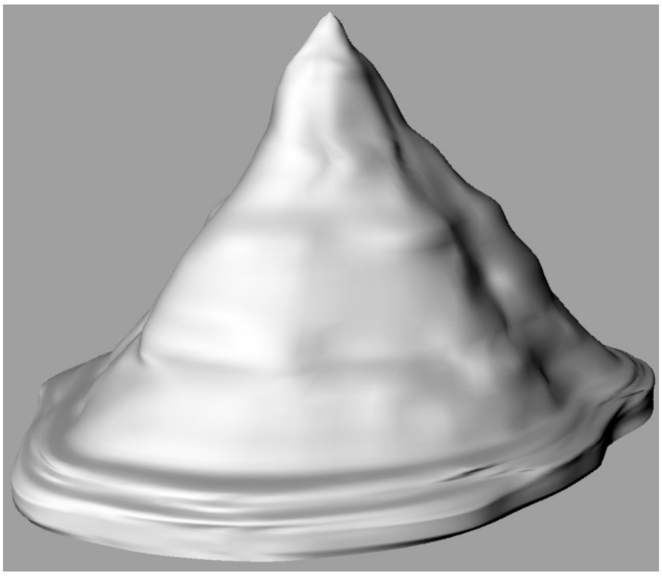


Figure 11. Rendered of the conical surface by using the proposed methodology.

4. CONCLUSIONS

As stated by Remondino (2003) “the mesh generation is the core part of almost all reconstruction programs”. But it is possible after a pre-processing to remove a certain amount of errors introduced by the scanning device, to reduce the input redundancy and the noise and gaps in the point clouds.

Thus, in this paper we had been presented a new methodology for 3D surface reconstruction from unorganized cloud of points. In the first step of the proposed methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. From the new set of

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points the contour of the object cross-section is defined by using NURBS. The analysis of consecutive cross-sections enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation (jagged surfaces or protrusions).

From all defined points the mesh can be done and then, using commercial software the object rendering can be made for its visualization.

Simulations results had been shown that the proposed method is very effective for clustering unorganized point clouds for meshing surfaces.

We can depict some important advantages of this proposed method: a) the gaps of the cloud of points must not be filled as others methods; b) the cloud of points must not be well organized, and c) it is not necessary an algorithm to reduce noises (they are eliminated by applying the reduction approach). These advantages had been shown in the presented examples.

5. ACKNOWLEDGMENTS

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