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OPTIMAL PATH PLANNING AND TASK ADJUSTMENT FOR COOPERATIVE FLEXIBLE MANIPULATORS

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Abstract. *Accuracy and energy consumption are key aspects to be considered when using serial manipulators in industrial environment. Affordable operational and maintenance costs are very important issues for automated processes. However, high performance robots also require increasing precision when performing sophisticated tasks. In this paper a method for optimal path planning and task adjustment of cooperative flexible manipulators is presented. The trajectory of two manipulators while manipulating a single object in a collaborative task and the object placement are written as an optimization problem. End-effector positioning and torque requirements are considered together in an optimal control formulation, aiming at improving accuracy and energy consumption during the path planning. The flexibility effects of manipulators working in a vertical workspace are taken into account, and joint limits are considered in a box-constrained objective function, to ensure the movement feasibility at the optimal configuration. Simulation results demonstrate the viability of the proposed methodology.*

Keywords: *Optimal path planning, cooperative flexible manipulator, optimal control, multibody dynamics.*

1. INTRODUCTION

The industrial productivity improvement can be achieved by reducing the weight of the robots and/or increasing their speed of operation. The first choice may lead to power consumption reduction while the second results in a faster work cycle. To successful achieve these purposes it is very desirable to build flexible robotic manipulators. In some situations is even necessary to consider the flexibility effects due to the joints and gear components of the manipulators for obtaining an accurate and reliable control.

Compared to conventional heavy robots, flexible link manipulators have the potential advantage of lower cost, larger work volume, higher operational speed, greater payload-to-manipulator-weight ratio, smaller actuators and lower energy consumption.

The study on the control of a flexible manipulator started in the field of the space robots research, as a space manipulator should be as light as possible in order to reduce its launching cost (Book, 1979)(Book, 1984). Uchiyama *et al.* (1990), Alberts *et al.* (1992), Krishnamurthy and Chao (1992), Dubowsky (1994), to cite some, also studied flexible manipulators used for space applications. Shi *et al.* (1998) discussed some key issues in the dynamic control of light weight robots for several applications.

As a consequence of the interest in using flexible structures in robotics, several papers regarding the design of controllers for the manipulation task of flexible manipulators are found in the literature (Choi and Krishnamurthy, 1994), (Chang and Chen, 1998) and (Latornell *et al.*, 1998)

In Tsujita *et al.* (2004), the trajectory and force controller of a flexible manipulator is proposed. From the point of view of structural dynamics, the trajectory control for a flexible manipulator is dedicated to the control of the global elastic deformation of the system, and the force control is dedicated to the control of the local deformation at the tip of the end-effector. Thus, preferably trajectory and force controls are separated in the control strategy.

Static and dynamic hybrid position/force control algorithms have been developed for flexible-macro/rigid-micro manipulator systems (Yoshikawa, Harada, and Matsumoto, 1996). The robust cooperative control scheme of two flexible manipulators in the horizontal workspace is presented in (Matsuno and Hatayama 1999). A passivity-based controller has been developed for the payload manipulation with two planar three degrees-of-freedom (3-DOF) flexible arms (Damaren 2000).

In (Miyabe, Konno and Uchiyama, 2004), the automated object capture with a two-arm flexible manipulator is addressed, which is a basic technology for a number of services in space. This object capturing strategy includes symmetric cooperative control, visual servoing, the resolution of the inverse kinematics problem, and the optimization of the configuration of a two-arm redundant flexible manipulator.

Focused on industrial applicability, in the present paper the optimal path planning of two flexible manipulators is addressed. The manipulators are requested to perform a cooperative task in a vertical plane. Under this condition the gravitational effects are taken into account.

An optimal control formulation to determine the optimal torque profile, leading the end-effectors to interact in a common task, is proposed. The corresponding optimal task placement is also a design variable. From the best of the author's knowledge, the optimal task placement and the optimal torque profile analysis in a unified optimal control formulation for flexible manipulators was not found in the literature.

As a result, the contributions of the current paper are the proposition of a new optimization formulation for dealing with such a problem, and the choice of a computationally efficient methodology to solve the model.

The paper is organized as follows. In Section 2, different strategies present in the literature to model a flexible manipulator are discussed, and the mathematical model adopted is described. Section 3 recalls the general optimal control formulation, and later, new performance indexes are proposed. The numerical results and discussion are presented in section 4. The conclusions and perspectives for future work are given in section 5.

2. MANIPULATOR MODEL

Different schemes for modeling of the manipulators have been studied by a number of researchers. The mathematical model of the manipulator is generally derived from energy principles and, for a simple rigid manipulator, the rigid arm stores kinetic energy due its moving inertia, and stores potential energy due its position in the gravitational field. A flexible link also stores deformation energy by virtue of its deflection, joint and drive flexibility. Joints have concentrated compliance which may often be modeled as a pure spring storing only potential energy. Drive components such as shafts or belts may appear distributed. They store kinetic energy due to their low inertia, and a lumped parameter spring model often succeeds well to consider such effect.

The most important modeling techniques for single flexible link manipulators can be grouped under the following categories: assumed mode method, finite element method and lumped parameter models.

In the assumed modes approach, the link flexibility is usually represented by a truncated finite modal series, in terms of spatial mode eigenfunctions and time-varying mode amplitudes. Although this method has been widely used, there are several ways to choose link boundary conditions and mode eigenfunctions. Some contributions in this field were presented by Cannon and Schmitz (1984), Sakawa *et al.* (1985), Bayo (1986), Tomei and Tornambe (1988), among others. Nagaraj *et al.* (2001), Martins *et al.* (2002, 2003) and Tso *et al.* (2003) studied single-link flexible manipulators using Lagrange's equation and the assumed mode method.

Regarding the finite element formulation, Nagarajan and Turcic (1990) derived elemental and system equations for systems with both elastic and rigid links. Bricout *et al.* (1990) studied elastic manipulators. Moulin and Bayo (1990, 1991) also used finite element discretization to study the end-point trajectory tracking for flexible arms and showed that a non-causal solution for the actuating torque enables tracking of an arbitrary tip displacement with any desired accuracy.

By using a lumped parameter model, Zhu *et al.* (1999) simulated the tip position tracking of a single-link flexible manipulator. Khalil and Gautier (2000) used a lumped elasticity model for flexible mechanical systems. Megahed and Hamza (2004) used a variation of the finite segment multibody dynamics approach to model and simulate planar flexible link manipulator with rigid tip connections to revolute joints. Machado *et al.* (2002) considers a lumped mass approach to model a single link manipulator.

2.1 The proposed formulation

In this work the description of each link as a spring-mass-damper system is proposed. It is supposed that the link can be described as a four mass body with satisfatory accuracy, as presented in Fig. 1.

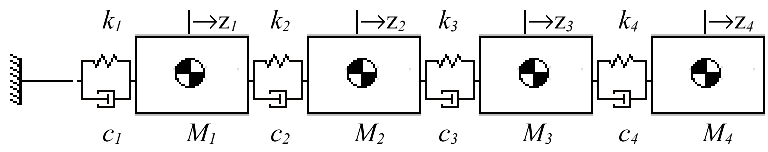


Figure 1 – Physical representation of the link.

According to this analogy, the first spring and damper constants, k_1 and c_1, are related to the joint behavior, and the other three sets of spring and damper represent link flexibility. A larger number of links are represented by including more elements in the system. Without loss of generality, each link can also be described by a larger or shorter number of elements.

It should be pointed out that for the present application the variables and the parameters presented in the model above should be interpreted accordingly, i.e., they are all related to angular dimensions.

A general time response for this mechanical system can be obtained from the following set of ordinary differential equations:

$$\mathbf{M}\ddot{\mathbf{z}}(t)+\mathbf{C}\dot{\mathbf{z}}(t)+\mathbf{K}\mathbf{z}(t)=\mathbf{u}(t) \quad (1)$$

In the context of a two-link manipulator, the variables to be taken into account are the position of each mass, as given by the position vector $\mathbf{z}=[z_1, \ldots, z_8]^T$, and torques in the joints as given by $\mathbf{u}=[u_1, u_2]^T$. Therefore, the state variable is formed by the mass positions z_i, (i=1,…8, for each manipulator: 4 elements to represent each link) and the control vector encompasses the torques applied to the joints, u_i (i=1,2 for each link; two joints).

In order to reduce the order of the above system, the state formulation is considered through the definition of the state variables $y_i = z_i$ and $y_{i+8} = \dot{z}_i$, i=1,…8.

When a two-link flexible manipulator is expressed by using this convention, the state formulation of the system dynamics is given by:

$$\dot{y}_i = \frac{1}{2}M_i g L_i \sin(y_i) + y_{i+8}, i=1,\ldots,8 \quad (2)$$

$$\dot{y}_9 = \frac{1}{j_1}[(-k_1-k_2)y_1 + k_2y_2 - (c_1+c_2)y_9 + c_2y_{10} + u_1] \quad (3)$$

$$\dot{y}_i = \frac{1}{j_{i-8}}[k_{i-8}y_{i-9} - (k_{i-8}+k_{i-7})y_{i-8} + k_{i-7}y_{i-7} + c_{i-8}y_{i-1} - (c_{i-8}+c_{i-7})y_i + c_{i-7}y_{i+1}], i=10,11 \quad (4)$$

$$\dot{y}_i = \frac{1}{j_{i-8}}[k_{i-8}y_{i-9} - (k_{i-8}+k_{i-7})y_{i-8} + k_{i-7}y_{i-7} + c_{i-8}y_{i-1} - (c_{i-8}+c_{i-7})y_i + c_{i-7}y_{i+1} - u_2], i=12 \quad (5)$$

$$\dot{y}_i = \frac{1}{j_{i-8}}[k_{i-8}y_{i-9} - (k_{i-8}+k_{i-7})y_{i-8} + k_{i-7}y_{i-7} + c_{i-8}y_{i-1} - (c_{i-8}+c_{i-7})y_i + c_{i-7}y_{i+1} + u_2], i=13 \quad (6)$$

$$\dot{y}_i = \frac{1}{j_{i-8}}[k_{i-8}y_{i-9} - (k_{i-8}+k_{i-7})y_{i-8} + k_{i-7}y_{i-7} + c_{i-8}y_{i-1} - (c_{i-8}+c_{i-7})y_i + c_{i-7}y_{i+1}], i=14,15 \quad (7)$$

$$\dot{y}_{16} = \frac{1}{j_8}[k_8y_7 - k_8y_8 + c_8y_{15} - c_8y_{16} + u_3] \quad (8)$$

where g = -9.81 m/s^2 is the gravitational constant, j_i is the moment of inertia of each element i, and u_1, u_2 and u_3 are the torques applied to the first and second joints and to the end-effector, respectively.

3. OPTIMAL CONTROL FORMULATION

3.1 Design variables

Optimal programming problems for continuous systems are included in the field of the calculus of variations. A continuous-step dynamic system is described by an n-dimensional state vector $\mathbf{x}(t)$ at time t. The choice of an m-dimensional control vector $\mathbf{u}(t)$ determines the time rate of change of the state vector through the dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \quad (9)$$

A general optimization problem for such a system is to find the time history of the control vector $\mathbf{u}(t)$ for $t_0 \leq t \leq t_f$, to minimize a performance index of the form:

$$J = \phi[\mathbf{x}(t_f)] + \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t), t)dt \tag{10}$$

subject to Eq. (9) with t_0, t_f, and $\mathbf{x}(t_0)$ specified.

In the present context, the interest is focused on the positions of the masses and the torques applied to the joints. Then, by considering $x_i = y_i$, i=1,…16, the state vector $\mathbf{x} = [x_1,\ldots,x_{16}]^T$ represents each mass position and velocity, and the control vector $\mathbf{u} = [u_1,u_2]^T$ is the joint torque. As a result, the dimension of the state vector is n = 16 and the control vector has dimension m = 2 (for each robot). The simultaneous analysis of the two manipulators doubles the dimension of the model. By using this formulation, the angular position of each mass is given by x_i, i=1,…,8. In this way, the system dynamics, Eq. (9), is written as:

$$[\dot{x}_i] = [f_i(\mathbf{x}(t), \mathbf{u}(t), t)], i = 1,...,16 \tag{11}$$

where the values for f_i are obtained by the right hand side of Equations (2)-(8), respectively.

3.2 Performance index

Three objective functions are considered in this work. Initially, the specification of a feasible project is modeled as an optimization problem. To achieve this purpose, let $\mathbf{p}_t=[x_{\text{target}}, y_{\text{target}}]^T$ be the cartesian target position where the end-effectors may intersect when performing a task. Then, a feasible project is the one whose torque profile is able to conduct each end-effector to a given Cartesian position in which the prescribed task will be performed. This profile is obtained at the optimum value of the following objective function:

$$\phi_1[\mathbf{x}(t_f)] = [\mathbf{e}_1(\mathbf{x}(t_f)) - \mathbf{p}_t]^2 + [\mathbf{e}_2(\mathbf{x}(t_f)) - \mathbf{p}_t]^2 \tag{12}$$

where $\mathbf{e}_1(\mathbf{x}(\text{t}))$ and $\mathbf{e}_2(\mathbf{x}(\text{t}))$ are the cartesian position of each robot end-effector, respectively. By using Eq. (12) as the only one performance index of the general formulation (Eq. (10)), the objective function is:

$$J_1 = [\mathbf{e}_1(\mathbf{x}(t_f)) - \mathbf{p}_t]^2 + [\mathbf{e}_2(\mathbf{x}(t_f)) - \mathbf{p}_t]^2 \tag{13}$$

The end-effector intersection at the specified target point and the torque minimization are the requirements to be achieved in the second analysis. The total torque T_i required from each robot joint i=1,2 is given by:

$$T_i = \int_{t_0}^{t_f} |\mathbf{u}_i(t)| dt\,. \tag{14}$$

In the optimization formulation, the torque required from the actuators of each manipulator to achieve the final position is approximated by a quadratic expression:

$$L(\mathbf{x}(t), \mathbf{u}(t), t) = \mathbf{u}_1^T(t)\mathbf{u}_1(t) + \mathbf{u}_2^T(t)\mathbf{u}_2(t) \tag{15}$$

To consider both end-effector positioning and torque consumption as objectives to be optimized, the following objective function is proposed:

$$J_2 = \sum_{r=1}^{2} w_r[\mathbf{e}_r(\mathbf{x}(t_f)) - \mathbf{p}_t]^2 + \sum_{r=1}^{2}\left[w_{r+2}\int_{t_0}^{t_f} \mathbf{u}_{1r}^T(t)\mathbf{u}_{1r}(t) + \mathbf{u}_{2r}^T(t)\mathbf{u}_{2r}(t)dt\right] \tag{16}$$

where w_i are weighting factors and r = 1,2 refers to the first and second robots, respectively.

The third analysis considers the end-effector positioning, the total torque and the best placement to perform the task. The last objective is expressed by considering the position of the interception $\mathbf{p}_t$ as a design variable. It can also be expressed implicitly when computing the distance between the two end-effectors at the final time. By using this approach, the objective function is:

$$J_3 = w_1[\mathbf{e}_1(\mathbf{x}(t_f)) - \mathbf{e}_2(\mathbf{x}(t_f))]^2 + \sum_{r=1}^{2}\left[w_{r+1}\int_{t_0}^{t_f} \mathbf{u}_{1r}^T(t)\mathbf{u}_{1r}(t) + \mathbf{u}_{2r}^T(t)\mathbf{u}_{2r}(t)dt \right] \tag{17}$$

A number of methods can be found in the literature to deal with optimal control (OC) problems (Betts, 2001), (Bryson, 1999), (Bertsekas, 1995). In the present work, the results are computed through a classical nonlinear programming (NLP) procedure. In this case, there is no need of extra parameter computations and the derivatives are numerically evaluated. This choice characterizes a strong point of the proposed methodology: it provides an efficient formulation to solve the multicriteria optimal control problem addressed in the present contribution.

Since the NLP procedure requires a finite number of points as design variables, the continuity of the OC variables along the time interval are obtained by interpolating the discrete set in time.

From the methods tested by the authors, the best solutions were obtained by using the BFGS Method (Luenberger, 1984). The algorithms were implemented in FORTRAN by using the optimization library DOT ® (Vanderplaats, 1995).

4. NUMERICAL SIMULATION

To perform the following numerical example, the time interval is bounded by $t \in [0,7]s$, represented by a set of $N=15$ steps. Each manipulator has a total mass of M_{total} = 55.4 *kg* and a total length L_{total} = 2 *m*. Follows that M_i = 6.925 *kg* and L_i = 0.25 *m*, for each element *i*=1,…,8.

The target point is $\mathbf{p}_t$ = (0,-1) and the manipulator's basis are placed at the points (-1.2, 0) *m* and (1.2, 0) *m*, as shown in Fig. 1. Manipulator 1 (on the left in Fig. 2) moves a payload of mass 8.075 kg.

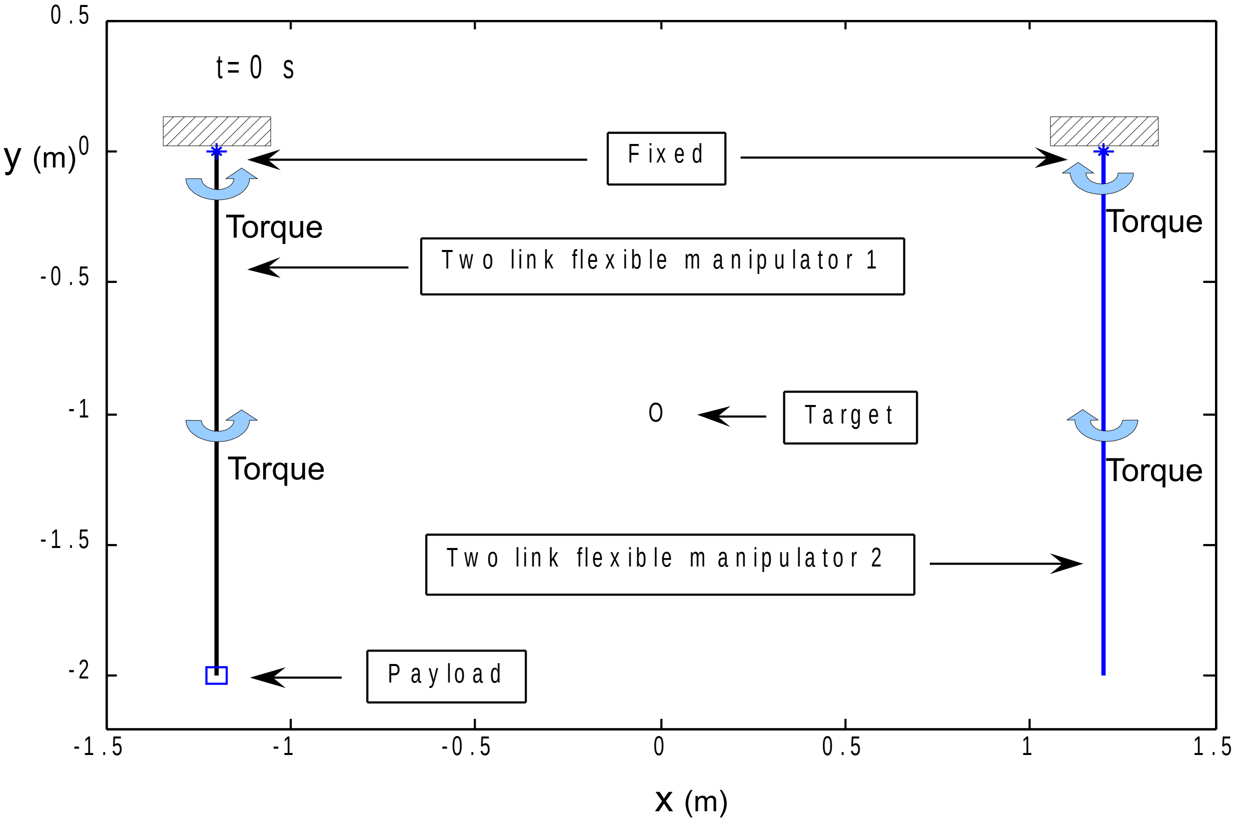


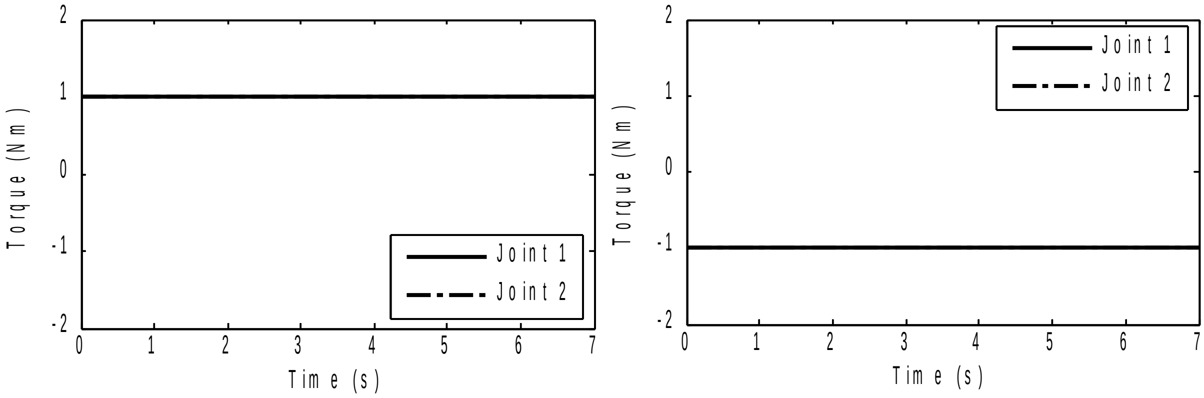
Figure 2. Workspace layout.

The first experiment is proposed aiming evaluate the viability to achieve a feasible control design.

Additionally, upper $\mathbf{u}_U(t)$ and lower $\mathbf{u}_L(t)$ bounds of the control are considered in the nonlinear programming formulation. In the following experiment $\mathbf{u}_U(t) = 200$ *Nm* and $\mathbf{u}_L(t) = -200$ *Nm*, $t_0 \leq t \leq t_f$. The target position is fixed at $\mathbf{p}_t = (0,-1)$.

In the absence of previous information about an feasible profile, the control variable is set to $\mathbf{u}_1(t) = 1.0$ *Nm* and $\mathbf{u}_2(t) = -1.0$ *Nm*, $t_0 \leq t \leq t_f$, as initial guess to the minimization of the objective function J_I (Eq. (13)). By using this initial torque profile, shown in Fig. 3, the distance between end-effectors of robot 1 (left), robot 2 (right) and target $\mathbf{p}_t$ are $d_1 = 1.562$ *m* and $d_2 = 1.562$ *m*, respectively.

The initial torque profile clearly does not lead to a satisfactory end-effector positioning, as shown in Fig. 4.



(a) (b)

Figure 3. Initial torque profile of (a) robot 1 and (b) robot 2.

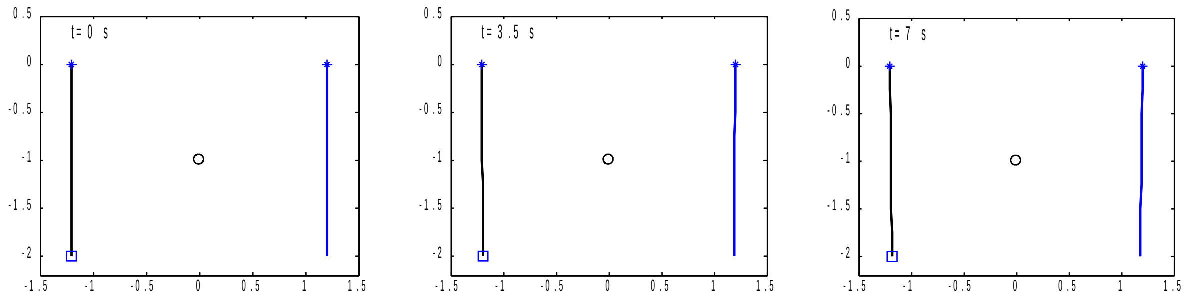


Figure 4. Robot movement given by initial torque profile.

After the optimization process, the end-effector is successful placed at the target position, as shown in Fig. 5. The distance between end-effectors of robot 1, robot 2 and target are $d_1 = 0.014$ *m* and $d_2 = 0.012$ *m*, respectively. The total torque required to provide the movement is $T_1 = 655$ *Nm* and $T_2 = 639$ *Nm*.

The corresponding torque profile is presented in Fig. 6.

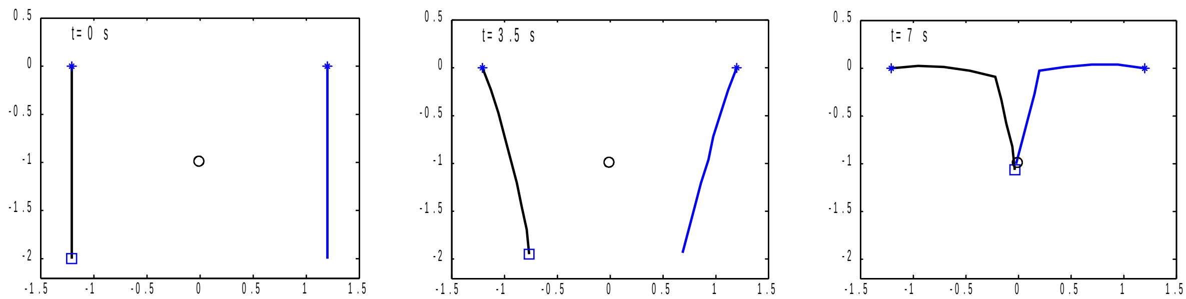
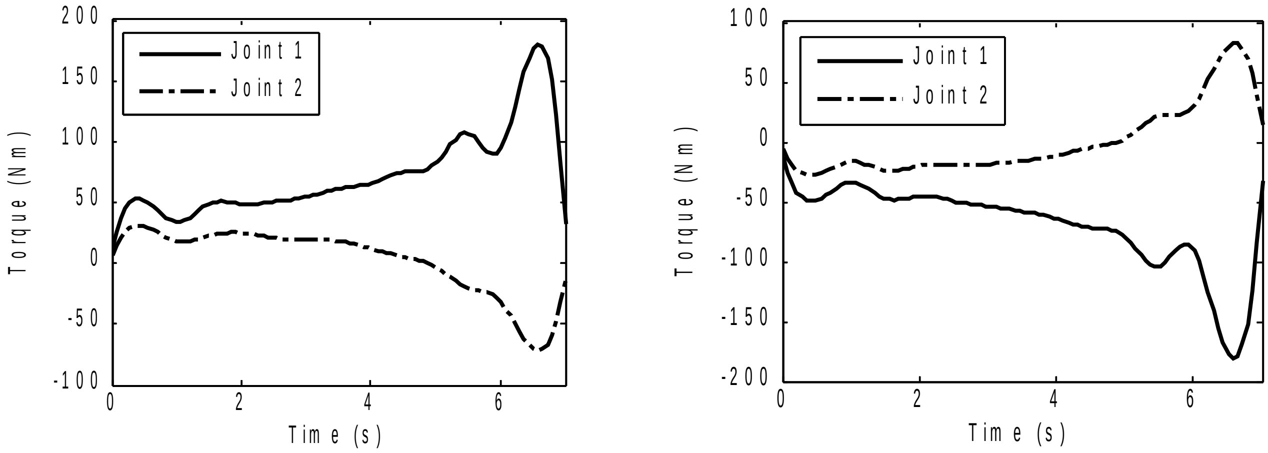


Figure 5. Robot movement given by optimal torque profile.



(a) (b)

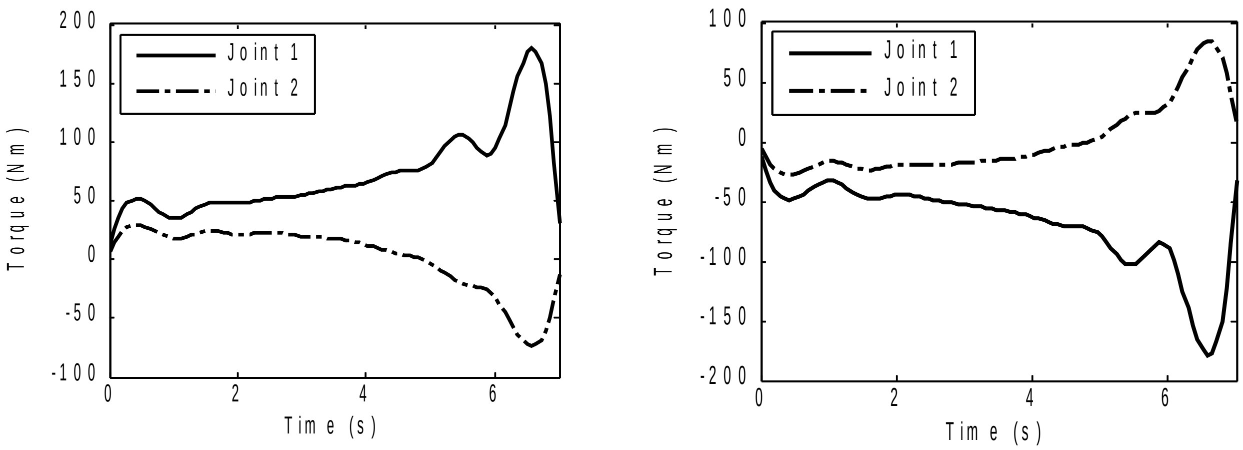
Figure 6. A choice of torque profile that lead (a) robot 1 and (b) robot 2 end-effectors to target position.

Since a feasible control design was found, in the second experiment the ability to optimize the torque consumption while positioning the end-effector is explored. For this experiment, the control variable is also set to $\mathbf{u}_1(t) = 1.0$ *Nm* and $\mathbf{u}_2(t) = -1.0$ *Nm*, $t_0 \leq t \leq t_f$, (shown in Fig. 3) as initial guess to the minimization of the objective function J_2 (Eq. (16)). A remark about this option is that no further information about the torque profile is required to perform the proposed methodology.

The weighting factors are necessary to correctly consider objectives with different magnitudes in the same scalar function. They where chosen in such way that each objective have a unitary value, i. e., $J_2(\mathbf{u}(t_0)) = 1+1+1+1 = 4$ (see Eq. (16)). Alternatively, a different choice can be made.

By performing the optimization of the performance index, the end-effector is successful placed at the target position. The distance between end-effectors of robot 1, robot 2 and target are $d_1 = 0.011$ *m* and $d_2 = 0.013$ *m*, respectively. The total torque required to provide the movement is $T_1 = 652$ *Nm* and $T_2 = 633$ *Nm*.

The corresponding torque profile is presented in Fig. 7.



(a) (b)

Figure 7. A choice of torque profile that lead (a) robot 1 and (b) robot 2 end-effectors to target position with minimal torque.

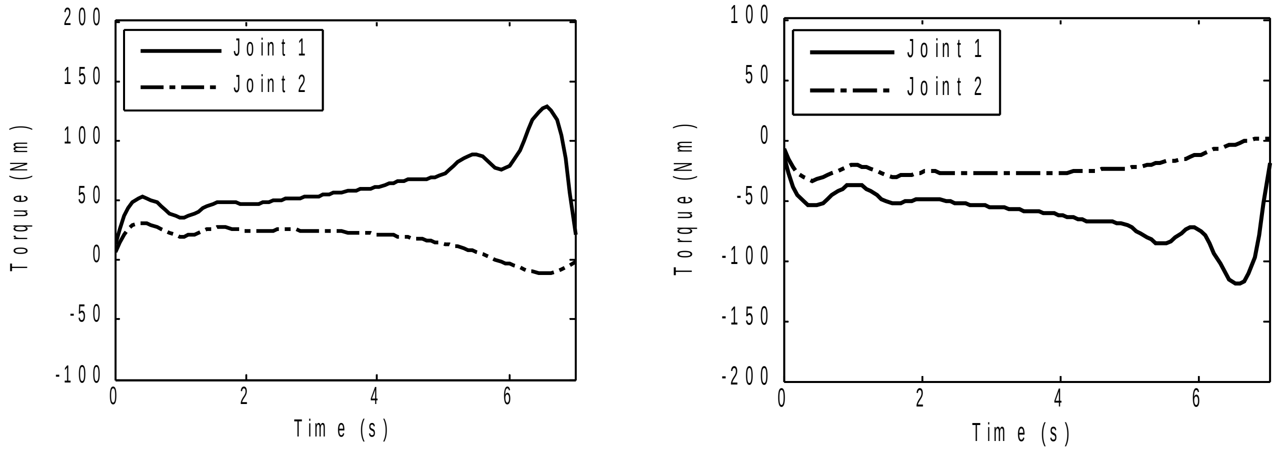
The optimal design has small changes when compared with those provided in the preceding example. In agreement with the multicriteria optimization theory, in particular with Pareto Optimality concept (Eschenauer *et al.*, 1990), changes in weighting factor may lead to different results.

The last experiment consists in analyze the required torque and end-effector positioning by changing the target placement. Now, the optimal position $\mathbf{p}_t$ is also a design variable, as expressed by J_3 (Eq. 16). Upper and lower bounds of the control are $\mathbf{u}_U(t) = 200$ *Nm* and $\mathbf{u}_L(t) = -200$ *Nm*, $t_0 \leq t \leq t_f$. The control variable is set to $\mathbf{u}_1(t) = 1.0$ *Nm* and

$\mathbf{u}_2(t) = -1.0$ *Nm*, $t_0 \leq t \leq t_f$, as in the preceding cases (Fig. 3). The weighting values where chosen in such way that each objective have a unitary value, i.e., $J_3(\mathbf{u}(t_0)) = 1+1+1 = 3$ (see Eq. (16)).

By performing the optimization of the performance index, a new target position established by the optimization process is $\mathbf{p}_t = (-0.03, -1.54)$ *m*, were the end-effector is successful placed. The distance between end-effectors of robot 1 and robot 2 is $d_{12} = 0.006$ *m*. The total torque required to provide the movement was successful decreased to $T_1 = 566$ *Nm* and $T_2 = 591$ *Nm*, respectively.

The corresponding torque profile is presented in Fig. 8. The robot movement is shown in Fig. 9.



(a) (b)

Figure 8. The optimal torque profile that lead (a) robot 1 and (b) robot 2 end-effectors to optimal target position.

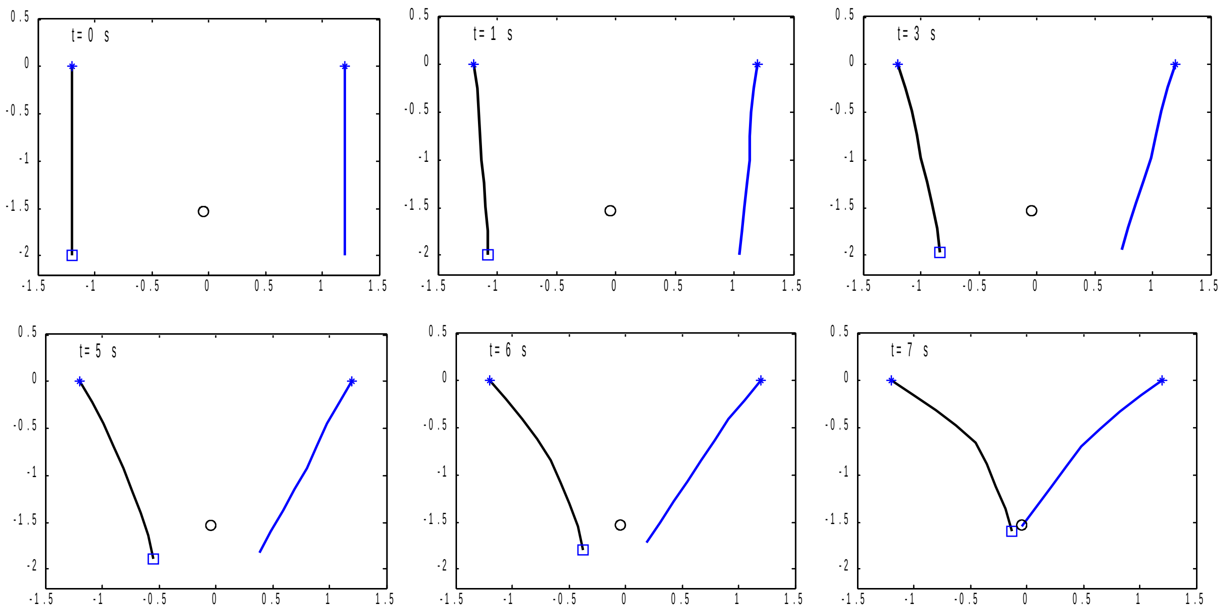


Figure 9. Optimal robot movement at several time instants.

5. CONCLUSION

In this contribution the path planning of cooperative flexible manipulators was addressed. The use of flexible manipulators is an interesting option that can reduce significantly the power consumption required to perform a given task. However, controlling such systems is not simple.

Initially, a short review about flexible manipulators and their areas of application was presented. Different approaches proposed in the literature to describe the kinematics and dynamics behavior of flexible robots were revisited. A formulation inspired in a spring-mass-damper system to represent a flexible link was addressed. This approach is justified by the interesting ratio between description accuracy of the system and computational cost of the analysis.

Following, different optimization objectives were considered. In the field of Optimal Control, each element position and velocity was chosen as state vector elements and the joint torques are the control vector elements. The first objective function was to find a feasible design, that is, a torque profile that leads each end-effector to the same target position. The second objective was optimizing the torque requirement while positioning each end-effector at the same (fixed) target position. The third objective was optimizing the target placement and torque requirement while positioning both the end-effectors at a common point. Each objective function was defined through a time-continuous optimal control formulation and solved by a nonlinear programming method.

Numerical experiments concerning two cooperative and flexible two-link planar manipulators were presented. The first numerical example demonstrates the viability in achieving a feasible design by using the proposed formulation. Due to the nonlinear nature of the formulation, various optimal designs can be found if different initial guesses are given. The use of a box-constrained nonlinear formulation ensures that each torque do not exceed a specified limit. The second numerical example extends the previous analysis, as the optimal torque profile is now also taken into account in the multicriteria optimization problem. The ability in reducing the torque requirement was shown in this numerical experiment. The improvement rate may differ according to the manipulator, task position and payload involved in the process. However, in an industrial environment where the robot repeats the same movement several times in each work cycle, even a small performance improvement can lead to important advantages. The effectiveness of target placement and torque optimization as a strategy to improve the overall performance of the system was shown in the third numerical example. This experiment illustrates the need to perform such an analysis when changes in the target position are allowed. This formulation also contemplates the case in which some manipulators may interact at the same time with a previously placed object, providing the best positioning for the task.

From a practical perspective, the present methodology can be used to define guidelines to several applications that require collaborative flexible manipulators.

A perspective of future development of this research consists in exploring necessary and sufficient conditions to achieve a global minimum and also to perform an analysis regarding the stability of the solution when different numerical methodologies are considered.

As the proposed methodology is efficient to obtain an improved manipulator trajectory while dealing with the flexibility effect and target placement, and is affordable from the computational point of view, the authors believe that it is a useful tool for robotic path planning design.

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