

## STATE CONTROL OF A WATER HYDRAULIC ACTUATOR

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**Abstract.** *This paper presents the control of a water hydraulic actuator. From the environment point of view, water hydraulic actuators present some advantages when compared to their counterparts with oil. In this work, the water hydraulic actuator is represented by a third order linear model with a pole at origin and two lightly damped poles. Aiming to study the control of this kind of actuator, some classical control techniques and the control in the state space are discussed. A state controller is designed by assigning a set of pole locations. Considering that not all state variables are measured, one designs a state observer. The system response with parametric uncertainties is analyzed. In the sequence, one analyzes the closed-loop stability when the velocity and acceleration are obtained by using filtering and time derivative. Simulation results illustrate the main characteristics of the proposed controllers.*

**Keywords:** *water hydraulic actuator, state control, state observer, filtering, parametric uncertainties*

### 1. INTRODUCTION

In the last few decades, the development and the technological advance have represented an important role in the society. Speaking about industries, the automation is a very important subject. Currently, most industries have in their production line a great number of automatic equipments that make possible an increase of productivity, economy of costs, and execution of tasks that offer risks to workers. Some of these automatic machines use positioning systems, which are driven by actuators: hydraulic, pneumatic or electric. The hydraulic actuators are very important when great forces need to be generated. Normally, the oil hydraulic actuators are employed. The main reason is that the knowledge of the technology has not been good enough to handle high pressures of low viscosity fluids with reasonable price (Verronen, 2000). However, there is an increase in the study of water hydraulic actuators.

Water hydraulic actuators present some advantages when compared to oil hydraulic actuators, such that: a clean work environment; a solution of low risk, mainly where there are explosion and fire risk; the water does not contain additives; the cost of the water is very low; easy maintenance; easy availability to find water; lesser loss of pressure in the system for not needing lubrication; accidental leakages are not dangerous to the people and the process; generally the ventilation of the engine is not necessary; they do not cause negative impact to the environment. On the other hand, they present some drawbacks: the price of the equipment is very high; the knowledge in water hydraulic is very small; corrosion problems exist, therefore the water presents a low index of lubrication; the cavitation is another barrier. To minimize the problem of the corrosion, materials with low coefficient of corrosion are used in the manufacture of the equipment used in the hydraulic system. A suitable combination of polymers and metals or ceramics has been a good solution to reduce the friction and thereby the fatigue wear (Rydberg, 2000). The cavitations cause effect as noises, vibrations, reduction of the efficiency and erosion. It is caused by the impact of the air bubbles of the fluid in the material of the equipment, consuming it.

In positioning applications, it is desired to control the hydraulic actuator so that it executes a task with optimized time and precision. Inside of this context, the present work has as objective to be a first study of these work's authors in the area of control of water hydraulic actuators. The idea is to reach, in future papers, at the same level of the development that has been done for algorithms to control oil hydraulic actuators, like in Cunha (2001, 2005) and Cunha et al. (2002, 2004).

This paper is divided as follows. Section 2 presents the linear water hydraulic actuator mathematical model. In section 3, the classical control of the water hydraulic actuator is discussed. Section 4 approaches the pole placement control. In section 5, a state observer is designed. Section 6 deals with the case where the velocity and acceleration are obtained by using a time derivative and filtering. In section 7, one presents the conclusions.

### 2. LINEAR WATER HYDRAULIC ACTUATOR MATHEMATICAL MODEL

The water hydraulic actuator under consideration is composed of a cylinder of double-action and single rod. This is driven by a four-way directional valve that allows controlling the piston movements in both directions. Although the hydraulic actuator is a nonlinear system, its model can be approximated with good results as a third order linear system relating the control input  $u$  and the piston position  $y$ .

The use of a third order linear model for representing hydraulic actuators has been used by many authors in the literature, for instance Paim (1997) where some classical controllers were designed and Guenther and De Pieri (1997) where a cascade controller was designed. In Makinen (2001), a water hydraulic actuator was approximated by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K_{qa} \omega_n^2}{s(s^2 + 2\delta_n \omega_n s + \omega_n^2)} \quad (1)$$

where  $Y(s)$  is the Laplace transform of the output (position of the end of the cylinder rod),  $U(s)$  is the Laplace transform of the input (voltage applied to the electronic card),  $K_{qa}$  is a hydraulic constant,  $\omega_n$  is the not damped natural frequency.

By using experimental tests, Makinen (2001) obtained the following parameters with a mass  $m = 850$  Kg:  $K_{qa} = 0.01m/sV$ ,  $\omega_n = 180 \text{ rad/s}$  and  $\delta_n = 0.1$ . Substituting these values into Eq. (1), one has

$$G(s) = \frac{324}{s^3 + 36s^2 + 32400s} \quad (2)$$

*Remark 1* - Makinen (2001) also modeled the friction forces and valve nonlinearities. Here, as the goal is to study the control of the system with a state controller considering the parametric uncertainties and the use of a state observer or time derivative to obtain velocity and acceleration, such nonlinearities will not be considered. Furthermore, when considering the nonlinearities, it is possible to use a nonlinear controller to cancel these nonlinearities like in Cunha et al. (2004).

Steps with two different setpoints were used in Makinen (2001) to verify the closed loop performance of the controllers. One from 0.275 m to 0.280 m and other from 0.250m to 0.280 m. Here, in the simulations, these two trajectories are also used, one called short trajectory and the other called long trajectory. An input saturation with limits of -10V and +10V was also included in the simulations.

### 3. CLASSICAL CONTROL OF THE WATER HYDRAULIC ACTUATOR

Classical control of oil hydraulic actuators has demonstrated to be inefficient when high performances are desired. The same analysis is valid to water hydraulic actuators. The system linear model presents one pole at origin and two complex poles lightly damped (see the root locus in Fig. 1). In this way, PID controllers do not modify substantially the poles location and, consequently, do not alter the closed loop performance significantly (Cunha, 2001). In the next section, a state controller is designed to change the original poles location.

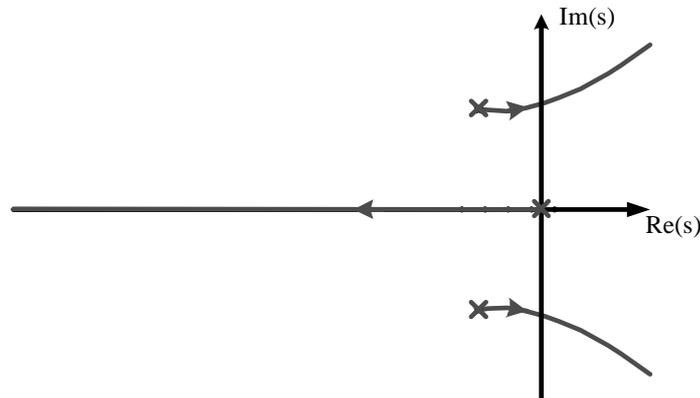


Figure 1 – Root Locus

### 4. POLE PLACEMENT CONTROL

Defining  $x_1$  as the piston position,  $x_2$  as the piston velocity and  $x_3$  as the piston acceleration, Eq. (2) can be written in the state form as  $\dot{x} = Ax + Bu$ ,  $y = Cx$ , where  $A$  is the system matrix,  $b$  is the input matrix and  $c$  is the output matrix:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -32400 & -36 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 324 \end{bmatrix} u \quad (3)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

In this section, one considers that all system state variables are available for measuring. In order to place arbitrarily all system poles, it is necessary that the system to be controllable. Calculating the controllability matrix, one obtains

$$M = \begin{bmatrix} 0 & 0 & 324 \\ 0 & 324 & -11664 \\ 324 & -11664 & -10077696 \end{bmatrix} \quad (4)$$

As the rank of the controllability matrix in Eq. (4) is 3, the system is completely controllable and can have its poles placed arbitrarily. After verify the controllability, one defines the control signal as

$$u = -Kx + v \quad (5)$$

where  $K$  is the state feedback matrix and  $v$  is the new control input.

Substituting Eq. (5) into the system equation, gives

$$\dot{x} = (A - BK) + Bv \quad (6)$$

$$y = Cx$$

The poles were chosen to be placed in  $\mu_1 = -100$ ,  $\mu_2 = -300$  and  $\mu_3 = 0$ . The new root locus is given by Fig.2.

The characteristic equation with poles new location is given by

$$\phi(\lambda) = \lambda^3 + 400\lambda^2 + 30000\lambda \quad (7)$$

The next step is to find out the state feedback matrix. Here, the  $K$  matrix is calculated by using the Ackermann formula (Ogata, 2003). To apply the Ackermann formula, it is necessary to calculate  $\phi(A)$ , given by

$\phi(A) = A^3 + \alpha_1 A^2 + \alpha_2 A + \alpha_3 I$ . Thus,

$$\phi(A) = \begin{bmatrix} 0 & -2400 & 364 \\ 0 & -11793600 & -15504 \\ 0 & 502329600 & -11235456 \end{bmatrix} \quad (8)$$

Then, the state feedback matrix is given by

$$K = [0 \ 0 \ 1] [B \ AB \ A^2B]^{-1} \phi(A) = [0 \ -7.4074 \ 1.1235] \quad (9)$$

The control law  $v$  is chosen as a proportional law:  $v = K_p e$ .

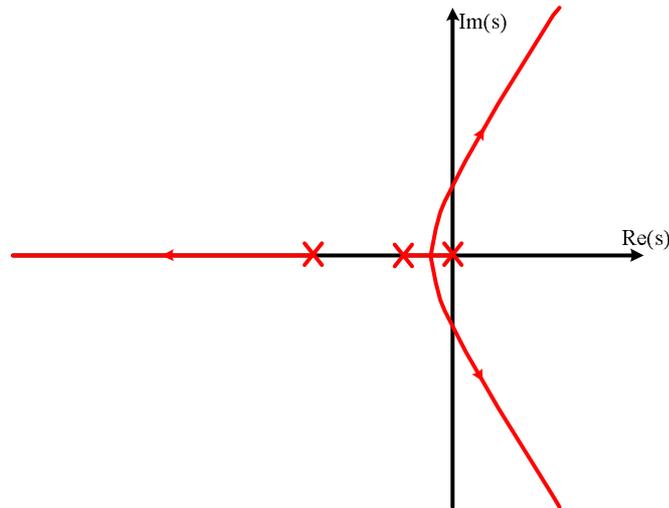


Figure 2. Root locus with the poles at new location

Figures 3 and 4 show the closed loop system response to the designed state controller. One can observe that the control signal saturates in both cases. But it lasts more time when the long trajectory is required.

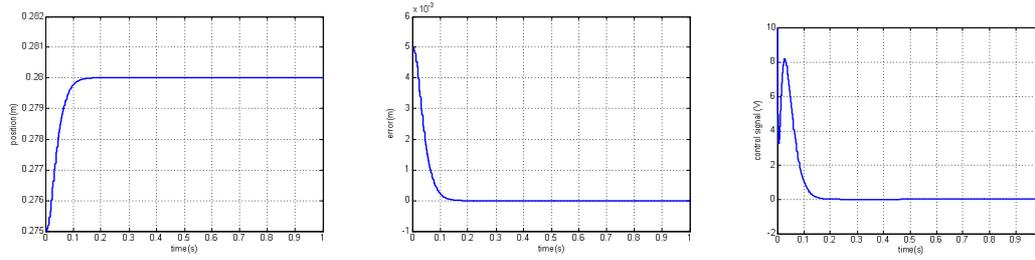


Figure 3 – Short Trajectory: position, error, and control signal

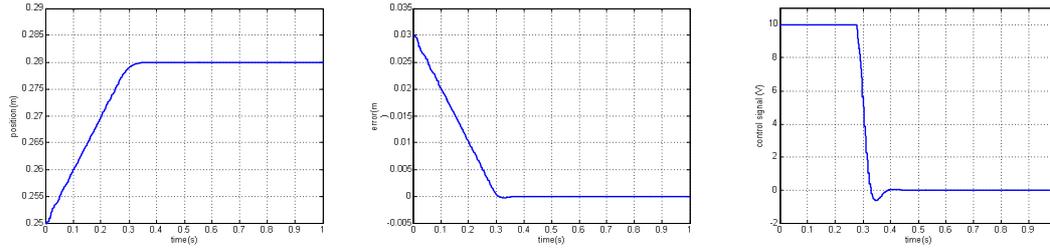


Figure 4 – Long trajectory: position, error, and control signal

## 5. STATE OBSERVER

In the previous section, one supposed that all state variables were available to measure. Normally, only a sensor to measure the position is used. Then, an alternative is to use a state observer. A state observer is an algorithm that estimates the state variable by measuring the output and the control variables. Here, the state observer is used to estimate the velocity and acceleration signals.

The observer mathematical model is given by

$$\dot{\hat{x}} = (A - K_e C)\hat{x} + Bu + K_e y \quad (10)$$

where  $\hat{x}$  is the estimated state,  $C\hat{x}$  is the estimated output, and  $K_e$  is the observer gain matrix.

The estimation error vector is the difference between the state real value and the estimates obtained by the observer:

$$e = x - \hat{x} \quad (11)$$

Calculating the time derivative of Eq. (10),  $\dot{e} = \dot{x} - \dot{\hat{x}}$ , one has

$$\dot{e} = (A - K_e C)e \quad (12)$$

By Eq. (12), one concludes that if the eigenvalues of the matrix  $(A - K_e C)$  are stable then  $e$  will converge to zero regardless of the initial error.

The first step is to check if the system is observable. It can be done by calculating the observability matrix, that is given by

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

As  $\text{rank}(O) = 3$ , then the system is completely observable. The next step is to find out the state observer gain matrix  $K_e$ . By using the Ackermann formula, one has

$$K_e = \phi(A) \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (14)$$

It is necessary to choose the state observer poles. They must be chosen two or five times faster than the system dominant poles. Choosing the observer poles at  $\mu_1 = -600$ ,  $\mu_2 = -700$  and  $\mu_3 = -800$  and applying the Ackermann formula, gives

$$K_e = [2064 \quad 1353296 \quad 220407744]^T \quad (12)$$

Figures 5-8 present the simulation results that allow one to compare the real variables with the observer estimated variables.

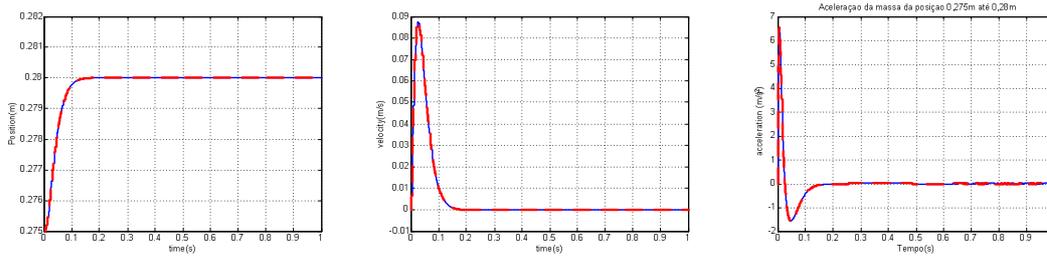


Figure 5 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – short trajectory

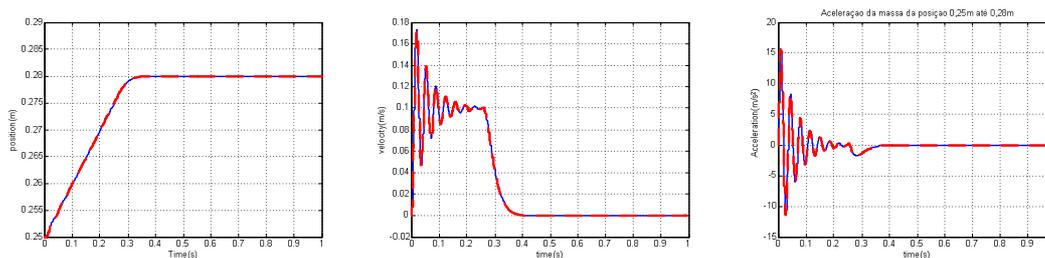


Figure 6 Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – long trajectory

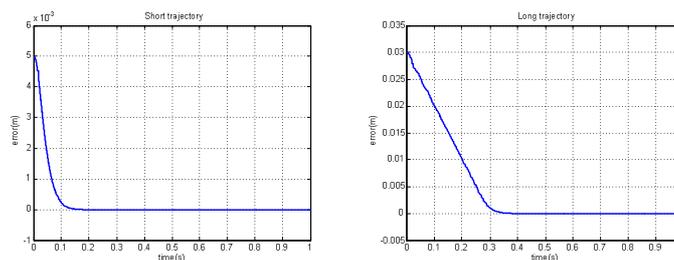


Figure 6 – Position error: short and long trajectories

Analyzing Figs. 5 and 6, one concludes that the error in the estimates were very small. In order to check the observer convergence, one supposes different initial conditions in the observer and in the actuator. The initial position in the observer is set as 0.26 m. Figures 7-11 illustrate this situation.

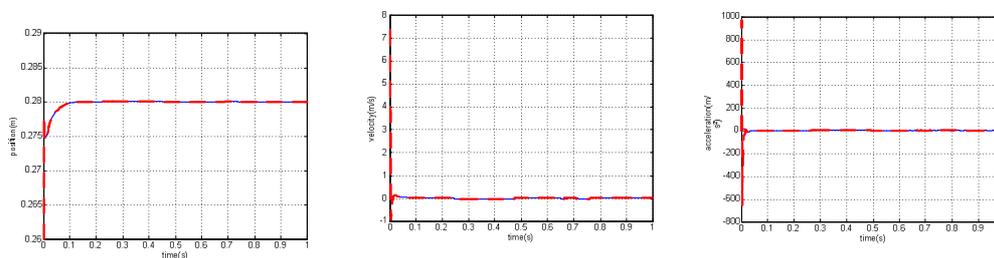


Figure 7 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – short trajectory

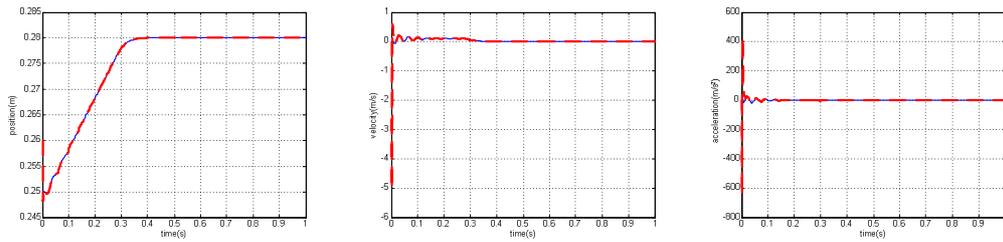


Figure 8 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – long trajectory

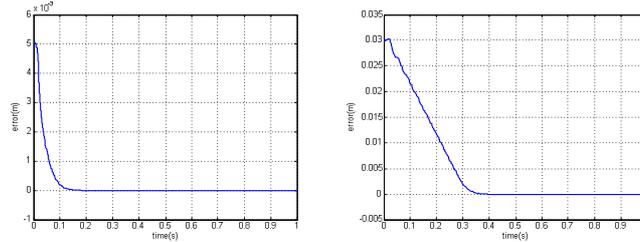


Figure 9 - Position error: short and long trajectories

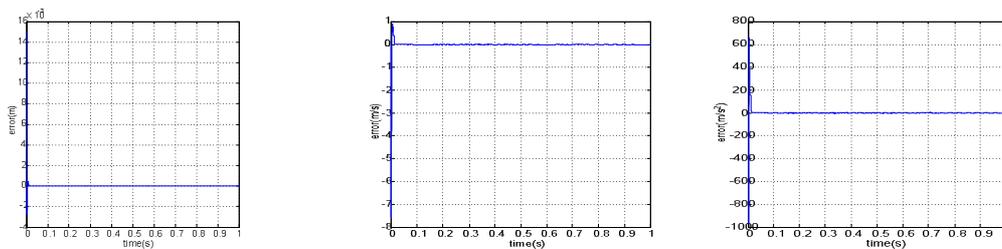


Figure 10 – Position, velocity and acceleration estimation errors – short trajectory

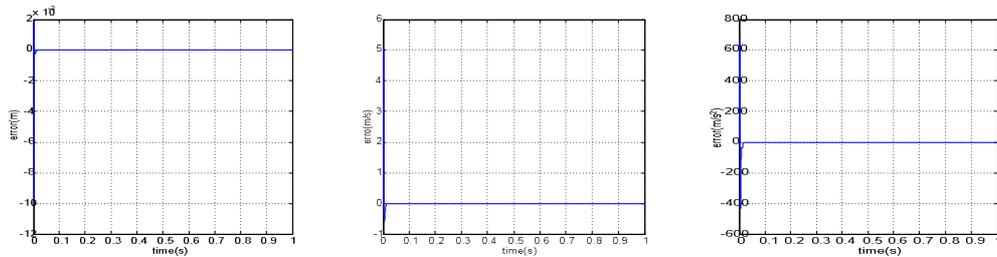


Figure 11 – Position, velocity and acceleration errors – long trajectory

The simulations confirm that the errors vector converge to zero regardless of the initial values. In order to verify the robustness of the designed controller with relation to parametric uncertainties, system errors are included in the simulations. Initially, one supposes a variation of 20% in the not damped natural frequency value ( $\omega_n$ ).

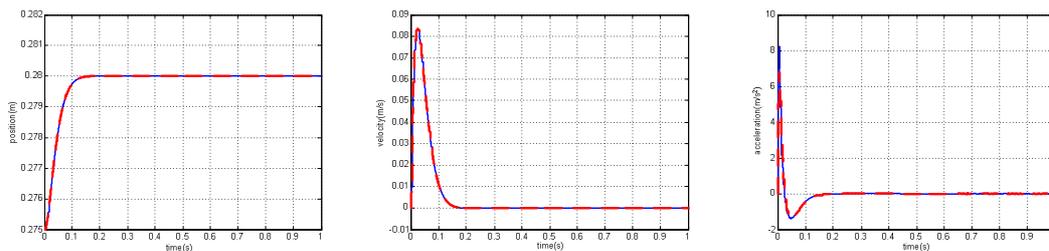


Figure 12 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – short trajectory

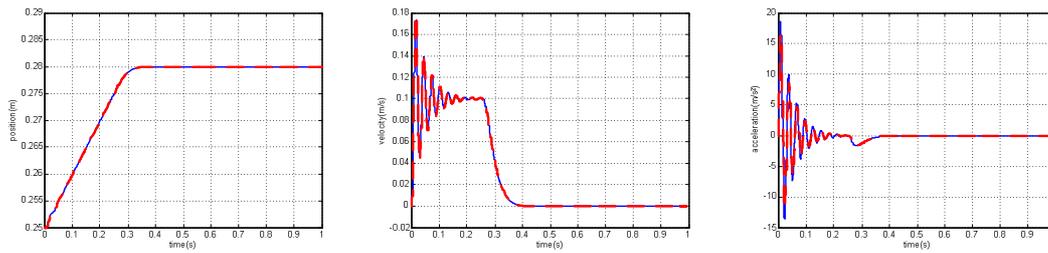


Figure 13 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – long trajectory

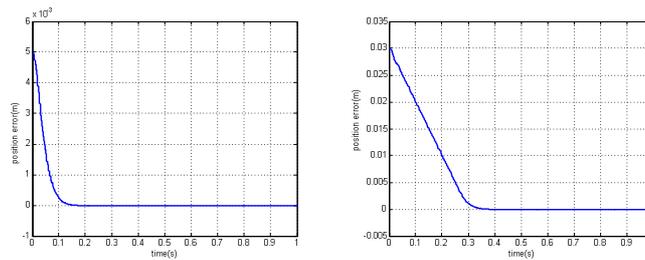


Figure 14 - Position error: short and long trajectories

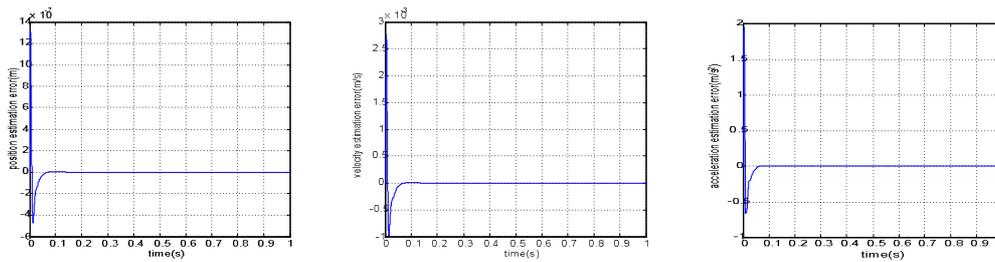


Figure 15 - Position, velocity and acceleration estimation errors – short trajectory

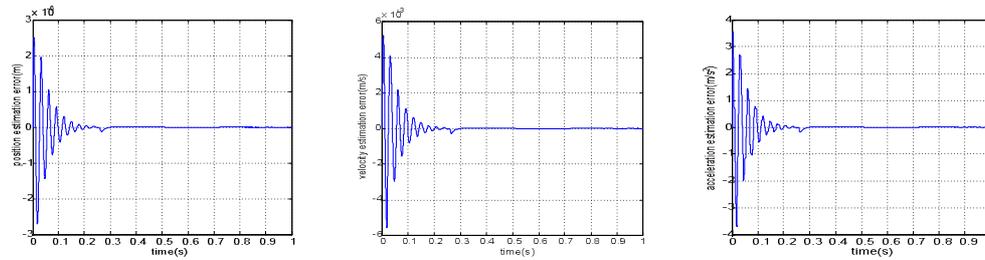


Figure 16 - Position, velocity and acceleration errors –long trajectory

Analyzing the simulation results (Figs. 12-16), one realizes that the parametric uncertainty in the not damped natural frequency does not cause significant errors in the variables estimated by the observer. The next step is to simulate a parametric uncertainty in  $K_{qa}$ . Figures 17 to 21 present the result for a parametric uncertainty of 20% in  $K_{qa}$ .

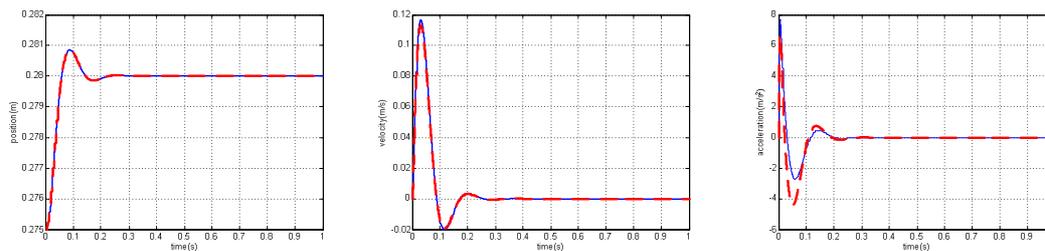


Figure 17 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – short trajectory

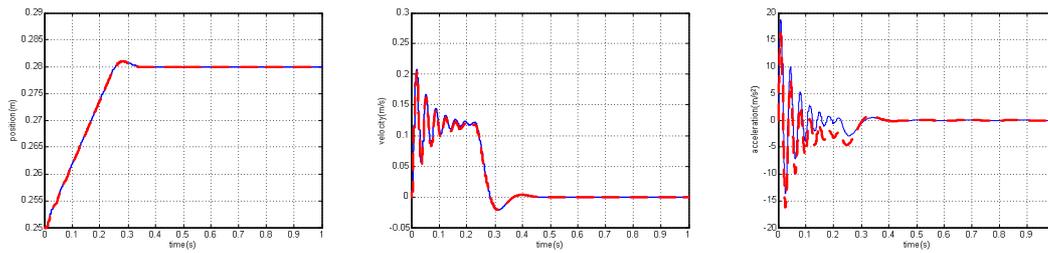


Figure 18 - Comparison between position, velocity and acceleration (blue lines) and the corresponding observer estimated values (red dashed line) – long trajectory

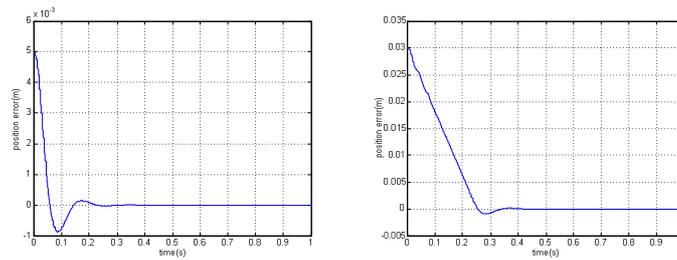


Figure 19 - Position error: short and long trajectories

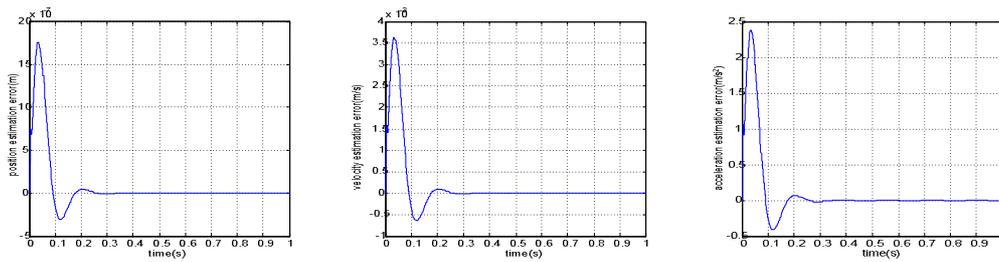


Figure 20 - Position, velocity and acceleration estimation errors – short trajectory

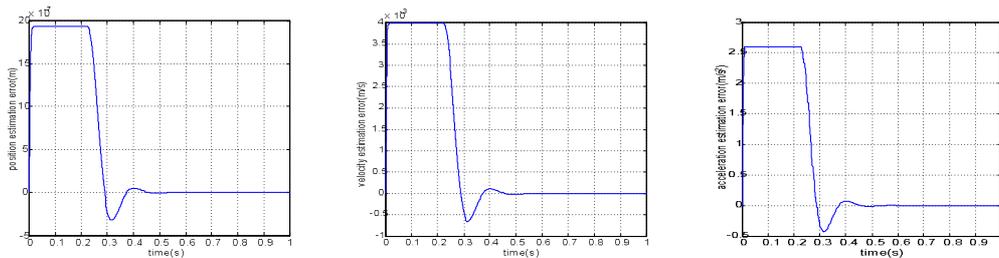


Figure 21 - Position, velocity and acceleration estimation errors – long trajectory

Analyzing the simulation results (Figs. 17-21), one realizes that parametric uncertainty in the parameter  $K_{qa}$  causes a significant error in the estimated state variables. It means that changes in  $K_{qa}$  will take to significant errors in the observed variables during a considerable time until the convergence to zero.

## 6. OBTAINING VELOCITY AND ACCELERATION BY USING FILTERING AND TIME DERIVATIVE

In many applications the velocity and acceleration signal are obtained by filtering and doing the time derivative of the position signal. The use of filter is necessary due to the noise that comes from sensors and could take the time derivative to have some spikes. Normally, the adjustment of the filter bandwidth is done by increasing the value of the cut frequency up to an acceptable quantity of noise without causing actuator vibration. However, as the theoretical analysis when with elaborated controllers in the closed loop is very difficult, the sense in adjust the filter bandwidth is more intuitive than based on the equations under analysis.

In this work, as one is dealing with a simple controller, this theoretical analysis is presented. The analysis presented in the sequence do not have a specific goal on water hydraulic actuators, but to make people of the control area to think about the implication of designing a controller by using some signal and implementing the controllers by obtaining the signals by using numerical time derivative.

The first step to analyze the stability of the closed loop system is to rewrite the system adding two state variables: velocity and acceleration obtained trough numerical time derivative. In this way, the new state model has five state variables:  $x_1$  - piston position;  $x_2$  - piston velocity;  $x_3$  - piston acceleration;  $x_4$  velocity obtained through filtering and derivative;  $x_5$  acceleration obtained through filtering and derivative. Such state equation can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -32400 & -36 & 0 & 0 \\ 0 & p_f & 0 & -p_f & 0 \\ 0 & p_f^2 & 0 & -p_f^2 & -P_f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 324 \\ 0 \\ 0 \end{bmatrix} u, \quad y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

where  $p_f$  is the filter bandwidth. Here, one analyzes the closed loop system in two different situations: (i)  $p_f = 100$  rad/s; (ii)  $p_f = 30$  rad/s.

(i)  $p_f = 100$  rad/s;

In this case, the closed loop eigenvalues are  $\lambda_1 = 0$ ,  $\lambda_2 = 0,96 + 207,59j$ ,  $\lambda_3 = 0,96 - 207,59j$ ,  $\lambda_4 = -203,76$  and  $\lambda_5 = -34,16$ . One concludes that the closed loop system presents two poles in the right-plane. Thus, the use of these filters bandwidth makes the closed loop system unstable.

(ii)  $p_f = 30$  rad/s

With this filter the closed loop poles are located in the half-left plane, in the following locations:  $\lambda_1 = 0$ ,  $\lambda_2 = -14,24 + 180,30j$ ,  $\lambda_3 = -14,24 - 180,30j$ ,  $\lambda_4 = -51,48$  and  $\lambda_5 = -16,03$ . The root locus is showed in Fig. 22 and the system response is showed in Fig. 23.

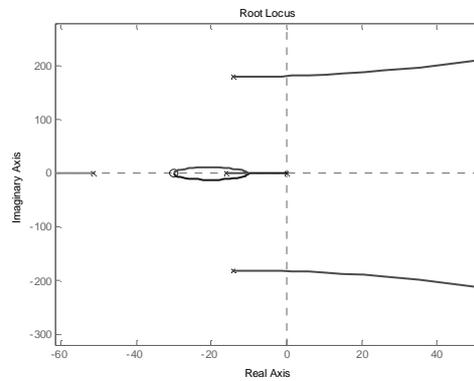


Figure 22 - Root locus with augmented system and a 30 rad/s filter

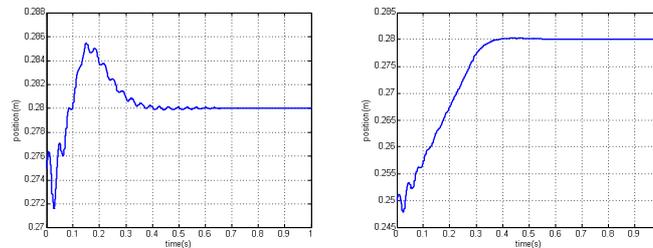


Figure 23 - Position with a filter of 30 rad/s

The advantage of the use of time derivative and filtering to obtain the velocity and acceleration signal in relation to the use of a state observer is that it does not depend on the system previous knowledge. However, as it was

demonstrated in these two examples, it can degrade the system response and can inclusive to take the system to instability.

## **7. CONCLUSIONS**

This paper presented a first study of these work's authors about the control of water hydraulic actuators. This study was based on a 3<sup>rd</sup> order linear model of a water hydraulic actuator. The classical control applied to the system was discussed. A state controller and a state observer were proposed. The closed loop system with the state controller and observer presented some robustness when subject to parametric uncertainties. An analysis of the closed loop system with the state controller and using the signals of velocity and acceleration obtained by filtering and doing the numerical time derivative was done. The system became unstable depending on the filter bandwidth. This is an interesting result because in this case it was possible to do a theoretical analysis that is not possible with more elaborated control algorithm.

Future work involves the inclusion of the friction forces and valve nonlinearities in the model and the development of new controllers to compensate such nonlinearities.

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