FAULT DETECTION USING STATE OBSERVERS WITH UNKNOWN INPUT, IDENTIFIED BY ORTHOGONAL FUNCTIONS AND PI OBSERVERS.

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Abstract *-In the present work a methodology of fault analysis in mechanical systems has been developed using Kalman Filter state observers, in which, the input to the observers are identified by Fourier, Legendre and Chebyshev orthogonal functions. Once the proportional-integral observer is presented to the unknown inputs, this observer is able to identify the inputs to the system and these are used to the fault detection by means of Kalman filter observer. Here the methodology of parameters and force identification can be seen using only the response of the system thought orthogonal functions. The methodology developed is applied on a composed structure of the assembled using the shake tables in the laboratory.*

1. Introduction

With the increase in production process, there is greater demand from industries for machines and equipment capable of executing more operations working in round the clock. As well as submitted the high dynamic forces. Usually these mechanisms are very expensive and therefore one of the major concerns of industry is to keep its equipment functioning without on necessary breakdowns. With this constant concern, in lately, development of new techniques has been verified for detection and localization of faults in mechanical systems submitted to dynamic loads. In order to guarantee continuous operation mechanical systems must be supervised and monitored so that the faults are diagnosed and repaired as fast as possible, if not so the disturbance in normal operation can to take to the deterioration of the system performance or even to the dangerous situations. Robust observers can reconstruct the unmeasured or estimate the motion of the system that can not be measured directly. Therefore, faults can be detected in the system by being able to monitor them through the reconstruction of the states. The existing methodologies employ state observers are usually used in control problems and detection of possible faults in sensors and instruments. The present work the state observers are used to fault detection in mechanical systems, using orthogonal functions or Proportional and Integral (PI) observer to estimate the unknown inputs. The Kalman filter observer is employed for the location and quantification of the faults. In previous works the identification of the faults using only state observers has been possible with the previous knowledge of the inputs; in this work the unknown input will be found using the orthogonal functions or PI observers.

Methods of identification of forces or parameters, with the objective of diagnosis of faults in mechanical systems, using orthogonals functions, have been developed since the end of 80's until the current days. These methods which have started with with Chun (1987), Melo and Steffen (1993) used the Fourier series for the identification of the structural parameters, and developed the inverse methodology for the identification of the forces. Pacheco (2001), in his doctoral thesis, employed some orthogonals functions for parameters identification through the comparisons between the functions. Pacheco and Steffen (2003) published a work where the orthogonals functions were used for identification of parameters in non linear systems. Melo (2004) studied the behavior of the error found in the identification of the parameters varying the number of terms of expansion of the orthogonal functions for some functions (Pezerat and Guyader, 2000). In recent work, Melo and Morais (2005a), had as objective to identify the forces and the parameters of the mechanical systems together, in the previos described works, the identification of the parameters alone was possible with the previous knowledge of the inputs.

It is physical and economically unfeasible, in some control systems, for transducers to be placed to measure all the variables of state. When analyzing the methodology of state observers, it has been found that some possess the capacity to reconstruct the inaccessible states. However, the necessary condition for this reconstruction is that the states are observable, Luenberger (1964), Luenberger, (1971) and Marano (2002). In the observers described by Luenberger the gain is determined through algorithms of allocation of eigenvalues and eigenvectors of the observer matrix under a certain criterion. A careful analysis must be made because not only that the speed of estimation, determined for the eigenvalues, is not very great but that sensitivity to the noise in the sensor is not very large also. This type of observer corresponds to a deterministic observer. Of course, the problem of the noise in the sensor leads stochastic observers who not only handle better the noise in the sensor (Muscolino, Cacciola and Impollonia, 2003), but are also characterized by having a gain that is optimized under a certain criterion, as it will be seen ahead. That optimized observer, or stochastic observer, is known as Kalman-Bucy (KF) filter (Valer, 1999 and). The filter of Kalman has demonstrated to be useful in many applications (Melo and Morais, 2005b), however, the interest here is in its application to faults detection.

2. Orthogonals Functions

A set of real functions $\phi_k(t), k = 1, 2, 3...$ is said to be orthonormal in the interval $[a,b] \in \Re$, if:

$$\int_a^b \varphi_m(t)\varphi_n(t)d(t) = \delta_{mn} \qquad \text{Where: } \begin{cases} \delta_{mn} = 0 \Rightarrow m \neq n \\ \delta_{mn} = 1 \Rightarrow m = n \end{cases} \tag{1}$$

Where δ_{mn} is the Kronecker Delta.

If a function $f(t)$ is continuous or partially continuous in the interval $[a,b]$, then $f(t)$ can be expanded in series of orthonormal functions, as follows:

$$f(t) = \sum_{n=1}^{\infty} c_n \phi_n(t) \tag{2}$$

Such series, called orthonormal, constitute generalizations of the Fourier series. Admitting that the sum in Eq. (2) converges to $f(t)$, we can multiply both members for $\phi_m(t)$ and integrate them in the interval $[a,b]$, with c_m as the generalized coefficients of Fourier.

The following property, related to the successive integration of the vector basis:

$$\int_0^t \underbrace{\cdots}_{n\,\text{times}} \int_0^t \{\phi(\tau)\}(d\tau) \cong [P]^n \{\phi(t)\} \tag{3}$$

Where $[P] \in \Re^{r,r}$ is a square matrix with constant elements, called operational matrix (Melo and Steffen, 1993), and $\{\phi(t)\} = \{\phi_0(t)\ \phi_1(t)\ ...\ \phi_r(t)\}^T$ is the vector basis of the orthonormal series. In fact, if a complete vector base is regarded, or is other words, if the series are not truncated, the relation ship obtained in Eq. (4) becomes an equality. However, in practice, it becomes not suitable, due to the high order of the matrix $[P]$ obtained. In the following sections, the vectorial basis and the operational matrix related to each type of orthogonal function considered in this paper are briefly reviewed (Pacheco, 2000).

Fourier series

Vectorial basis in the interval $[0,T]$	Operational matrix integration
$\{\varphi(t)\}=\{\varphi_0(t) \quad \varphi_1(t) \quad ..\varphi_s(t) \quad \varphi_1^*(t) \quad ..\varphi_s^*(t)\}^T$ $\varphi_n(t)=\cos\frac{2n\pi\, t}{T}$, $n=0,1,2,...,s$ $\varphi_n^*(t)=\sin\frac{2n\pi\, t}{T}$, $n=1,2,...,s$ $r=2s+1$	$[P]=\begin{bmatrix} \frac{T}{2} & \{0\}_{1xs} & -\frac{T}{\pi}\{\tilde{e}\}_s^T \\ \{0\}_{sx1} & [0]_{sxs} & \frac{T}{2\pi}[\tilde{I}]_{sxs} \\ \frac{T}{2\pi}\{\tilde{e}\}_s & -\frac{T}{2\pi}[\tilde{I}]_{sxs} & [0]_{sxs} \end{bmatrix}_{rxr}$ $\{\tilde{e}\}=[1 \quad 1/2 \quad 1/3 \quad \dots \quad 1/s]$ $[\tilde{I}]_{sxs}=diag\,\{1 \quad 1/2 \quad 1/3 \quad \dots \quad 1/s\}$

T = Period of sampling and s = number of terms of Fourier in sines and cosines

Legendre polynomials

Recursive formula on the interval $t\in[0,\mathrm{t_f}]$	Operational matrix integration
$(n+1)p_{n+1}(t)=(2n+1)\left(\frac{2t}{t_f}-1\right)p_n(t)-np_{n-1}(t)$ $,\mathrm{n}=1,2,3,\dots,r-1$ $p_0(t)=1$ $p_1(t)=2t/t_f-1$	$[P]=\frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \cdots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{-1}{2r-3} & 0 & \frac{1}{2r-3} \\ 0 & 0 & \cdots & 0 & \frac{-1}{2r-1} & 0 \end{bmatrix}$

r = number of terms truncated

Chebyshev polynomials

Recursive formula on the interval $t\in[0,\mathrm{t_f}]$	Operational matrix integration
$T_{i+1}(t)=2\left(\frac{2t}{t_f}-1\right)T_i(t)-T_{i-1}(t)$ $i=1,2,\dots,r-1$ $T_0(t)=1$ $T_1(t)=\frac{2t}{t_f}-1$	$[P]=\frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1/4 & 0 & 1/4 & 0 & \dots & 0 & 0 & 0 \\ -1/3 & -1/2 & 0 & 1/6 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{(-1)^{r-1}}{(r-1)(r-3)} & 0 & 0 & 0 & \dots & \frac{-1}{2(r-3)} & 0 & \frac{1}{2(r-1)} \\ \frac{(-1)^r}{r(r-2)} & 0 & 0 & 0 & \dots & 0 & \frac{-1}{2(r-2)} & 0 \end{bmatrix}$

r = number of terms truncated

2.1. Identification of mechanical systems through orthogonal functions

The proposed identification method can be exploited either on the free or forced time domain responses, as functions of displacements, velocities or accelerations. Since the formulations for these three kinds of responses are quite similar (Melo and Morais, 2004), only the formulation for forced systems in terms of displacements will be presented.

The development of the method starts with the equation of motion of a forced mechanical system of *N* degrees of freedom:

$$[M]\left\{\ddot{x}(t)\right\}+[C]\left\{\dot{x}(t)\right\}+[K]\{x(t)\}=\{f(\mathrm{t})\} \tag{4}$$

Where [M], [C] and [K] are the inertia, damping and stiffness N-order matrices respectively; $\{x(t)\}$ is the vector of displacement time responses and $\{f(t)\}$ is the vector of exciting forces.

Integrating Eq. (4) twice in the interval [0,t], it becomes:

$$\begin{aligned}&[M]\left(\{x(t)\}-\{x(0)\}-\{\dot{x}(0)\}t\right)+[C]\left(\int_0^t\{x(\tau)\}d\tau-\{x(0)\}t\right)+[K]\int_0^t\int_0^t\{x(\tau)\}d\tau^2=\\&=\int_0^t\int_0^t\{f(\tau)\}d\tau^2\end{aligned} \tag{5}$$

The signals $\{x(t)\}$ and $\{f(t)\}$ can be expanded in the truncated series of *r* orthogonal functions as follows:

$$\{x(t)\}=[X]\{\phi(t)\} \qquad \text{and } \{f(t)\}=[F]\{\phi(t)\} \tag{6}$$

Where: $[X]\in\Re^{N,r}$ is the matrix of the coefficients of expansion of $\{x(t)\}$

$[F]\in\Re^{N,r}$ Is the matrix of the coefficients of expansion of $\{f(t)\}$

Substituting Eq. (6) in Eq. (5) and applying the integral property given by Eq. (3), the following system of algebraic equations can be obtained, (Melo and Morais, 2005a).

$$\begin{bmatrix}[\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \left\{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\right\} & [\mathrm{C}] & [\mathrm{K}]\end{bmatrix}\begin{bmatrix}[X]\\ \{e\}^T\\ \{e\}^T[P]\\ [X][P]\\ [X][P]^2\end{bmatrix}=[F][P]^2 \tag{7}$$

Eq.(7) can be rewritten as

$$\begin{bmatrix} [F] & [M]\{x(0)\} & [M]\left\{\dot{x}(0)\right\} + [C]\{x(0)\} \end{bmatrix} \begin{bmatrix} [P] \\ \{e\}^T \\ \{e\}^T [P] \end{bmatrix} = [M][X] + [C][X][P] + [K][X][P]^2 \tag{8}$$

And so, the Eq. (7) and the Eq. (8) can be represented as:

$$[H][J] = [E] \tag{9}$$

Identifying H to the Eq. (8) we can to determine the structural parameters of the system. And doing the same to the Eq. (9) we can determine the system inputs.

3. General structure of the State Observer: Kalman Filter

Considering a linear system, invariant and observable in the time:

$$\bar{S} : \begin{cases} \dot{x}(t) = A\, x(t) + B\, u(t) + L\, \xi(t) \\ y(t) = C\, x(t) + \eta(t) \end{cases} \tag{10}$$

Where: *x(t)* is the state vector *n x 1*, *u(t)* is the input vector *p x 1*, *y(t)* is the output vector *k x 1*, *A* is the matrix of system *n x n* (dynamic matrix), *B* is the matrix of distribution *n x p* (matrix of inputs), *C* is the matrix of measures *k x n*, being *n* the order of the system, *p* the number of inputs u(t), and *k* the number of outputs *y(t)*. The vector ξ is the noise of excitement in the state and represents a disturbance in the system and the vector η is called noise in the sensor, (Inouye and Suga, 1999). Due to the stochastic nature of the vectors ξ and η, in the Kalman Filter, they have certain statistical properties, corresponding to the white Gaussian noise, stationary (invariant in the time) and not correlated between themselves. Now we can define the matrices Ξ and Θ, called intensity of the noise ξ and η, respectively, symmetrical and defined positive:

$$\Xi = \Xi^T \geq 0,\ \Theta = \Theta^T > 0 \tag{11}$$

Given the assumptions above, the problem of optimum estimate of the state vector x in presence of white noises (as vectors of state as the measured variable) can be formulated to find the optimum value (filter of Kalman) that generates an estimate $\overline{x}$ for the real state vector x, so that minimizes the covariance of the error estimation (eq.12) is minimized:

$$e(t) = \overline{x}(t) - x(t) \tag{12}$$

$$\Im_{KF} = \mathrm{E}\left[e(t)e^{T}(t)\right] \tag{13}$$

Kalman and Bucy have proved that the best structure for the Kalman filter (among all the possible structures, linear and nonlinear) when the dynamics of the system is linear and the noises are white and Gaussian, amounts to be:

$$\bar{S}_{KF} : \left\{\dot{\bar{x}}(t) = A\ \bar{x}(t) + B\ u(t) + \mathrm{K}_{KF}\ \left(y(t) - C\ \bar{x}(t)\right)\right. \tag{14}$$

In which K_{KF} is the matrix of the state observer, $\{\bar{x}(t)\}$ is the state vector of the observer.

3.1. Filter Algebraic Riccati Equation (Fare)

The solution of the optimization problem can be found in literature. Since in the present work the main interest is the application of the control methodologies, we present it here without any proof. The optimum gain K_{KF} for the Kalman filter is given by the following relation:

$$\mathrm{K}_{KF} = S_{KF} C^{T} \Theta^{-1} \tag{15}$$

In which S_{KF} is defined like a symmetrical and positive matrix satisfying the Riccati equation for the Kalman filter (FARE):

$$S_{KF} A^{T} + A\ S_{KF} + L\ \Xi\ L^{T} - S_{KF} C^{T} \Theta^{-1} C\ S_{KF} = 0 \tag{16}$$

4. Method of the State Observers with unknown inputs

The state observers where all the inputs of the system must be known and available have some utility in the case of only one input to the control system. For the cases of unknown inputs or disturbances which cannot be measured or its measurement is very difficult or simply impossible, the performance of the observer can very be poor. In this work a methodology of diagnose of faults has been developed using observers of state in which its input is considered unknown or partially known in which the proportional and integral observer is used to estimate the unknown inputs, and, the gain of this observer is determined by the gain given by the Kalman Filter. After the identification of the unknown inputs these are used for the detection of possible faults that are occurring in the systems. For this, the Kalman Filter was used to generate unknown states.

A very convenient representation for systems with these characteristics is as indicated by the following equation:

$$S:\begin{cases}\dot{x}(t)=A\,x(t)+B\,u(t)+B_d\,v_d(t)\\ y(t)=C\,x(t)\end{cases} \tag{17}$$

In which: $x(t)$ is a state vector *n x 1*, $u(t)$ is a input vector *r x 1*, $y(t)$ is a output vector *m x 1*, $v_d(t)$ is a vector of disturbance or unknown input *p x 1*, A is a matrix of system *n x n* (dynamic matrix), B is a matrix of distribution *n x r* (matrix of input), C is the matrix of measures *m x n* and B_d is the matrix distribution of disturbance *p x n*, being *n* the order of the system, *r* the number of inputs *u(t)*, *m* the number of outputs *y(t)* and *p* the number of disturbance $v_d\,(t)$.

The problem of state estimation of a linear and invariant the system with both known and unknown inputs has been subject of researching during the last decades, deserving considerable importance, because in real systems, there are many situations where the disturbance are present or some inputs are inaccessible, precluding the use of conventional observers in which all the inputs are known. Therefore, an observer capable of estimate the state for linear system with partially unknown inputs, not sensible to disturbance, can be of great utility.

The main idea is designing an observer to estimate the disturbance v_d. The Fig. 1 suggests the function of this observer.

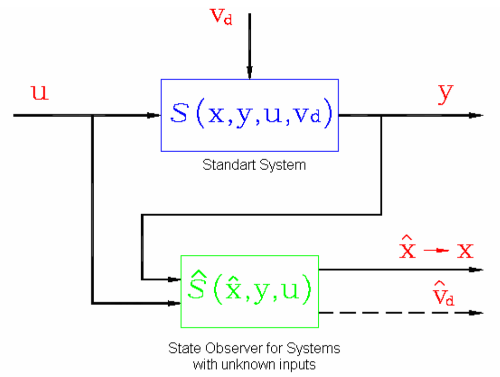

Figure 1: Observer with unknown inputs

4.1. Modeling of the Observer with Unknown Inputs

According this approach, we verify that the dynamics of the disturbance vector satisfies the following differential equation:

$$v_d(t)=C_d\,w(t) \tag{18}$$

$$\dot{w}(t)=A_d\,w(t) \tag{19}$$

In which w represents the disturbance state in the matrix A_d and the matrix C_d indicates if the disturbance is dependent of the state. The choice for these matrices depends on the kind of disturbance. For example, in the case where the disturbance v_d *is* constant, a convenient choice this is that matrix $A_d = 0$ and $C_d = I$ (I is a identity matrix). Arranging the Eq. (17) with (18) and (19) we get augmented:

$$S_a : \begin{cases} \dot{x}_a(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v_d(t) \end{bmatrix} \end{cases} \tag{20}$$

It has been verified in the equation above that w is not controllable through of u. But, in general, it is observable (Valer, 1999) and with this, it is possible to design an observer for this system that estimate both variables x and w. Thus, an observer of full order for this new system will be:

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{w}}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} (y(t) - C\,\hat{x}(t)) \tag{21}$$

In which the matrix $K = \begin{bmatrix} K_1^T & K_2^T \end{bmatrix}^T$ has been added to guarantee stability for the observer. In full equation, we have:

$$\hat{S}_{ed/ds} : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_d C_d \hat{w}(t) + K_1(y(t) - C\hat{x}(t)) \\ \dot{\hat{w}}(t) = A_d \hat{w}(t) + K_2(y(t) - C\hat{x}(t)) \end{cases} \tag{22}$$

4.2. Proportional-Integral Observer

When the disturbance spectrum does not contain high frequencies, the observer in the section 4.1 can be used considering $A_d = 0$ and $C_d = I$ getting a simplification in the model. In this case the part corresponding to the estimation the disturbance vector becomes a bank of integrators and the part corresponding to the estimation of the state vector becomes in proportional and integral to the residual: $y(t) - C\hat{x}(t)$. This observer is called proportional-integral or PI and has superior properties whom compared with the full-order proportional observer. The proportional-integral observer is capable of estimate any disturbance (constant, linear and nonlinear) but it has to be slower than the time constant of integral action and the number of measurements can not be less that the number of disturbance causing. By increasing the integral gain it is possible to reject the faster disturbances, however, the negative effect of decreasing the stability of the observer. Using the Eq. (22), we have for the case of proportional-integral observer:

$$\hat{S}_{pi}:\begin{cases}\dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t)+B_d v_d(t)+K_P(y(t)-C\hat{x}(t))\\ \dot{\hat{v}}_d(t)=K_I(y(t)-C\hat{x}(t))\end{cases} \tag{23}$$

Or equivalent:

$$\hat{S}_{pi}:\left\{\dot{\hat{x}}_a(t)=A_a\hat{x}_a(t)+B_a u(t)+K_a(y(t)-C_a\hat{x}_a(t))\right. \tag{24}$$

In which: $\hat{x}_a=\begin{bmatrix}\hat{x}\\ v_d\end{bmatrix}$, $A_a=\begin{bmatrix}A & B_d\\ 0 & 0\end{bmatrix}$, $B_a=\begin{bmatrix}B\\ 0\end{bmatrix}$, $C_a=\begin{bmatrix}C & 0\end{bmatrix}$, $K_a=\begin{bmatrix}K_P\\ K_I\end{bmatrix}$

The necessary and sufficient condition for the existence of the observer is that the pair (A_a, C_a) is at the least, observable. Thus it is possible where the eigenvalues of the following matrix onto the complex plan:

$$\hat{A}_a=A_a-K_aC_a=\begin{bmatrix}A-K_PC & B_d\\ -K_IC & 0\end{bmatrix} \tag{25}$$

In this work the gain of observer PI is determined by the gain acquired from Kalman Filter presented in section (3.1).

4.3. Example

In this section an example is presented of the determination of an unknown input in a robotic arm as shown in Fig. 2.

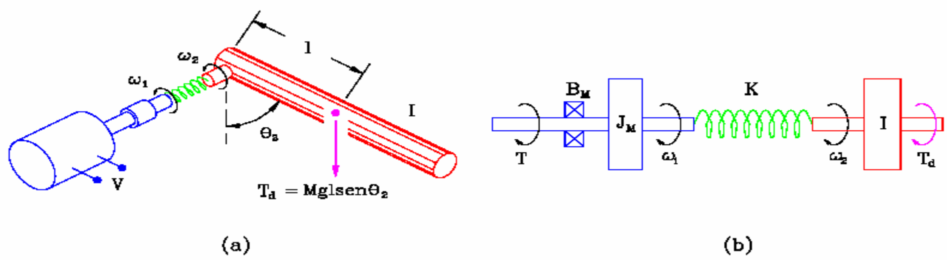

Figure 2: flexible arm of a robot with unknown input (disturbance) from the weight.

One mathematical model can be represented by the state equation, Eq. (17), in which the matrices are given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/J_M & K/J_M & -B_M/J_M & 0 \\ K/I & -K/I & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ K_e/J_M \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } x = [\theta_1 \quad \theta_2 \quad \omega_1 \quad \omega_2]^T$$

In which:

$\theta_1(t)$: Angular displacement of the robot arm ($\theta_1(0)$ =15°)

$\theta_2(t)$: Angular displacement in the output of the reduction box ($\theta_2(0)$ =15°).

$\omega_1(t)$: Angular speed of the robot arm ($\omega_1(0)$ =0).

$\omega_2(t)$: Speed in the output of the reduction box ($\omega_2(0)$ =0).

V: Voltage of armour of motor DC (5 V and 3 *rad/s square shaped wave).*

I: Inertia of the arm robot (= 0.4Kg *m2).*

K: Torsional stiffness of the spring (= 1 *N m/rad).*

J_M: Inertia equivalent of the motor including reduction box (= 0,0424 *kg m2*)

B_M: Viscous friction in the motor (= 0,0138 *N m s/rad).*

K_e: Momentum gain for the motor (= 0,0403 *N m/V*).

Simulating the system the Runge Kutta method has been employed, in which it is considered as unknown output a nonlinear force from the weight of the arm and equal to T_d = Mgl sin(θ_2) with M = 1 *Kg*, g = 9,8 *Kg ms* $^{-2}$ and l = 0,3 *m*. For the PI observer, the nonlinear force is considered as being an interferential input to the system. In the Fig. 3 the real input and the estimated by the observer are presented.

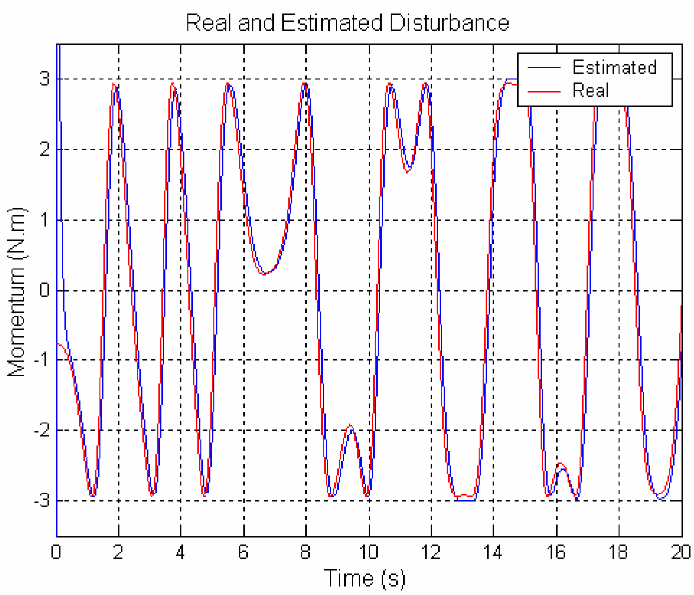

Figure 3: Unknown input estimated through PI observer

4.4. Design of State Observers

The design of a system is presented in the Fig. 4 working along side the state observers, including the known excitement force *u(t)*, the unknown inputs v_d *(t)*, the measured outputs *y(t)*, the PI observers used to identify the unknown input, the global and robust observers to the parameters subject to faults s_1…, s_n and a logical decision unit. The global state observer is responsible for the detection of the fault, while the robust state observer is responsible for their location. The global observer is a copy of the original system, and analyzes all the system searching for possible faults. The robust state observer can detect the fault if it occurs in the parameter for which it was designed. We have to design a bank of robust observers, each one in relationship to a parameter to be monitored, in order to obtain a good location of the fault.

When the system is operating adequately, without indications of faults, the global state observer answers equal the real system. When one component of the system in focus starts to fail, the state observer immediately feels the influence of this process. The global and robust observers are modeled, in this work, using the methodology of the Kalman Filter for its good behavior under noise in the system. They are included in a bank of observers and the RMS values of the differences between the real displacement (measured) signals and the ones generated by the observers are analyzed in a logical decision unit that evaluates progression trend of the fault and sets in motion, if necessary, an alarm system. The alarm system can also be triggered under a parameter variation and this is on line process.

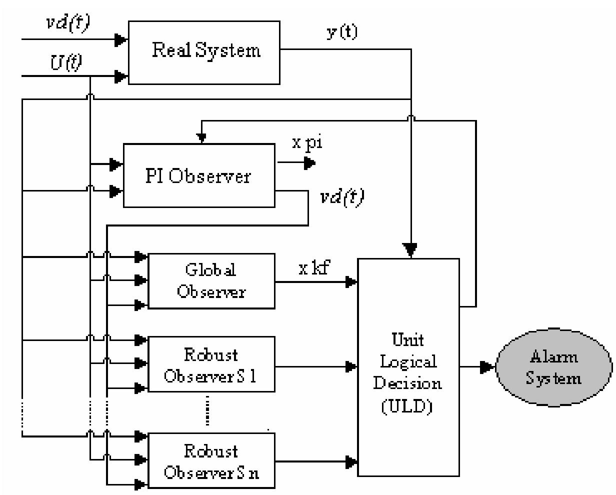

Figure 4: System of Robust Observation.

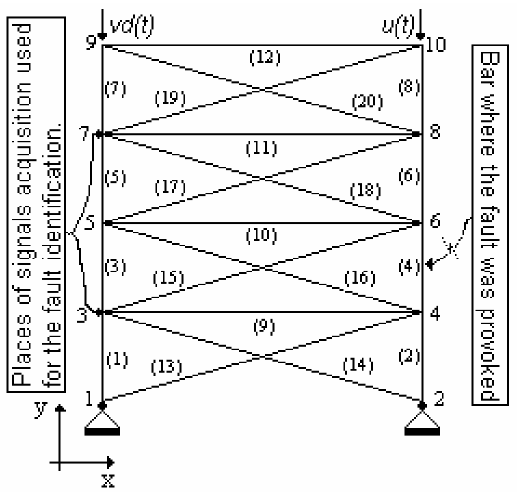

Figure 5- Truss Structure with 20 bars

5. Simulation and results for a 20-bars truss structure

To validate the methodology of identification and location of faults applied to the mechanical systems using the state observer, Kalman filter, with unknown forces identified through PI observers, has been simulated a 20-bars truss structure as shown in the Fig. 5. For this, we used the finite elements method to be able to simulate the structure, in which each bar represents a composite element comprising two joints and each joint having two degrees of freedom (d.o.f.), the x and y displacements. Considering that the structure has restrictions in the joints 1 and 2, we have a system with 16 d.o.f., as shown in the Fig. 5. The system was excited in the join 9 and 10 in the direction of y with 300N and 500N harmonic forces and 250 rad/s and 3700 rad/s frequencies, in that order. The force applied in join 9 is considered unknown and will be determined by the PI observer, as to be seen in the Fig. 7.

All the elements that compose the truss are isoperimetric with the following properties: ρ = 7850 kg/m3, E = 200GPa, height = 2.0cm, width = 3.0cm. All the bars in the x direction are 2.0m long and in the y direction is 0.5m long. During the simulation considered a low proportional damping for the matrix of mass and the stiffness of the system given by: *C=1.0e-10*K+1.0e-04*M*. The output of this system was evaluated through the fourth order method of Runge-Kutta with 4096 points in the interval of 1.0s. We used the output in x-direction (joints 3 and 7) and a 30% reduction in the area of bar 4 to simulate the fault. To validate the robustness of the Kalman filter in the presence of noises in the signals, added, to the input, a white noise has been added with energy, 5% of the value of the energy of the input sign *u(t)*.

A bank of robust observers is generated for the parameters subjected to the faults with 10% variation in the area of each bar. It has been considered, in this work, that all the bars of the system are susceptive to the occurrence of a possible fault. Fig. 7 depicts the inverse values of RMS differences found between the “measured” signal in the structure and the signals generated from the global observers (0% of fault) and from the robust observers, reducing in 10% the value of each parameter subject to fault. In Fig. 6 the locating the fault provoked in bar 4 with 30% of reduction in this parameter can be promptly seen.

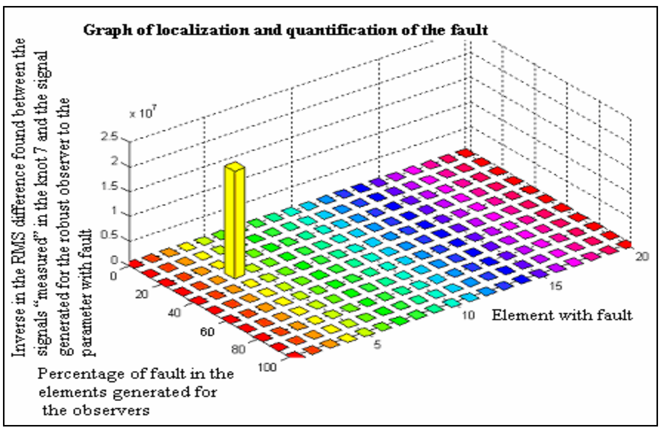

Figure 6: Bank of robust observers generated

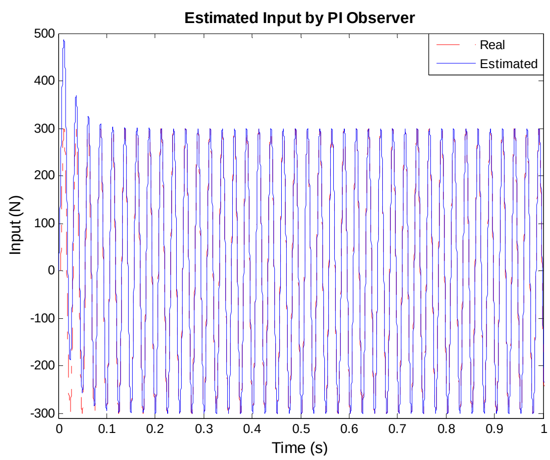

Figure 7: Estimated input by PI observer

6. Experimental Results

A dynamic system constituted of shake tables has been built using metallic stainless steel blades to represent the stiffness of the system; aluminum plates for the tables and rubber to simulate viscous damping. The rubbers are fixed between the blades, as it can be observed in the Fig. 8. The structure has been modeled like a system of three degrees of freedom with discrete parameters. The structural parameters have been determined using techniques of experimental modal analysis. For this, the parameters of mass, damping and stiffness have been evaluated for each table separately. In the Tab. (1) the results are presented. The bottom table has been excited with a harmonic force and the signals have been acquired during 1,0 s and with 2048 points in this interval, using DASYLAB software with four channel for signals acquisition, the three first channels for acquisition of the of displacement signals and the last channel for determination of the excitement force. Has been the signal used as the answers measured by the accelerometers have been integrated two times, using the Bruell´s Nexus Conditioner/Amplifier of signals.

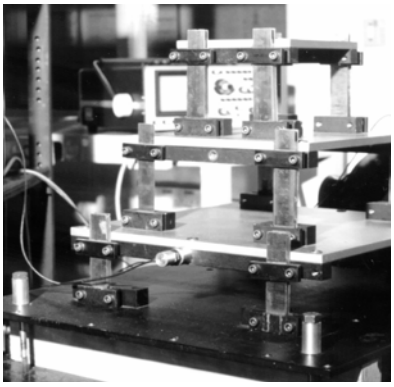

Figure 8: Test Rig-Vibratory System

Table 1: Space parameters identified for the structure from analyze modal experimental classic.

Table	Lower	Intermediate	Upper
M (Kg)	6,644	4,619	1,889
K (KN/m)	275,367	114,489	104,993
C (Ns/m)	100,042	36,360	29,660

Excitement force to the system has been measured to compare and verify the method efficacy. In the Fig. 9 has been measured the inputs estimated by means of the Fourier, Legendre and Chebyshev orthogonal functions are shown. It Has been used during the identification of the inputs 100 terms of expansion, as exposed in the work (Melo *et al*, 2004).

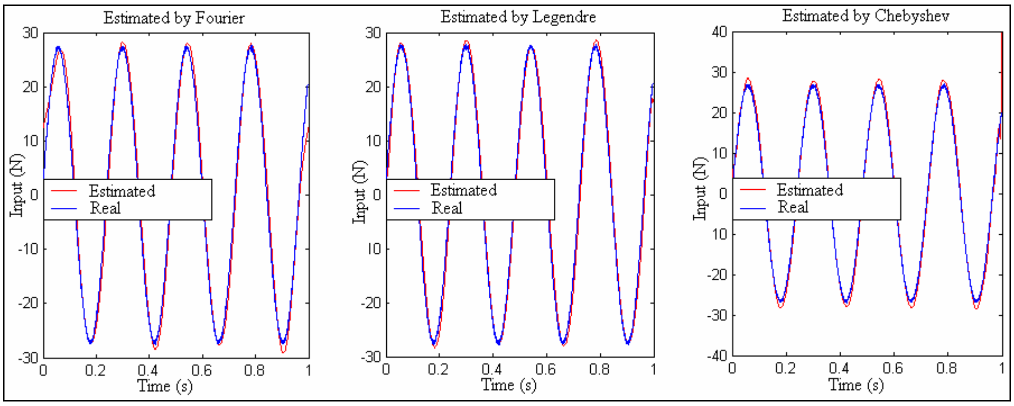

Figure 9: Input identified through Fourier, Legendre and Chebyshev methods, respectively.

For the fault detection only the output of the displacement measured in the lower table, has been used. We initiated the process of identification and location of the fault when a plate of the upper table has been removed and verified a reduction of 8,9% has been found in the stiffness. A bank of robust observers was presented for the parameters subjected to the faults with 1% of variation in the stiffness. In the Fig. 10 the inverse values of differences RMS are presented, found between the measured signal in the structure and the signals generated for the global observers (0% of fault). For the robust observers, we reduced in 1% the value of each parameter subject to fault. In Fig. 10 the fault provoked in the upper table could be located and quantified in the region of 9% of reduction in the stiffness by way the inputs identified through Fourier, Legendre and Chebyshev methods, respectively.

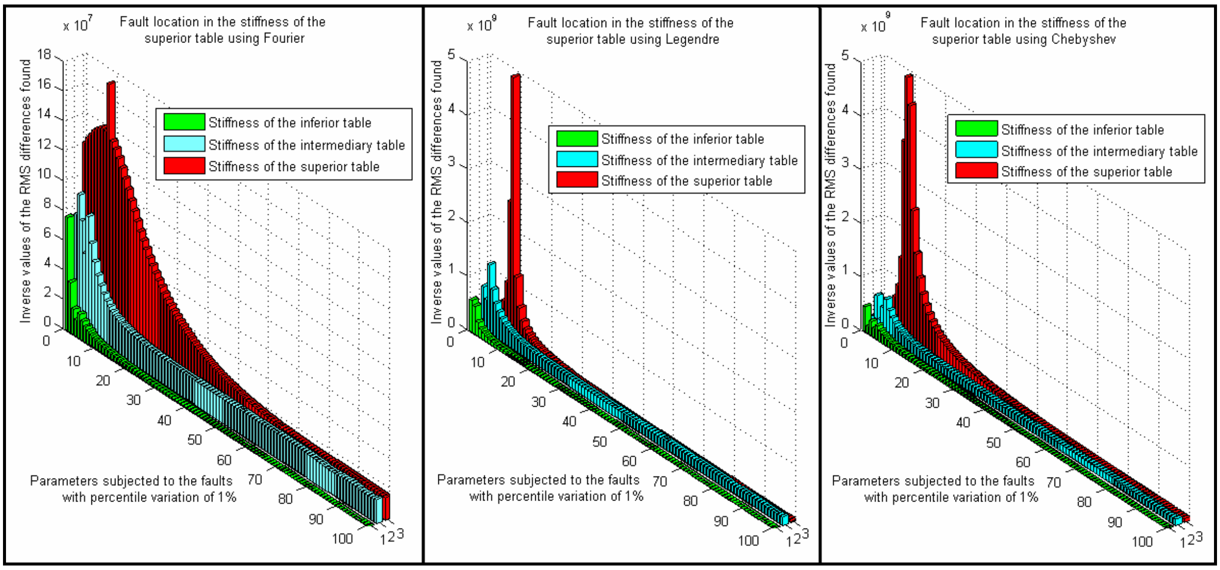

Figure 10: Fault detection and location through Fourier, Legendre and Chebyshev methods.

In according the Figure 10, we had good results obtained during the force and parameters identifications.

7. The last consideration

In the present work has been developed a methodology of diagnosis of faults deploying state observers. The Kalman Filter has been used to the construction of bank of observers, and in this case the observer needs all the inputs known or with white noise because it is the only kind of interference which can be used to design the Kalman filter. The inputs are identified by the means of orthogonal functions or proportional and integral observer. It was presented a robotic arm, in which, it has been possible to identify the external force due the arm mass using PI observer. We used a 20-bars truss structure to detect fault, in which two inputs have been considered, being that one of them unknown and identified by PI observer. The experimental validation of the methodology has been carried through in a simple system of three degrees of freedom, with force estimated using orthogonal functions. The fault was precisely identified for all functions used. Regarding the computational time necessary for the assembly of the bank of robust observers to the parameters subject to faults, this is somewhat high, but actually the bank of state observers is assembled just once, in such way that for the on-line acquisition of signals in a structure, reassembling the bank of observers is not necessary any longer. In fact, thanks to this, we can conclude that method presented is quite suitable processes of on-line detection of faults.

7. References

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