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A variational boundary element approach for strain gradient elasticity

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ABSTRACT

The mathematical modeling of microdevices, in which structure and microstructure have approximately the same scale of magnitude, as well as of macrostructures of markedly granular or crystal nature (microcomposites), demands a nonlocal approach for strains and stresses. The present paper starts from Mindlin's theory of the strain gradient theory, based on three additional constants for homogeneous materials (besides the Lamé's constants), to arrive at a proposition made by Aifantis with just one additional parameter – and subsequently applied mainly by Beskos and collaborators in the context of a boundary element formulation. It is shown that a hybrid variational approach – as proposed by Pian and generalized by Dumont for finite and boundary elements – provides the natural conceptual framework to properly deal with the interelement compatibility of the normal displacement gradients, in which "corner nodes" are not an issue. Nonsingular fundamental solutions – domain interpolation functions – are presented for two-dimensional (2D) and three-dimensional (3D) problems, with the generation of families of finite elements that may be implemented in a straightforward way. The singular fundamental solutions needed in a boundary element formulation are rederived and conceptually assessed. Since the experimental data available in the technical literature are still scarce and the numerical results are mostly questionable, consistency is assessed by means of patch tests and by investigating the spectral properties of the matrices derived for some 2D plane strain application problems, as well as for the simple truss element. It is also shown that the solutions available in the literature for bending and torsion of rods are not free from inconsistencies. New formulations for these degenerated elasticity cases are proposed.

Keywords: Gradient elasticity, variational methods, Hellinger-Reissner potential, hybrid boundary element method