

# CASCADE NONLINEAR CONTROL WITH PARAMETER ADAPTATION APPLIED TO A PNEUMATIC SERVODRIVE

**Sobczyk S., Mário R., sobizique@yahoo.com.br**

**Perondi, Eduardo A., perondi@mecanica.ufrgs.br**

Mechanical Engineering Department - Federal University of Rio Grande do Sul  
Rua Sarmento Leite, 425, 90050-170, Porto Alegre-RS, Brazil.

**Cunha, Mauro A. B., mauro@cefetrstche.br**

Automation and Control Research Group - Pelotas Federal Center for Technological Education  
Praça Vinte de Setembro, 455, 96015-360, Pelotas-RS, Brazil.

***Abstract.** This work focuses on two adaptive nonlinear control schemes to be applied to a pneumatic actuator. Such controllers are developed according to the cascade methodology. The efficiency of the proposed control schemes rely on their diffeomorphism rule, which requires the previous knowledge of a number of parameters, especially the maximum mass flow rates in and out of the cylinder chambers, which are very difficult to determine experimentally. Also, that rule is subject to errors due to the uncertainties associated with those parameters. For such reasons, the proposed algorithms are equipped with adaptive capabilities for the estimation of the aforementioned maximum mass flow rates, in order to allow their identification with reduced errors and without need of extensive parameter identification processes. The convergence properties of those parameters and the stability characteristics of the controlled system are discussed. Simulation results illustrate the main properties of the proposed controllers.*

**Keywords:** pneumatic systems, nonlinear control, cascade control, parameter adaptation

## 1. INTRODUCTION

Pneumatic positioning systems are very attractive for many applications because they are cheap, lightweight, clean, and easy to assemble, besides presenting a good force/weight ratio. In spite of these advantages, pneumatic positioning systems present some undesirable characteristics which limit their use in applications that require a fast and precise response. These undesirable characteristics are mainly derived from the high compressibility of the air as well as from friction effects, which are highly nonlinear and very difficult to model accurately.

To overcome such problems, a cascade control strategy has been developed in which the pneumatic positioning system is interpreted as an interconnected system: a mechanical subsystem driven by the force generated by a pneumatic subsystem (Perondi and Guenther, 2002). Further work based on the same approach led to the incorporation of the variable structure control methodology in that framework, providing improved robustness to the original controller (Sobczyk et al., 2006).

Both control algorithms depend upon the same diffeomorphism rule to determine the electric control signal applied to the servovalve from the corresponding theoretical control law. Among other parameters, such diffeomorphism rule depends on the maximum air mass flow rates through the servovalve. Such parameters are very difficult to determine experimentally, requiring an extensive identification process (Vieira, 1998). For this reason, this work proposes a parameter adaptation scheme to be applied to the aforementioned control laws. Such scheme is intended to make a continuous update of the maximum air mass flow rate estimates, so that such estimates converge to the actual value of those parameters in the real plant. With the use of such an adaptation scheme, it is expected that long parameter identification processes can be avoided and that parametric uncertainties of those quantities vanish to zero. Then, application of the proposed control schemes to different servovalves would be facilitated.

The stability properties of both control schemes are discussed. The convergence properties of the parameter estimates are also studied. The pneumatic servodrives' dynamic behaviour is simulated with the control algorithms, both with and without use of the parameter adaptation scheme. The results are then compared, so that the efficiency of the proposed adaptation scheme is evaluated.

This paper is organized as follows. Section 2 is dedicated to present the theoretical model of the actuator, while, in Section 3, the control algorithms are described. The adaptation rule of the maximum air mass flow rates is presented in Section 4. In Section 5, the stability and convergence properties of the controlled system are discussed. Simulation results are presented in Section 6. Finally, the main conclusions are outlined in Section 7.

## 2. DYNAMIC MODEL

The mathematical model of the system is based on two basic processes: **(i)** the relationship between the air mass flow rate and the pressure changes in the cylinder chambers, obtained by energy conservation laws; **(ii)** the equilibrium of the forces acting on the piston, given by the Newton's second law. For a schematic view of the pneumatic positioning system with its attached load, refer to Fig. 1.

Applying Newton's second law to the piston-load assembly yields:

$$M\dot{y} + F = A(p_1 - p_2) \quad (1)$$

where  $M$  is the mass of the piston-load assembly,  $F$  is the sum of both external and friction forces,  $A$  is the cylinder cross-sectional area, and  $p_1$  and  $p_2$  are the pressures in the two chambers (see Fig. 1).

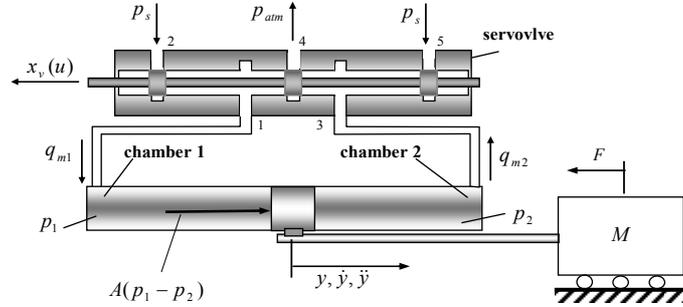


Figure 1. Pneumatic positioning system attached to a generic load.

The mass flow rate of air into a given chamber is  $q_m = (dm/dt)$ . So, for the two considered chambers, one finds  $q_{m1}$  and  $q_{m2}$ . Such mass flow rates are nonlinear functions of the servovalve control voltage  $u$  and of the cylinder pressures, that is,  $q_{m1} = q_{m1}(p_1, u)$  and  $q_{m2} = q_{m2}(p_2, u)$  (Vieira, 1998). Thus, using energy conservation arguments, it is possible to show that the pressure dynamics in chambers 1 and 2 are given, respectively, by:

$$\dot{p}_1 = -\frac{Ar\dot{y}}{Ay + V_{10}} p_1 + \frac{RrT}{Ay + V_{10}} q_{m1}(p_1, u) \quad (2)$$

$$\dot{p}_2 = \frac{Ar\dot{y}}{A(L - y) + V_{20}} p_2 + \frac{RrT}{A(L - y) + V_{20}} q_{m2}(p_2, u) \quad (3)$$

where  $r$  is the ratio between the specific heat values of the air,  $\dot{y}$  is the piston velocity,  $R$  is the universal gas constant,  $T$  is the air supply temperature,  $y$  is the piston position,  $V_{10}$  and  $V_{20}$  are the dead volumes of air in the lines and at the extremities of both chambers, and  $L$  is the cylinder stroke.

Equations (1), (2) and (3) correspond to a fourth order nonlinear dynamic model of the pneumatic system, without direct friction modeling. Defining the differential pressure applied to the piston as  $p_\Delta = p_1 - p_2$ , Eq. (1) is rewritten as:

$$M\dot{y} + F = Ap_\Delta \quad (4)$$

The time rate of the differential pressure  $\dot{p}_\Delta$  is calculated by subtracting expressions (2) and (3):

$$\dot{p}_\Delta = \hat{h}(p_1, p_2, y, \dot{y}) + \hat{u}(p_1, p_2, y, u) \quad (5)$$

where  $\hat{u} = \hat{u}(p_1, p_2, y, u)$  and  $\hat{h} = \hat{h}(p_1, p_2, y, \dot{y})$  result from the separation of the terms affected by the servovalve control voltage  $u$  and the terms which are functions only of the states of the system. The regrouped terms are given, respectively, by:

$$\hat{u}(p_1, p_2, y, u) = RrT \left[ \frac{q_{m1}(p_1, u)}{Ay + V_{10}} - \frac{q_{m2}(p_2, u)}{A(L - y) + V_{20}} \right] \quad (6)$$

$$\hat{h}(p_1, p_2, y, \dot{y}) = -rA\dot{y} \left[ \frac{p_1}{Ay + V_{10}} + \frac{p_2}{A(L - y) + V_{20}} \right] \quad (7)$$

Equations (4) and (5) describe the dynamics of the whole system. Equation (4) models the mechanical subsystem driven by a pneumatic force. Equation (5) describes the pneumatic subsystem dynamics, in which the force to be applied to the mechanical subsystem is controlled by the input voltage  $u$ .

### 3. CONTROL STRATEGY

It is presented here the cascade control strategy, associated to two other control techniques. The cascade approach consists in interpreting the whole system as two interconnected subsystems: mechanical and pneumatic. Its employment allows one to design entirely different control laws for each subsystem. Both control schemes discussed in this work make use of the same control law in the mechanical subsystem, differing only in the control laws applied to the pneumatic subsystem. Such laws are: proportional control with feedback linearization; variable structure control with boundary layer. From now on, those control laws for the pneumatic subsystem shall be designated as *proportional controller* and *variable structure controller*, respectively. The next sections describe each of the aforementioned control approaches: the cascade technique, the control law for the mechanical subsystem, and both control laws for the pneumatic subsystem.

#### 3.1 Controller Overview: the Cascade Approach

The essence of the Cascade approach consists in interpreting one state of the system as a control input to another subsystem. In the case of the pneumatic actuator, the differential pressure term in Eq. (4) is regarded as an input to be applied to the mechanical subsystem. By means of a suitable control law, it is possible to determine the desired value  $p_{\Delta d}$  for that pressure difference, which causes the piston with the assembled load to track a given trajectory. Then, another control law is used in the pneumatic subsystem, so that the actual differential pressure tracks  $p_{\Delta d}$ . In order to employ this methodology, it is necessary to rewrite Eq. (4) in such a manner that the desired differential pressure term is explicitly given. With that purpose, the differential pressure tracking error is defined as:

$$\tilde{p}_{\Delta} = p_{\Delta} - p_{\Delta d} \quad (8)$$

Using Eq. (8), the expressions (4) and (5) may be rewritten as:

$$M\ddot{y} = Ap_{\Delta d} + d(t) \quad (9)$$

$$\dot{p}_{\Delta} = \hat{h}(p_1, p_2, y, \dot{y}) + \hat{u}(p_1, p_2, y, u) \quad (10)$$

where  $d(t)$  is an input disturbance given by

$$d(t) = A\tilde{p}_{\Delta} - F \quad (11)$$

The system given by Equations (9) and (10) is now written in an interconnected form, suitable for the cascade technique. Equation (9) can be interpreted as a mechanical subsystem driven by a desired force  $g_d$  and subjected to an input disturbance  $d(t)$  as defined in Eq.(11). Equation (10) represents the dynamics of the pneumatic subsystem.

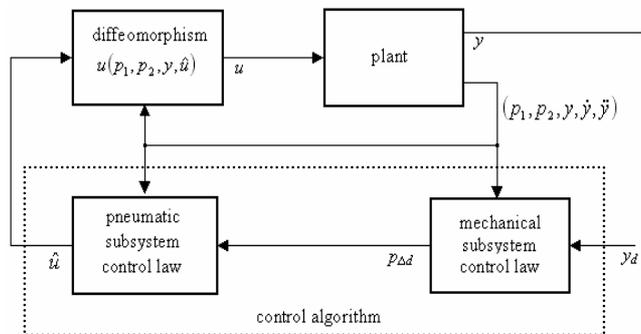


Figure 2. Complete control strategy.

The design of the cascade controller for the system (9), (10) is illustrated in Fig. (2), and can be summarized as follows:

(i) Compute a control law  $g_d(t) = Ap_{\Delta d}(t)$  for the mechanical subsystem (Eq.(9)) in such a way that the piston displacement ( $y(t)$ ) achieves a desired trajectory  $y_d(t)$  in the presence of the disturbance  $d(t)$  ;

(ii) Compute a control law  $u(t)$  in such a way that the pneumatic subsystem (Eq.(10)) applies to the mechanical subsystem a force  $g(t) = Ap_{\Delta}(t)$  that tracks the desired force  $g_d(t)$  as close as possible.

In this paper, the mechanical subsystem control law  $g_d(t)$  is based on the controller proposed by Slotine and Li (1988). The control law  $\hat{u}(t)$  is synthesized according to one of the two aforementioned approaches (proportional or variable structure), and the control signal  $u(t)$  is obtained from  $\hat{u}(t)$  through a diffeomorphism. So, once the desired control law  $\hat{u}(t)$  is computed, an inversion of Eq.(6) yields the corresponding signal  $u(p_1, p_2, y, \hat{u})$  to be applied to the servovalve. The mass flow functions  $q_{mi}(p_i, u)$  used in that expression will be discussed in Section 4.

### 3.2 Tracking Control in the Mechanical Subsystem

Based on Slotine and Li (1988), the following control law to obtain trajectory tracking in the mechanical subsystem is proposed:

$$g_d = M\ddot{y}_r - K_D s \quad (12)$$

where  $K_D$  is a positive constant,  $\ddot{y}_r$  is the time derivative of a reference velocity  $\dot{y}_r$ , and  $s$  is a measure of the velocity tracking error. In fact,  $\dot{y}_r$  and  $s$  can be obtained as follows:

$$\dot{y}_r = \dot{y}_d - \lambda \tilde{y}; \quad \tilde{y} = y - y_d; \quad s = \dot{y} - \dot{y}_r = \dot{\tilde{y}} + \lambda \tilde{y} \quad (13)$$

where  $\lambda$  is a positive constant, and  $y_d$  is the desired position. All terms marked with ( $\sim$ ) indicate error terms, as in the case of  $\tilde{p}_{\Delta}$  in Eq. (8).

### 3.3 Tracking Control in the Pneumatic Subsystem

#### Proportional Controller

The first version of the control law to be applied to the pneumatic system is given as follows:

$$\hat{u} = \dot{p}_{\Delta d} - \hat{h}(p_1, p_2, y, \dot{y}) - K_p \tilde{p}_{\Delta} - As \quad (14)$$

where  $\dot{p}_{\Delta d}$  is the time derivative of the desired differential pressure,  $\hat{h}(p_1, p_2, y, \dot{y})$  is the part of the pneumatic subsystem that is independent of  $\hat{u}(t)$  in Eq. (10), and  $K_p$  is a positive constant. Basically, this law represents a proportional controller with feedback linearization. A detailed discussion of this control law can be found in Perondi and Guenther (2002).

#### Variable Structure Controller

In order to obtain improved robustness with respect to parametric uncertainties, the control law given by Eq. (14) was reformulated, making use of the variable structure control methodology. Such approach is characterized by great levels of chattering in the control signal that is applied to the servovalve. That is an undesirable feature, because it reduces the utile lifespan of the servovalve. In order to avoid that, the boundary layer concept (Slotine and Li, 1991) was also employed. Thus, the resulting control law that is applied to the pneumatic subsystem becomes:

$$\hat{u} = -\hat{h}(p_1, p_2, y, \dot{y}) - As - ksat(\tilde{p}_{\Delta}/\Phi) \quad (15)$$

where  $sat(\tilde{p}_{\Delta}/\Phi)$  is defined as:

$$sat(\tilde{p}_{\Delta}/\Phi) = \begin{cases} -1 & \tilde{p}_{\Delta}/\Phi < -1 \\ \tilde{p}_{\Delta}/\Phi & -1 \leq \tilde{p}_{\Delta}/\Phi \leq 1 \\ 1 & \tilde{p}_{\Delta}/\Phi > 1 \end{cases} \quad (16)$$

and  $\Phi$  is a positive constant that defines the *boundary layer* of the controller.

In practical terms,  $\Phi$  is a quantification of the tolerance degree to the variation of  $\tilde{p}_\Delta$  without implying maximum control action. As  $\Phi$  increases, it is necessary a larger variation of  $\tilde{p}_\Delta$  to cause saturation of the control signal. Thus, for small values of  $\tilde{p}_\Delta$ , the discontinuous part of that modified control law acts as a proportional controller. Under such conditions, the resulting control signal is smoothed and chattering is avoided.

A detailed discussion of this control law is given in Sobczyk et al. (2006).

#### 4. ADAPTATION OF MAXIMUM ABSOLUTE AIR FLOW RATES

As mentioned in Section 3, once the desired control law  $\hat{u}$  is determined, it is necessary to obtain the actual control signal  $u$  to be applied to the servovalve by a numeric inversion of Eq.(6). In order to attain that inversion, it is necessary to know the air mass flow functions  $q_{mi}(p_i, u)$ , for both processes (filling and exhausting) in each chamber. In this work, such functions are modeled as (Perondi and Guenther, 2003):

$$q_{mi}^{process}(p_i, u) = [q_{mi}^{process}]_{\max} f_{pi}^{process}(p_i) f_{ui}^{process}(u) \quad (17)$$

where  $[q_{mi}^{process}]_{\max}$  is the maximum absolute value of the air mass flow rate into or out of the  $i$ -th chamber during the considered process. Likewise with respect to chambers and processes,  $f_{pi}^{process}(p_i)$  is a third order polynomial function of the chamber pressure, such that  $0 \leq f_{pi}^{process}(p_i) \leq 1$ , and  $f_{ui}^{process}(u)$  is a third order polynomial function of the chamber pressure. This function is also defined so that  $0 \leq f_{ui}^{process}(u) \leq 1$ . It is assumed that, for each chamber and each process, both polynomial functions are known, but the maximum air mass flow rates are unknown.

**Remark #1:** In order to simplify the notation of the expressions to follow, the terms that represent the maximum absolute values of the air mass flow rates shall be designated as  $Q_i^j$ , where  $i$  stands for identifying the chamber (1 or 2) and  $j$ , the process (1 for filling, 2 for exhausting). In a similar manner, the polynomial functions of the pressure and of the control signal shall be identified as a single term  $f_i^j$ , that stands for the product of both functions, and the air mass flow rate as a whole shall be written as  $q_i^j$ . With that notation, for instance, the air mass flow function into chamber one during its filling process is written as  $q_1^1 = Q_1^1 f_1^1$ .

The air mass flow rate functions used in the diffeomorphism rule (by inverting Eq. (6)) are based on the estimates of their maximum values  $\hat{Q}_i^j$ . Thus, the terms of the estimated air mass flow rates become:

$$\hat{q}_i^j = \hat{Q}_i^j f_i^j \quad (18)$$

The rule for updating the maximum values estimates for each air mass flow rate is now described.

First, the following vector/matrix quantities are defined:  $\hat{\mathbf{Q}} = [\hat{Q}_1^1 \quad \hat{Q}_1^2 \quad \hat{Q}_2^1 \quad \hat{Q}_2^2]^T$ ,  $\mathbf{Q} = [Q_1^1 \quad Q_1^2 \quad Q_2^1 \quad Q_2^2]^T$ , and  $\mathbf{f} = \text{diag}[f_1^1 \quad f_1^2 \quad f_2^1 \quad f_2^2]$ , where *diag* stands for a diagonal matrix. With these definitions, the vectors of the air mass flow rates as functions of time, both estimated and actual, can be written as  $\hat{\mathbf{q}} = \hat{\mathbf{f}}\hat{\mathbf{Q}}$  and  $\mathbf{q} = \mathbf{f}\mathbf{Q}$ , respectively.

Then, defining  $\mathbf{P}(t)$  such that

$$\mathbf{P}(t) = \left[ \int_0^t \mathbf{f}^2(\tau) d\tau \right]^{-1} \quad (19)$$

the updating rule for the estimates of the maximum absolute air flow rates into and out of both chambers is given by:

$$\dot{\hat{\mathbf{q}}} = -\mathbf{P}(t) \mathbf{f} \mathbf{e} \quad (20)$$

where  $\mathbf{e} = \hat{\mathbf{q}} - \mathbf{q}$  is the estimates error vector for the air mass flows into and out of the chambers.

Equations (19) and (20) shall be used in the parameter convergence analysis.

## 6. CONVERGENCE PROPERTIES

This section is dedicated to discuss the convergence properties of the pneumatic positioning system in closed loop. In Section 6.1, the stability properties of the system are stated when both versions of the cascade controller are employed and all parameters necessary for the inversion of Eq. (6) are known. In Section 6.2, it is proven that, when the proposed algorithm is used and the desired trajectory satisfies the persistent excitation condition, the vector of the estimation errors of the maximum absolute air mass rates converge to the origin in the state space.

### 6.1 Stability Properties – Closed Loop System

Consider pneumatic positioning system, controlled by means of both versions of the cascade control law, given by  $\Omega_1 = \{(9), (10), (12), (14)\}$  (proportional controller) and  $\Omega_2 = \{(9), (10), (12), (15)\}$  (variable structure controller). It is assumed that the desired piston position and its derivatives, up to 3rd order, are continuous functions. It is also assumed that all parameters necessary to evaluate Eq. (6) are known. For this system, the properties of the tracking errors are stated below.

**Main result #1** – Consider the case in which there are parametric uncertainties in the system model, given by  $\Delta\hat{h}$ , and there is the combined effect of external and/or friction forces  $F$ . Given an initial condition, for both proposed control laws, the controllers' gains can be chosen in order to obtain the convergence of the trajectory tracking errors vector  $\mathbf{p} = [\tilde{y} \quad \dot{\tilde{y}} \quad \tilde{p}_\Delta]^T$  to a residual set  $R$  as  $t \rightarrow \infty$ . The upper limit for the norm of the set  $R$  depends on the upper bounds for both  $F$  and  $\Delta\hat{h}$  and on the controller gains.

The proofs for this proposition are based on Lyapunov's Second Method (Slotine and Li, 1991), by using the following non-negative function:

$$2V = Ms^2 + P\tilde{y}^2 + \tilde{p}_\Delta^2 \quad (21)$$

The complete proofs for this statement are given in Perondi and Guenther (2002) for the proportional controller and in Sobczyk et al. (2006) for the variable structure controller.

### 6.2 Convergence Properties – Parameter Estimates

Consider the case in which the maximum absolute values of the air mass flows into and out of both chambers, which are necessary to compute the inversion of Eq. (6), are not known. Consider, also, that the desired trajectory that the controlled system is required to track satisfies the persistent excitation condition. In this case, the convergence properties of the errors in the maximum values of the absolute air mass estimates are stated below.

**Main result #2** – Let  $\tilde{\mathbf{Q}}$  be the maximum absolute air mass estimate errors vector, defined as  $\tilde{\mathbf{Q}} = \hat{\mathbf{Q}} - \mathbf{Q}$ . If the adaptation algorithm described by Equations (19) and (20) is employed, and the persistent excitation condition is satisfied, then  $\|\tilde{\mathbf{Q}}\| \rightarrow 0$  as  $t \rightarrow \infty$ .

**Proof:** Consider the definition for  $\mathbf{P}(t)$  given in Eq. (19). It follows from that expression that

$$\frac{d}{dt} \mathbf{P}(t)^{-1} = \mathbf{f}^2 \quad (22)$$

Using this result combined with Eq. (20), it can be shown that:

$$\frac{d}{dt} [\mathbf{P}(t)^{-1} \tilde{\mathbf{Q}}(t)] = 0 \quad (23)$$

Separating variables and integrating Eq. (23) yields:

$$\tilde{\mathbf{Q}}(t) = \mathbf{P}(t)\mathbf{P}(0)^{-1}\tilde{\mathbf{Q}}(0) \quad (24)$$

From Eq. (19), it can be seen that if the system is persistently excited, all terms of  $\mathbf{P}(t)$  tend to zero as  $t \rightarrow \infty$ . Then, from Eq. (24), it follows that  $\|\tilde{\mathbf{Q}}(t)\| \rightarrow 0$  as  $t \rightarrow \infty$ , which completes the proof.

**Remark #2:** A formal definition of the persistent excitation condition can be found in Slotine and Li (1991). Usually, such condition requires that the excitation function contains at least one sinusoidal component.

**Remark #3:** It should be observed that the convergence properties described in Sections 6.1 and 6.2 are separate, and that the convergence of  $\tilde{\mathbf{Q}}$  to zero does *not* necessarily imply the convergence of  $\mathbf{p}$  to a residual set. In other words, no proof is given in this work that the simultaneous convergence of  $\tilde{\mathbf{Q}}$  and  $\mathbf{p}$  should necessarily occur.

## 7. SIMULATION RESULTS

Most parameters used in the simulations are assumed to correspond to their exactly known values in the real system:  $A=4.19 \times 10^{-4}$  [m<sup>2</sup>],  $r=1.4$ ,  $R=286.9$  [J kg/K],  $T=293.15$  [K],  $L=1$  [m],  $V_{10} = 1.96 \times 10^{-6}$  [m<sup>3</sup>], and  $V_{20} = 4.91 \times 10^{-6}$  [m<sup>3</sup>]. The desired trajectory  $y_d$  is a sinusoidal curve, with amplitude of 0.45 [m] and frequency of 2 [rad/s]. The payload mass  $M$  is always 1.5 [kg]. The gains used in each controller are given in Table 1. Those gain values are determined by means of the pole placement method when the closed loop system is approximated by a linear transfer function (see Sobczyk, 2005, for details).

Table 2. Gain values for the each controller version.

Controller Version/ Gains	$K_p$	$K_d$	$k$	$\lambda$	$\Phi$
Proportional	40	100	—	50	—
Variable Structure	—	100	2000	50	20

All simulation tests were performed without applying external forces, but the friction effects are present in the open loop model. Once the proposed control schemes do not address friction effects directly, they represent an unmodelled dynamics to be dealt with by the controller. In this paper, the friction force is described according to the LuGre friction model proposed by Canudas et al. (1995). This model describes complex friction behavior, such as stick-slip motion and Stribeck effects.

The results presented in Figures 3, 4, 5 and 6 show the efficiency of the proposed controller in guaranteeing system convergence to the desired trajectory in presence of the unmodelled friction effects, for each of the control laws applied to the pneumatic subsystem and for different initial values of the estimated maximum absolute air mass flows  $\hat{Q}_i^j$ . In Figures 3 and 4, such quantities are initially assumed by the controllers as being 50% inferior to their actual values in the plant. In Figures 5 and 6, those initial estimates are 50% greater than the actual values. The maximum absolute air mass flows that are regarded as “actual” are those experimentally identified by Perondi and Guenther (2003). In each figure, the graphic on the left illustrates the obtained trajectory tracking error, whereas the right-hand image presents the electric control signal applied to the servovalve in a normalized form.

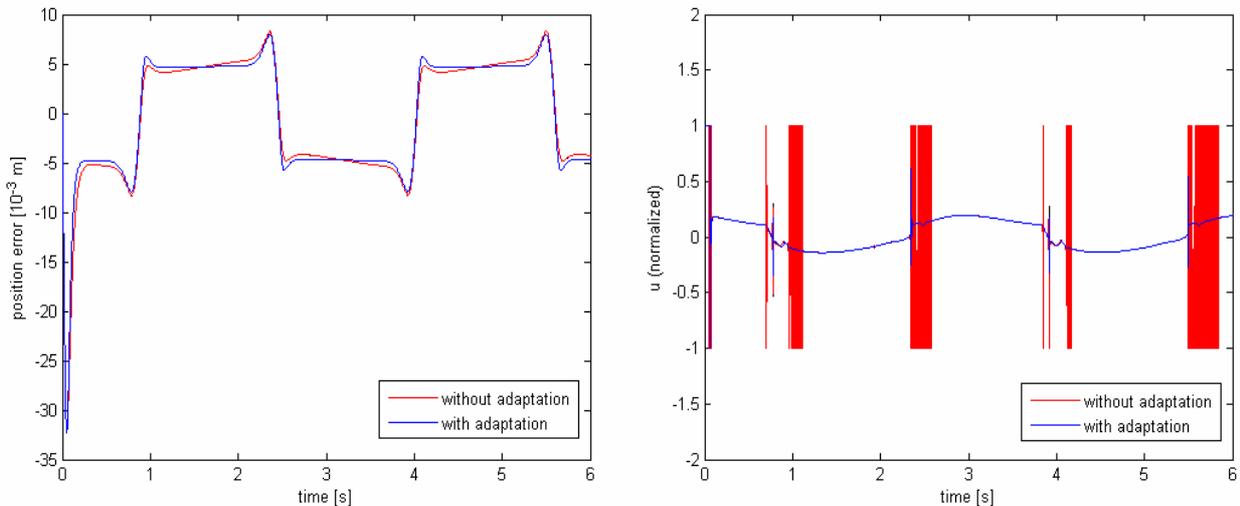


Figure 3. Performance characteristics when parameters are underestimated in 50% – proportional controller.

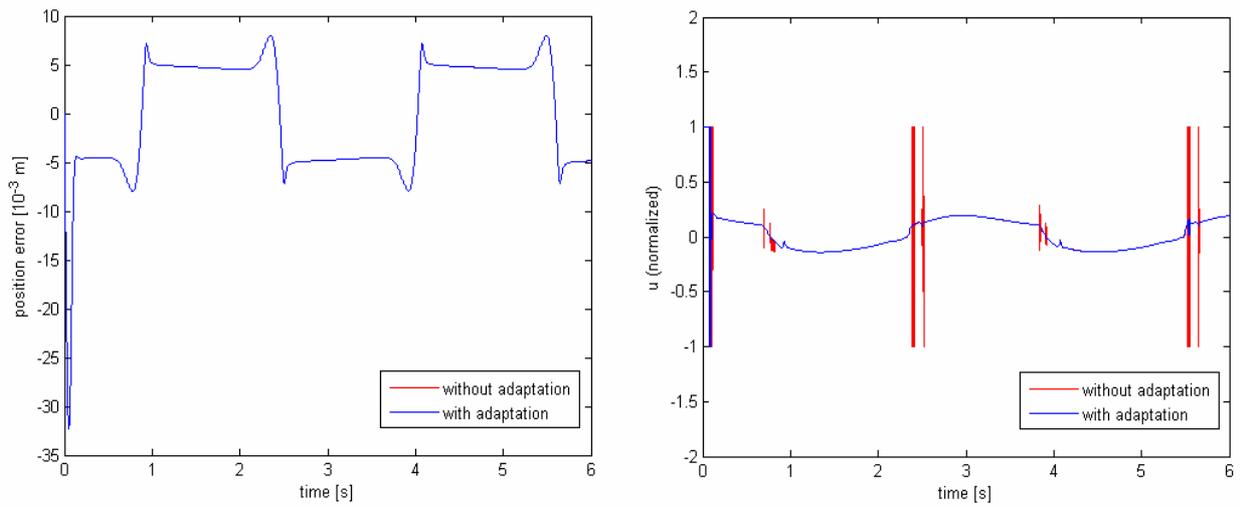


Figure 4. Performance characteristics when parameters are underestimated in 50% – variable structure controller.

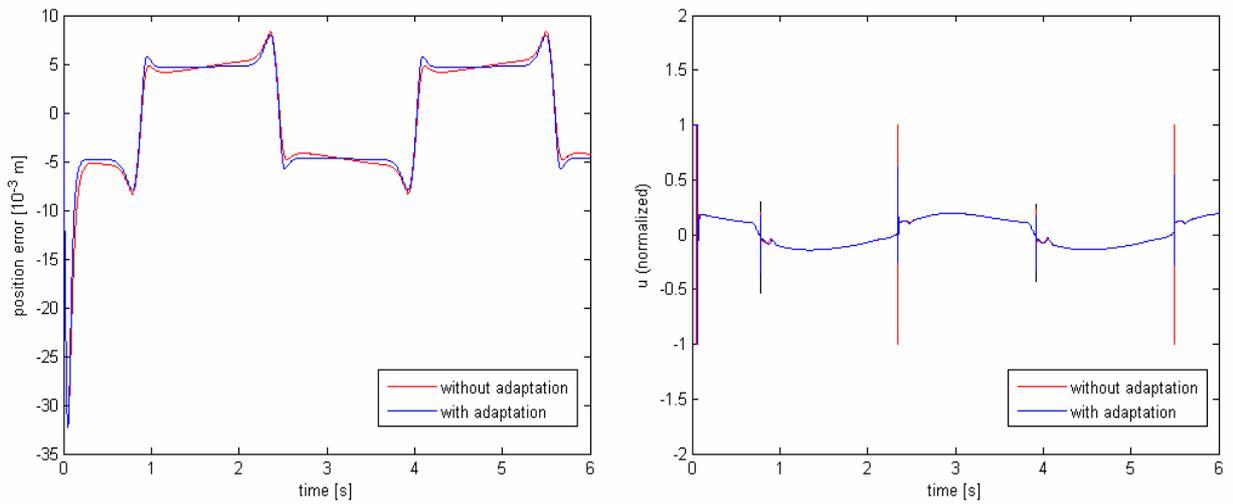


Figure 5. Performance characteristics when parameters are overestimated in 50% – proportional controller.

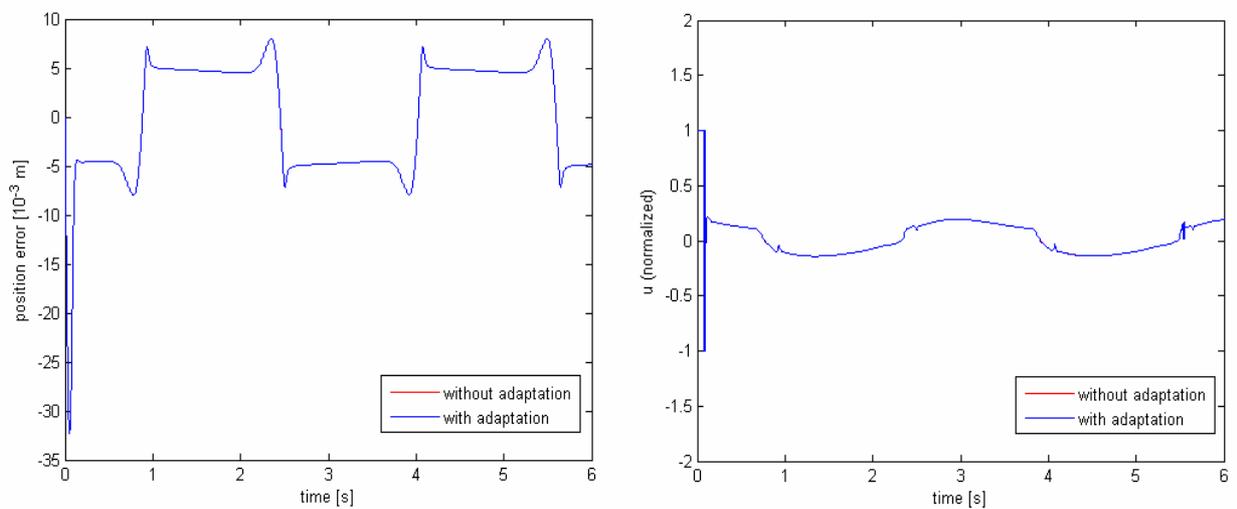


Figure 6. Performance characteristics when parameters are overestimated in 50% – variable structure controller.

For both controller versions, it is seen that the performances of the closed loop system with respect to trajectory tracking are very similar. The amplitudes of the tracking errors are about 8 [mm] in each case. Such results are compatible with those reported in other works that are also focused on pneumatic actuators, as in Lee et al. (2002) and Smaoui et al. (2006). Thus, the proposed control schemes present acceptable performance properties with respect to trajectory tracking errors. As for the electric control signals applied to the servovalve, it can be seen that parametric uncertainties in the estimation of the maximum absolute air mass flow rates are more serious when such values are underestimated, and that overestimations hardly interfere with the controlled system's performance at all. Also, it can be observed that the proportional controller is more subject to deterioration in its performance than the variable structure controller (notice the increased chattering of the electric control signal). Finally, it should be noticed that the proposed algorithm for adaptation of the maximum absolute values of the air mass flow rates is actually effective in reducing the chattering of the electric control signal. Such reduction is desirable because it helps in prolonging the servovalve's utile lifespan.

Figures 9 and 10 illustrate the convergence behavior of the estimated parameters. It can be observed that the actual values of all parameters are reached in about 2 milliseconds at most, which is a much shorter time than that required for the convergence of the trajectory errors.

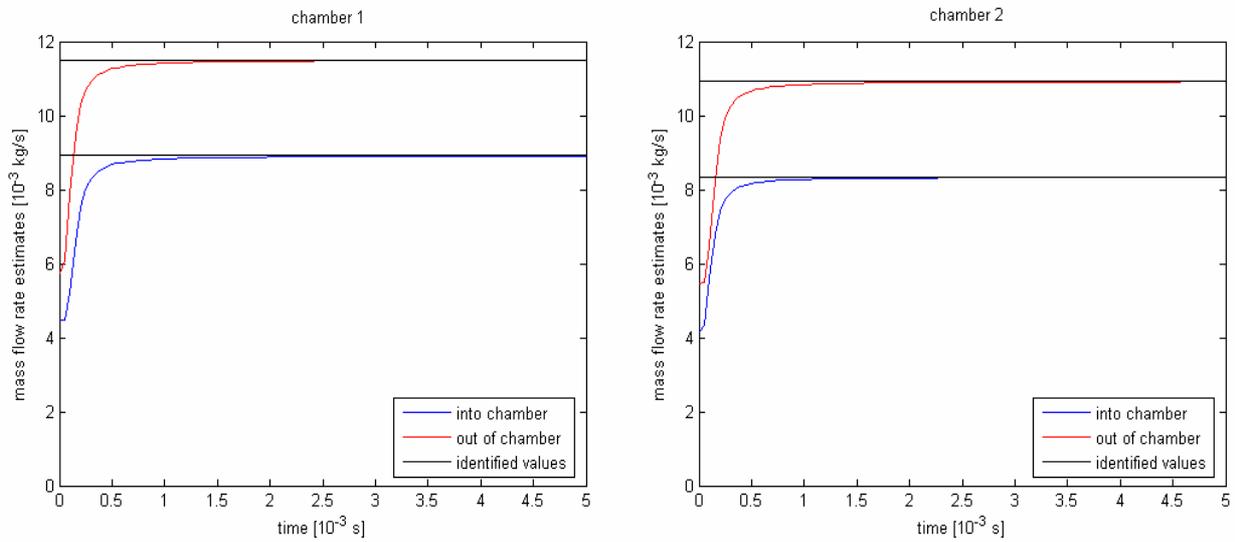


Figure 7. Parameter convergence when initial estimates are 50% less than the identified values.

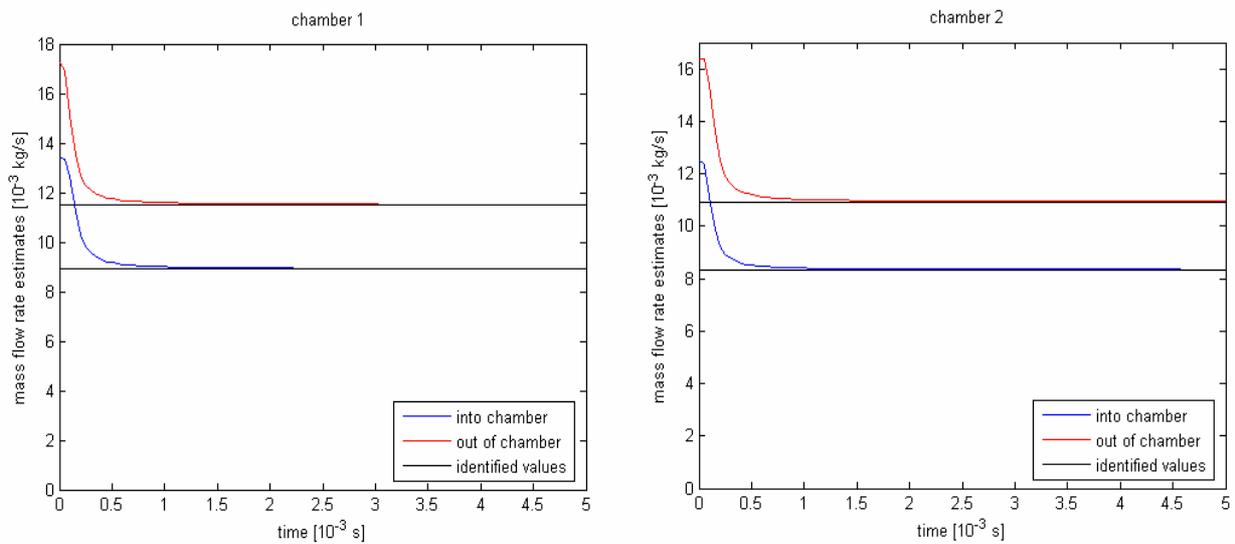


Figure 8. Parameter convergence when initial estimates exceed the identified values by 50%.

## 8. CONCLUSIONS

In this paper, two control schemes that were developed in previous works to be applied to a pneumatic positioning system were discussed. It was seen that both of them rely on the efficiency of the same diffeomorphism rule, which depends on the absolute maximum values of the air mass flow rates in and out of the actuator's chambers. As the experimental determination of such parameters is very difficult, application of the proposed controllers to different servovalves becomes complicated. For that reason, an adaptive scheme was proposed, so that the initial estimates of those quantities would converge to their correct values as the controlled system was operated.

The convergence of the estimated parameters to their correct values was demonstrated. It was seen that, for such convergence to hold, it is necessary that the controlled system be subject to permanent excitation. It was also mentioned that, when the correct values of the aforementioned parameters are previously known, the tracking errors in the closed loop system can be proved to converge to a limited region in the state space, even in the presence of external forces and friction effects.

Simulation results confirm the theoretical statements. It was observed that, when initial estimates are inferior to the correct values of the absolute maximum air mass flow rates, the proposed adaptation algorithm is effective in helping to avoid chattering of the electric control signal that is applied to the servovalve.

Future works will include experimental implementation of the proposed algorithms. A complete stability proof for the controlled system, including the simultaneous convergence of both tracking and parametric errors to a limited region in the state space, is currently under research

## 9. REFERENCES

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## 10. RESPONSIBILITY NOTICE

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