Parameter Estimation Using Digital Image Correlation and Inverse Problems

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Abstract

Digital Image Correlation (DIC) is an optical technique developed for full field and non contact measuring of surface displacement and deformation. This technique requires a digital imaging system to record images of the surfaces before and after deformation. In the present work DIC is used to obtain the displacement field of specific regions of a beam under bending. Such fields are used to obtain the constitutive parameters of this beam. The parameter estimation is performed by means of the minimization of an error function comprising of the difference between the displacement fields obtained from the experiment and from the model. The inverse problem is solved by means of the classical Levenberg-Marquardt nonlinear parameter estimation technique. Due to the fact that there is some level of uncertainty concerning the coordinates of the reference point of the analyzed region it is also considered the estimation of a geometric parameter of the system.

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1. INTRODUCTION

Optical methods, such as shearography, laser speckle interferometry, and moiré were developed for measuring surface displacement and deformation [1,2,3]. All of these methods have been combined with the image processing and electronic systems and developed in commercial scientific instruments. However, some types of equipment based on these optical methods are still very expensive and require stable environment as well as laborious data reduction processes. Recently, with the development of high resolution Charge Coupled Device (CCD) cameras the use of digital image correlation (DIC) method [4,5] has been increased considerably.

The use of inverse problems in experimental mechanics has had an enormous increase in the last decade with applications in different areas such as heat transfer, modal analysis, structural health and monitoring and many others. There has always been a demand to seamlessly integrate experimental information with analytical or numerical models. It is needless to say that such effort for integration has gained an important ally with the advent of experimental techniques that are capable of generating full field displacement, as the presented previously. Basically, the objective in achieving this integration is to estimate the boundary conditions and the characterization of material proprieties [6,7].

The aim of this work is to estimate the elasticity modulus of an aluminum beam using a simple experimental setup. The analysis presented in this work takes into account the digital image correlation method to generate full field displacement and the Levenberg-Marquardt method to solve associated inverse problem. In order to validate the parameter estimation technique based on DIC method it will be considered the problem of estimating the elastic parameters of a free-cantilevered aluminum beam.

2. DIGITAL IMAGE CORRELATION (DIC) METHOD

The DIC method is an optical-numerical full-field surface displacement measuring technique, which is nowadays widely used in experimental mechanics. This technique is based on a comparison between two images of the specimen coated by a random speckle pattern in the underformed and in the deformed state. Its special merits are non-contact measurement, simple optic setup, no special preparation of specimens and no special illumination.

The basic principle of DIC method is to search for the maximum correlation between small zones in the underformed and deformed images, as illustrated in Figure 1, from which the displacement at different positions in the zone of interest can be obtained. The simplest image-matching procedure is the cross-correlation, which can determine the in plane displacement field (u,v) by matching different zones of two images.

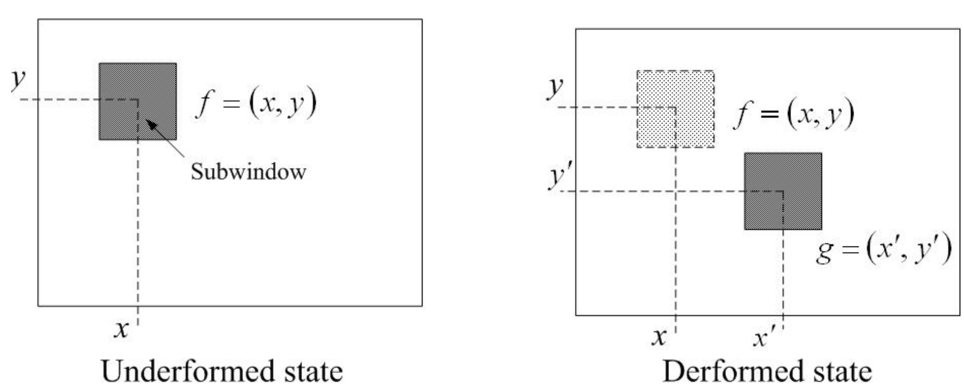


Figure 1. Schematic diagram of the deformation relation.

A commonly used cross-correlation function is defined as below:

$$C(u,v)=\frac{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[f\left(x_i,y_j\right)-\bar{f}\right]\left[g\left(x'_i,y'_j\right)-\bar{g}\right]}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[f\left(x_i,y_j\right)-\bar{f}\right]^2}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[g\left(x'_i,y'_j\right)-\bar{g}\right]^2}} \quad (1)$$

where

$$\begin{aligned} x' &= x+u+\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy \\ y' &= y+v+\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy \end{aligned} \quad (2)$$

$f(x,y)$ is the gray level value at coordinate (x,y) for the underformed or original image and $g(x',y')$is the gray level value at coordinate (x',y') for deformed or target image, $\bar{f}$ and $\bar{g}$ are the average gray values and u and v are the displacement component for the subset centers in the *x* and *y* directions, respectively.

The camera uses a small rectangular piece of silicon, which has been segmented into H x V array of individual light sensitive cell, also known as pixel. Each pixel store a certain gray scale value ranging from 0 to 255, in accordance with the intensity of the light reflected by the surface of specimen.

Relative to specimen preparation, the aim is to create a speckle pattern on the specimen surface, the smaller the grains, the higher the spatial resolution.

The results will depend on CCD pixel resolution, speckle size and DIC software considered.

3. PARAMETER ESTIMATION

In order to fully characterize a mechanical system it is required to estimate a set of unknown parameters which is representative to its dynamics. Therefore, for the sake of simplicity, it is defined a vector $\mathbf{p}$, which contains information concerning all the unknown parameters of a system, as follows

$$\mathbf{p} = \{p_1, p_2, \ldots, p_{N_p}\}^T \tag{3}$$

where N_p corresponds to the number of unknown parameters. In the inverse problem formulation one considers that the set of parameters $\mathbf{p}$ is unknown and that there is available a set of experimental data concerning the response of the system $\mathbf{y}^E(\mathbf{x},t)$ to a certain excitation/stimulus. The basic idea behind the inverse problem formulation is to find the set of parameters $\mathbf{p}$ that best correlates the response $\mathbf{y}(\mathbf{x},t)$, which is obtained from the mathematical model of the system under study, with the experimental response $\mathbf{y}^E(\mathbf{x},t)$, when they are subject to the same excitation/stimulus. Therefore, it is required to define a function S to measure the difference between these two responses $\mathbf{y}^E(\mathbf{x},t)$ and $\mathbf{y}(\mathbf{x},t)$. If one assumes the hypothesis that the measurement errors have zero mean, constant variance, Gaussian distribution and that they are additive and non-correlated, the error function S that provides the minimum variance estimates is the ordinary least squares norm defined as follows [8,9]

$$S(\mathbf{p}) = [\overline{\mathbf{Y}} - \mathbf{Y}]^T [\overline{\mathbf{Y}} - \mathbf{Y}] \tag{4}$$

where $(\bullet)^T$ indicates the transpose of $(\bullet)$ and $\bar{\mathbf{Y}}$ and $\mathbf{Y}$ contain information about the experimental and the estimated responses of the system respectively and are defined as follows

$$\bar{\mathbf{Y}}^T = \{\bar{\mathbf{Y}}_1{}^T, \ldots, \bar{\mathbf{Y}}_{N_t}^T\} \tag{5}$$

$$\mathbf{Y}^T = \{\mathbf{Y}_1^T, \ldots, \mathbf{Y}_{N_t}^T\} \tag{6}$$

where N_t corresponds to the total number of measured instants of time. The column vectors $\bar{\mathbf{Y}}_j$ and $\mathbf{Y}_j$ contain experimental and estimated information respectively and they are organized such that $[\bar{\mathbf{Y}}_j]_s$ and $[\mathbf{Y}_j]_s$ represent measurements of the s-*th* sensor taken at the j-th instant of time. Therefore, once the error function had been properly defined, the inverse problem consists in determining the set of parameters which minimizes such a function, viz.

$$\min_{\mathbf{p}} S(\mathbf{p}) \qquad \mathbf{p} \in P \tag{7}$$

where every constraint associated to the inverse problem is represented by the solution set P.

The inverse problem defined in (7) will be solved, in the present article, by means of the Levenberg-Marquardt method, which corresponds to a powerful iterative method for solving nonlinear least squares problems of parameter estimation [8,9]. Aiming at minimizing the functional S in equation (7) one has to obtain the derivative of $S(\mathbf{p})$ with respect to the set of unknown parameters $\mathbf{p}$, and then equals such derivative to zero, i.e., the optimality condition is given as follows

$$\frac{\partial S(\mathbf{p})}{\partial p_j} = 0, \qquad j = 1,\ldots,N_p \tag{8}$$

The optimality condition in (8) can be rearranged in matrix notation as follows

$$\nabla S(\mathbf{p}) = -2\,\mathbf{J}\,[\overline{\mathbf{Y}} - \mathbf{Y}(\mathbf{p})] = 0 \tag{9}$$

where the matrix $\mathbf{J}$ is the *Sensitivity Matrix* whose components, named as sensitivity coefficients, are defined as follows

$$J_{ij} = \frac{\partial Y_i}{\partial p_j}, \qquad i = 1,\ldots,N_s \times N_t \quad \text{and} \quad j = 1,\ldots,N_p \tag{10}$$

The iterative procedure of the Levenberg-Marquardt method is given by

$$\mathbf{p}^{k+1} = \mathbf{p}^k + [\mathbf{J}^{k^T}\mathbf{J}^k + \mu^k \Gamma^k]^{-1}\mathbf{J}^{k^T}[\overline{\mathbf{Y}} - \mathbf{Y}(\mathbf{p}^k)] \tag{11}$$

where μ is a stabilization parameter, Γ is a diagonal matrix and the superscript $(\bullet)^k$ denotes the iteration number. The purpose of the term $\mu^k\,\Gamma^k$ in equation (11) is to reduce the oscillations or instabilities due to the ill-conditioning associated to the problem. The decrease of these instabilities or oscillations can be achieved by adopting a matrix $\mu^k\,\Gamma^k$ whose components are relatively large as compared to the components of the matrix $\mathbf{J}^{kT}\mathbf{J}^k$ [8]. At the beginning of the iterative process a large parameter μ is chosen and the Levenberg-Marquardt Method tends to the Steepest Descent Method. The parameter μ is gradually reduced as the iterative process approaches the solution of the problem and then the Levenberg-Marquardt Method tends to the Gauss Method. The parameter μ^k is chosen such that $S(\mathbf{p}^{k+1}) < S(\mathbf{p}^k)$ remains valid at every iteration. The stopping criteria adopted for the iteration process are the ones suggested by Dennis and Schnabel [8] as follows

$$S(\mathbf{p}^{k+1}) < \varepsilon_1 \tag{12}$$

$$\| \mathbf{J}(\mathbf{p}^{k+1})^T [\overline{\mathbf{Y}} - \mathbf{Y}(\mathbf{p})] \| < \varepsilon_2 \tag{13}$$

$$\| \mathbf{p}^{k+1} - \mathbf{p}^k \| < \varepsilon_3 \tag{14}$$

where $\varepsilon_1, \varepsilon_2$ and ε_3 are user-prescribed and $\| \bullet \|$ corresponds to the Euclidean norm. Different versions of Levenberg-Marquardt method can be found in the literature, depending on the choice the diagonal matrix Γ and on the form chosen for the variation of the parameter μ [8]. For the present work it has been chosen the matrix Γ as follows

$$\Gamma^k = diag[\mathrm{J}^{k^T}\ \mathbf{J}^k] \tag{15}$$

2. EXPERIMENTAL SETUP

In order to validate the parameter estimation technique using digital image correlation method it will be considered the problem of estimating the elastic parameters of a free-cantilevered aluminum beam subject to a concentrated load applied at its free tip. It should be mentioned that before starting the parameter estimation process it is desirable to obtain a good prior estimate of the elastic parameters of the beam under analysis by means of another technique in order to have a reference value of the unknown parameters. Among a number of possible techniques that could have been chosen to obtain the prior estimate of the elastic parameters the authors decided for the classical modal analysis one. More specifically, it was decided to estimate the elastic parameters of the material under analysis by formulating an inverse problem which takes into account the first natural frequencies of a beam made of the same material as the one that is being analyzed. Hence, this section is devoted to describe these two experiments just mentioned.

2.1 Free-Free Beam

Aiming at obtaining the natural frequencies of the beam made of the material under analysis the authors decided for an experimental setup of free-free structure. An sketch of the free-free beam is shown in figure (2). The beam was suspended by lightweight elastic cords and it was instrumented with a piezoelectric accelerometer located at one of its ends. The model of the accelerometer is PCB 353M197 and its mass is equal to 9.71 g. The system was excited with an impact hammer and its 5 first natural frequencies were measured and they are shown in table 1.

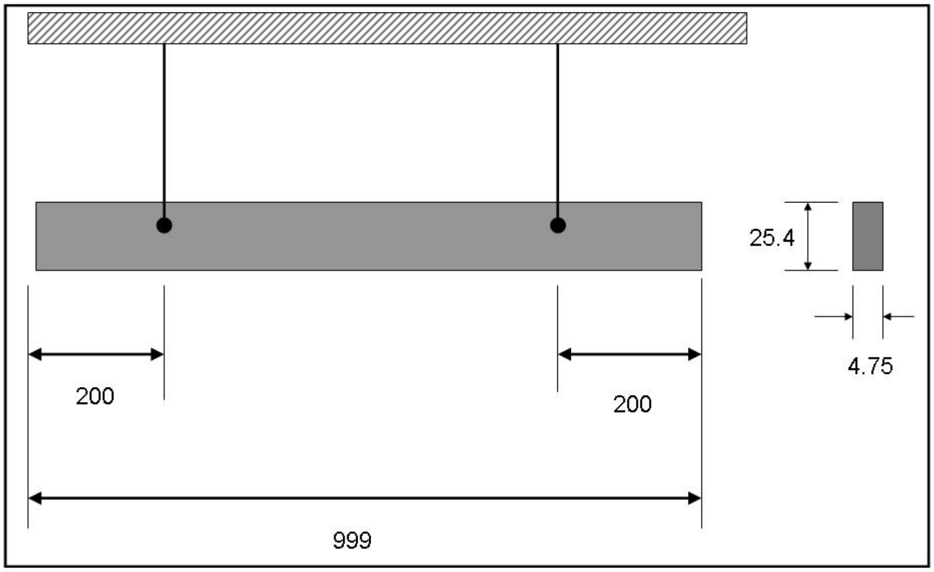


Figure2. Sketch of the free-free beam. Dimensions in mm.

Table 1. Measured natural frequencies of the free-free aluminum beam.

Mode k	f_k (Hz)
1	23.25
2	64.5
3	127.5
4	212
5	317

These five natural frequencies were used as an input data for the inverse problem associated with the estimation of the elasticity modulus of the beam E. It was used the classical Levenberg-Marquardt nonlinear parameter estimation technique. The results are shown in table (2) and they will be used in the next subsection as a reference value for validating the results obtained by means of the digital image correlation based parameter estimation technique.

Table 2. Estimated E for two different error functions.

Error Function	$\sum_{j=1}^{5}(f_j - f_j^{Exp})$	$\sum_{j=1}^{5}(f_j - f_j^{Exp})/f_j^{Exp}$
$\hat{E}$ **(GPa)**	**68.7**	**68.3**

2.2 FREE-CANTILEVERED BEAM

The experimental arrangement involves a fixed-free cantilever beam with loading device, a CCD camera set perpendicularly to the specimen and a computer, as shown in Figure 2. The cantilever beam was covered with painted speckles (random black and white pattern) and the CCD used to record the speckle image of the specimen before and after load has a resolution of 640x480 pixels. In this experimental configuration one pixel of the CCD camera corresponds to an area of about 68.3×68.3 μm on the object.

Definition of the cantilever beam:

Material: aluminum;

Elasticity modulus: Estimated value obtained using classical modal analysis.

Inertia moment: $I = 6.17\times10^{-11}\, m^4$;

Dimension: $L\times b\times h = 101\times4.7\times5.4\ mm^3$;

Applied force: $F = 22.01\,N$.

Figure 3. Experimental arrangement of DIC method.

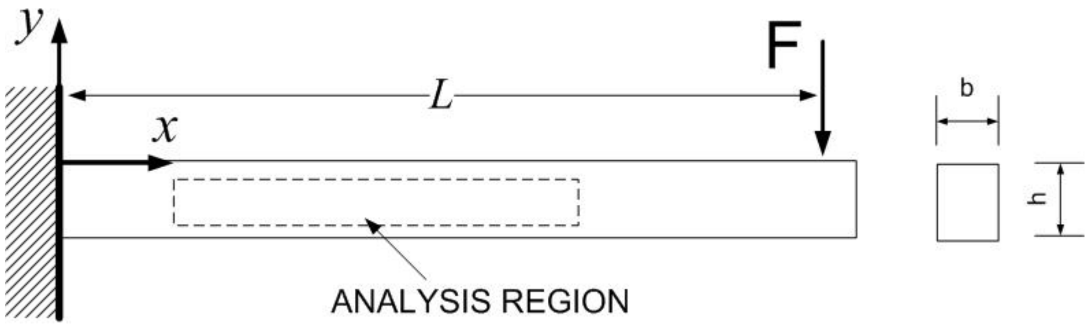
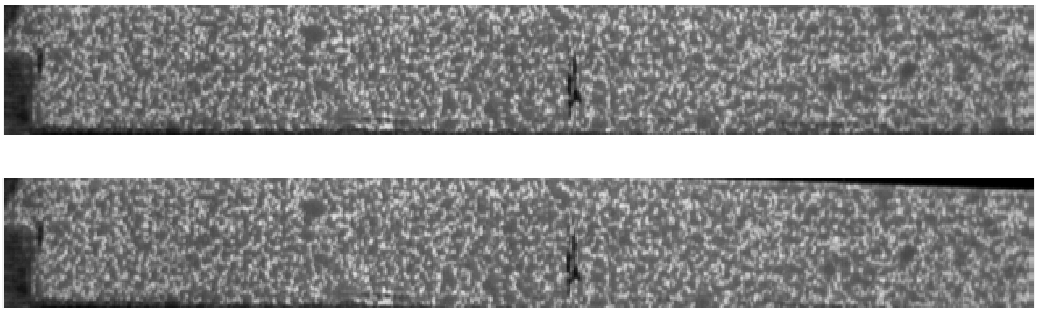


Figure 4. Cantilever beam details.

These two image of the beam in the same analysis region, illustrated in Figure 4, considering different loads of 0 and 22.01 N, were used to determine full-field displacement by means of a computer software, which has been developed in our laboratory (Laboratório de Óptica Não Linear & Aplicada / UFF). Figure 5 illustrates the texture pattern of underformed and deformed coating specimen surface.



(a)

(b)

Figure 5. Pattern of coating specimen, (a) underformed, (b) deformed.

Figure 6 shows the full-field displacement $v(x,y)$ along of axis x when the cantilever beam is subjected to applied load.

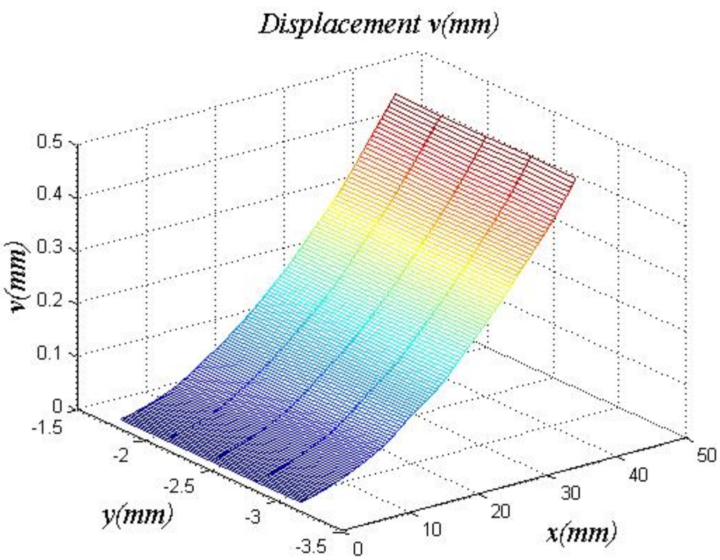


Figure 6. 2D displacement, $v(x,y)$

3. RESULTS

As a preliminary set of results it is considered a region of analysis near the fixed end of the beam and only the experimental data concerning the vertical displacement field *v* will be considered. The authors should mention that there is some level of uncertainty associated to the exact axial position where the region of analysis starts. Nevertheless, for the first case (C1) it is considered that the left face of the analyzed region is positioned at $x = 0$. The estimation process was formulated considering the elasticity modulus as the only one unknown parameter. It was used the Levenberg-Marquardt parameter estimation technique considering the following parameterization $E = p_1 x 10^9$ and the initial guess $p_1^0 = 30$. The provided result is $p_1 = 49.6$ which represents a discrepancy of approximately 27% with respect to the values shown in table (2). Such level of discrepancy can still be decreased by formulating a more complete inverse problem as it will be shown. In the second case to be analyzed (C2), the authors consider that the left face of the analyzed region is positioned at $x = x_s$ and take x_s into account as an unknown when the inverse problem is formulated. For the estimation process it was considered the following parameterization $E = p_1 \text{ x } 10^9$ and $x_s = p_2 \text{ x } 10^{-3}$ and the initial guesses $p_1^0 = 50$ and $p_2^0 = 0.01$. The estimated parameters are $p_1 = 61.3$, which represents a discrepancy of approximately 10% with respect to the values shown in table (2), and $x_s = 3.4$. The experimental and estimated responses for case C2 are graphed in figure (7) and it can be concluded from figure (7) that there is agreement between them.

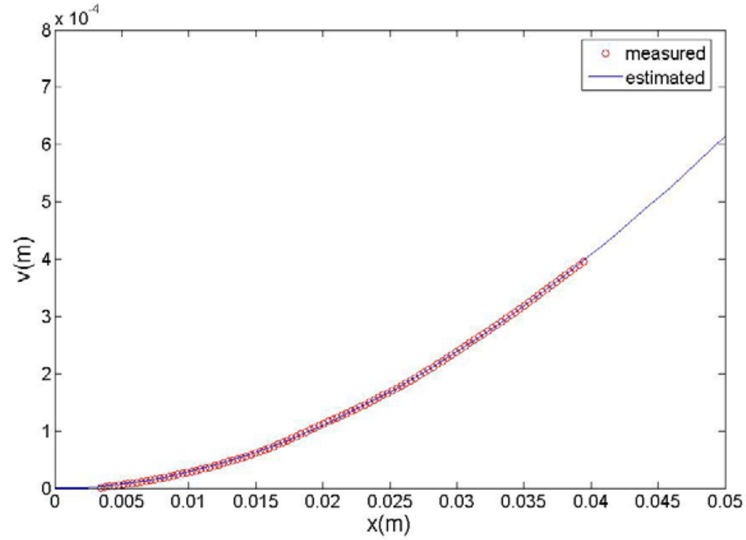


Figure 7. Estimated and experimental response for case C2.

4. CONCLUDING REMARKS

This is a preliminary work aimed at using experimental information obtained from digital image correlation to estimate constitutive parameters. Digital Image Correlation was used to obtain the displacement field of specific regions of a beam under bending and such fields were used to obtain the elasticity modulus of this beam. The parameter estimation was performed by means of the minimization of an error function comprising of the difference between the displacement fields obtained from the experiment and from the model. The inverse problem was solved by means of the classical Levenberg-Marquardt nonlinear parameter estimation technique.

Due to the preliminary characteristic of the present work the authors should state that there are some specific points that should be tuned. More specifically, at this stage the direct measurement of the starting point of the analyzed region, which is characterized by x_s, is the most crucial point. Therefore it is necessary to obtain a more straightforward means of obtaining x_s.

The main contribution of this work is to provide an alternative means of estimating the elastic parameters by means of an experimental setup which is relatively low-cost when compared to other experimental setup based on digital image techniques. For future work the authors aim to approach the following problems: (i) estimate the Poisson's ratio of homogeneous materials, (ii) estimate constitutive parameters of composite materials and (iii) estimate parameters of heterogeneous materials.

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