A LES SUBFILTER VISCOSITY MODEL BASED ON TAYLOR STATISTICAL DIFFUSION THEORY

Gervásio Annes Degrazia, gervasiodegrazia@gmail.com
Guilherme Sausen Welter, gswelter@gmail.com
Franciano Scremin Puhales, fpuhales@gmail.com
Silvana Maldaner, silvana.maldaner@gmail.com
Otávio Costa Acevedo, acevedo@pesquisador.cnpq.br
Universidade Federal de Santa Maria, Santa Maria, RS, Brasil

Umberto Rizza, u.rizza@isac.cnr.it
Istituto di Scienze dell' Atmosfera e del Clima, Consiglio Nazionale delle Ricerche, Lecce, Italy

Abstract. *A turbulent subfilter viscosity for Large Eddy Simulation (LES) based onTaylor's statistical diffusion theory is proposed. This viscosity is described in terms of a velocity variance and a time scale, both associated to the inertial subrange. This new subfilter viscosity contains a cutoff wavenumber Kc, representing a sharp division between large and small wavenumbers of a turbulent flow and, henceforth,Taylor's subfilter viscosity is in agreement with the sharp Fourier filtering operation, frequently employed in LES models.*

1. INTRODUCTION

The planetary boundary layer (PBL) is a physical system presenting a variety of complex states characterized by the turbulence phenomenon. Understanding the turbulence patterns and its structural details has a fundamental importance in large and small meteorological scales and atmospheric dispersion. From the numerical point of view, the PBL turbulence has been investigated employing Large-Eddy Simulation (LES) models (Smagorinsky, 1963; Lilly, 1967; Leonard, 1974; Deardorff, 1973; Moeng, 1984; Lesieur & Metais, 1996; Khanna & Brasseur, 1998; Weil, Sullivan, Moeng, 2004 and Basu & Porta-Agel, 2006). In LES, only the energy-containing eddies of the turbulent motion are explicitly resolved and the effect of the smaller, more isotropic eddies (typical eddies of the inertial subrange), needs to be parameterized (Muschinski, 1996). Modeling these residual turbulent motions, which are also termed subfilter-scale (SFS) motions (Pope, 2004), is in large part a phenomenological procedure based on heuristic arguments (Sullivan *et al.*, 1994). The basic equations in the LES models are the incompressible Navier-Stokes equations for a horizontally homogeneous boundary layer. There solved turbulent flow random variables are determined by the application of a low-pass spatial filter presenting a characteristic width, known as the turbulent resolution length scale (Pope, 2004), smaller than the scales of the resolved turbulent motions. The filter scale is within or at least close to the inertial subrange of the turbulent energy spectrum (Mason, 1994). According to Wyngaard (1982), "the parameterization of the residual stress term in the large-eddy equation is dynamically essential; it causes the transference of kinetic energy to smaller scales. Thus, its parameterization is a key step in developing a large eddy model".

Considering fully developed turbulence as a state with a quasi infinite number of degrees of freedom (myriads of excited energy modes), the parameterization of residual stress means to reproduce the physical effects of a large number of higher frequency harmonics (turbulent scales), which, by virtue of the filtering operation are not explicitly resolved in a numerical simulation. A widely employed parameterization for the residual stress tensor is described by (Smagorinsky, 1963; Sullivan *et al.*, 1994 and Lesieur & Metais, 1996).

$$\tau_{ij} = -\nu_T \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{1}$$

where u_i and u_j are the resolved velocity components, and i, j = 1, 2, 3 corresponding to the (x,y,z) cartesian directions respectively and ν_T is the subfilter turbulent viscosity proposed by Deardorff (1973, 1980), which is expressed as

$$\nu_T = c_k \; l_0 \sqrt{e} \tag{2}$$

where c_k = 0.1 is a constant, l_0 is a mixing-length scale related to the filter operation (Mason, 1994) and e is the turbulent kinetic energy of the subfilter scales (SFS). The concept of subfilter viscosity is based on the fact that the energy transfer from the resolved to the subfilter scales is similar to the molecular mechanism represented by a viscosity. More recent applications of LES have almost exclusively employed a subfilter turbulent viscosity (Wyngaard, 2010). Recently, based on Heisenberg's turbulent spectral energy transfer theory, Degrazia *et al*. (2007)

proposed a subfilter eddy viscosity for LES models expressed in terms of the cutoff or limiting wave number for the inertial subrange.

The purpose of the present study is to derive a new subfilter eddy viscosity based on the cutoff energy from Taylor statistical diffusion theory. An additional aim is to employ this new subfilter viscosity in a LES model to obtain the vertical velocity skewness in a convective PBL. This is accomplished introducing the new model for the subfilter viscosity in the LES code of Moeng and Sullivan (Moeng, 1984, Sullivan *et al.*, 1994)

2. TAYLOR'S SUBFILTER TURBULENT VISCOSITY

The starting point of the derivation consists in considering the structure of the 3D turbulent spectrum in geophysical flows, such as the PBL, where the Reynolds numbers is very large (limit of infinite Reynolds number). In such situations, the turbulent energy spectra can be subdivided in three major spectral regions: energy-containing, inertial and dissipation subranges (Hinze, 1975).

In the energy-containing subrange the large eddies make the main contribution to the turbulent kinetics energy and the density spectrum shows its maximum, so it is possible to choose its corresponding frequency n_e to characterize the evolution time of the energetic eddies.

In the dissipation subrange, where the molecular viscosity ν plays its major role, it is also possible to associate a frequency n_d with the time scales that provide the main contribution to the dissipation. Therefore, for fully developed turbulence, the finite positive dissipation in the region of frequencies below the region of maximum dissipation will be negligibly small compared with the energy flux by inertial effects. In such an inertial subrange, the effect of molecular viscosity would then vanish ($\nu_T >> \nu$). As a consequence we may associate a frequency n_c, where n_c is a cutoff or a limiting frequency for the inertial subrange, with the scale of the eddies that provide the main contribution to this inertial energy flux, that is

$$n_T << n_c << n_d \tag{3}$$

With this assumption ($n_e << n_c$), the turbulence in the inertial subrange is statistically independent of the range of energy-containing eddies and a relationship for the subfilter turbulent viscosity ν_T can be obtained. Thus, following Taylor's statistical diffusion theory, a subfilter turbulent viscosity can be obtained from the following general formulation

$$\nu_T = \sigma_i^2 \, T_c \tag{4}$$

where T_c is the Lagrangian time scale associated to the frequency n_c given by (Hanna, 1981)

$$T_c = \frac{1}{6} \frac{\beta}{n_c} \tag{5}$$

Where T_c is defined as the ratio of the Lagrangian to the Eulerian time scales and σ_i^2 is the turbulent velocity variance for the inertial sub range. Wandel & Kofoed-Hansen (1962) have shown rigorously that for isotropic turbulence $\beta = \frac{\sqrt{\pi}}{4} \frac{\mu}{\sigma_l}$ resulting

$$\nu_T = \frac{\sqrt{\pi}}{24} \frac{u \, \sigma_l}{n_c} \tag{6}$$

where u is the mean wind speed. Substituting $k_c = \frac{2\pi}{u} n_c$, where k_c is the cutoff or limiting wave number for the inertial sub range, into Eq. (6) yields

$$\nu_T = \frac{\pi^{3/2}}{12} \frac{\sigma_l}{k_c} \tag{7}$$

Considering Kolmogorov's turbulent energy spectrum in the inertial sub range, represented by the following form (Kolmogorov, 1941)

$$E(k) = \alpha_k \, \varepsilon^{3/2} \, k^{-5/3} \tag{8}$$

where ε is the turbulent dissipation rate and α_k =1.4is the Kolmogorov constant (Sagaut, 1998), an expression for σ_l is given by the relation

$$\sigma_l = (3\,\alpha_k)^{1/2} \left(\frac{\varepsilon}{k_c}\right)^{1/3} \tag{9}$$

Finally, employing (9) in Eq. (7), results

$$\mu_T = 0.95 \; \varepsilon^{1/3} \, k_c^{-4/3} \tag{10}$$

suggesting that Taylor's subfilter turbulent viscosity, as given by (10), presents an identical functional form (differing only by a constant) to the one derived firstly by Muschinski & Roth (1993), from the Heisenberg's turbulent spectral energy transfer theory and expressed as (Degrazia *et al*., 2007)

$$\mu_T = 0.44 \; \varepsilon^{1/3} \, k_c^{-4/3} \tag{11}$$

Subfilter eddy viscosity as given by Eq. (11) is identical to the spectral eddy diffusivity proposed by Lesieur & Metais (1996), (Degrazia *et al*, 2007,p. 7061). Differently from Eq. (11),Taylor subfilter viscosity given by the Eq. (10) presents a numerical coefficient that is more than 100% larger.

The subfilter viscosity models as represented by the formulations (10) and (11) are classified as models based on the energy at cutoff. According to Sagaut (1998), the relevance for these models lies on the fact that "*...the information is contained in the resolved field, but localized in frequency and therefore theoretically more pertinent for describing the phenomena at cutoff than quantities that are global and thus not localized in frequency. Such models ensure that the subgrid viscosity will be null if the flow is well resolved, i.e. if the highest frequency mode captured by the grid is zero. This type of model thus ensures a better physical consistency than those models based on large scales.*" Taylor's and Heisenberg's theory (Eqs. (10) and (11)) establishes that the energy transfer from wavenumbers smaller than a particular value to larger ones can be represented as the effect of a turbulent viscosity. This introduces a separation between scales at any arbitrary wavenumber in the inertial subrange. This scale division is indeed naturally relevant in the LES methodology, where a cutoff wavenumber is arbitrarily chosen in the inertial subrange introducing then an artificial sharp division (connected with the sharp Fourier cutoff filter), to which Taylor's and Heisenberg's approach seems well suited to be applied.

Furthermore, the presence of the cutoff wavenumber in Eqs. (10) and (11) allows to select in LES models filter widths that ensure the condition $k_c >> k_p$ in the proximity to the ground (such criterion guarantees that the resolved turbulent motions are dominant), where k_p is the peak wavenumber of the turbulent spectrum (Armenio *et al*. 1999, Eq. (4); Degrazia *et al.* 2009, Eq. (3)).

3. TEST OF THE TAYLOR VISCOSITY MODEL WITH NUMERICAL EXPERIMENTS

In order to provide a numerical experiment employing Taylor's subfilter turbulent parameterization proposed in section 2, the LES model firstly developed by Moeng (1984) and later modified by Sullivan *et al* (1994), is utilized with a variable vertical grid spacing Δz for $z < 0.1\, z_i$ (Degrazia *et al*, 2009). A (4, 4, 2) km box domain with 256 points in each direction (x,y,z) has been used. During the simulation the kinematic heat flux was held constant with a magnitude $\overline{w'\theta'} = 0.24$ Kms^{-1}, and the geostrophic wind with a barotropic profile set to 10 ms^{-1}. The initial values for the CBL heigth and the surface potential temperature were set respectively to z_{i0} = 1000 m and θ = 300 K.

The wavenumber k_c in Eqs. (10) and (11) is defined as the ratio π/Δ where Δ is the characteristic cutoff length of the filter (Sagaut, 1998). For neutral ($\overline{w'\theta'} = 0$) or unstable ($\overline{w'\theta'} > 0$) conditions, l is equal to the low-pass LES filter width ($l = \Delta$) (Weil, 2004) where this filter width is given by (Moeng & Wyngaard, 1988)

$$\Delta = \left[\left(\frac{3}{2}\right)^2 \Delta x \, \Delta y \, \Delta z \right]^{1/3} \quad (12)$$

where Δx, Δy and Δz are the computational mesh size in the three coordinate directions and the constant $\left(\frac{3}{2}\right)^{2/3}$ accounts for the de-aliasing. The turbulence dissipation rate ε is parameterized as (Moeng, 1984)

$$\varepsilon = c_\varepsilon \frac{e^{3/2}}{\Delta} \quad (13)$$

where c_ε is a constant and equal to 0.93.

Finally, the substitution of Eq. (13) into Eq. (10) yields

$$\nu_T = 0.2\, e^{1/2} \, \Delta \quad (14)$$

4. THIRD MOMENTS AND VERTICAL VELOCITY SNAPSHOTS

The vertical velocity skewness is an important turbulent statistical quantity for the investigation of contaminant diffusion (Weil, 1990; Rotach, 1996 and Arya,1999). In a CBL with non-divergent horizontal flow, the vertical velocity has a zero mean value, but a strongly negative mode (the most frequent value of the vertical velocity). This indicates that within the CBL, the probability density function has a positive skewness (Lamb, 1982). The vertical velocity skewness profile for Taylor's subfilter viscosity is shown in Fig.1. Therefore, for Taylor's subfilter viscosity simulation the positive magnitude of the skewness increases from 0.3-0.45 near the surface to values between 0.6 and 0.9 in the height range 0.2 $< z/z_i <$ 0.6, attaining the maximum(values greater than 1) at the entrainment interfacial layer, at the vicinity of z/z_i =0.9.

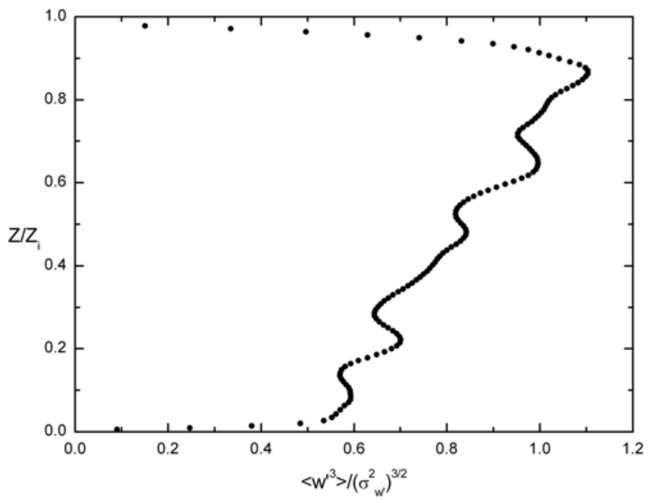

Figure 1. Vertical profile of the vertical velocity skewness simulated from the LES model using Taylor's subfilter viscosity

Thusly comparing the results generated for the simulation with observational data from AMTEX (Lenshow *et al.*, 1980) and Minnesota (Wingaard, 1988) experiment it can be seen that there is a fairly good agreement over the lower portion of the CBL. This good agreement occurs also near the surface where normally LES simulations are ill-defined presenting contradictory results (Sullivanetal, 2003). The vertical velocity skewness profile generated by

Taylor's subfilter viscosity simulation presents similar form and pattern as the skewness of the vertical velocity obtained by wind tunnel simulations of the CBL (Fedorovich *et al*., 1996).

The skewness represents a measure of the asymmetry of the probability density function of a random variable. A positive value of skewness indicates that large positive values of the variable are more frequent than the large negative values. Figure 1 shows that the vertical velocity skewness for a well developed CBL is positive and grows continuously with height. In this aspect, the structure of the vertical velocity field generated by the simulation depicted in Fig. (2) (configurations of the vertical velocity on the horizontal (x - y) planes at height of z/zi = 0.025; 0.05; 0.1; 0.2; 0.5; 0.9), is responsible at the growth of the positive values of the velocity skewness. Figure2 shows a vertical flow with spatial coherence, in which the width of updraft/downdraft regions grows with distance from the surface with well-defined patterns that display air intense updraft movement (the range of positive vertical velocity values increases toward the entrainment interfacial layer).

In fact, the contours of vertical velocity on horizontal planes show that from the surface to elevated CBL regions the down drafts occupy a larger area than the updrafts (dominant down draft movements). Therefore, at low levels of the convective surface layer (z/z_i = 0.025; 0.05) there is an irregular configuration of updrafts and downdrafts characterizing by vanishing skewness. Fig. 2 shows that for a height of z/zi = 0.025 the downdraft movements are slightly dominant presenting low magnitudes of negative velocity. Thusly, in the convective surface layer the LES simulation indicates a more isotropic pattern without the presence of a strongly organized transport. On the other hand, for more elevated regions of the CBL (z/zi =0.1; 0.2; 0.5; 0.9); there are well defined updraft/downdraft cells characterized by greater positive skewness. Fig 2 reveals that at these heights the downdrafts are dominant and associated to higher magnitudes of negative velocity when compared with those of the surface layer.

From the above analysis it can be seen that for a certain height ($z/z_i \cong 0.1$) in the CBL, here is a transition of the vertical flow field to a new order. This new order is caused by a flow structural reorganization that leads to a different spatial symmetry marked by an organized and intense vertical flow occurring above the convective surface layer. Such behavior, as discussed by Moeng & Rotunno (1990), is also evidenced at the upper levels of the CBL as seen in Fig.2 ($z/zi \cong 0.9$) that shows few but vigorous updrafts (maximum value of the skewness). Therefore, the skewness profile of the vertical velocity generated from the LES simulation employing Taylor's subfilter viscosity provides partial, but useful, information about the vertical pattern and structure of the CBL and can be used in Lagrangian dispersion models (Anfossi &Physick, 2005).

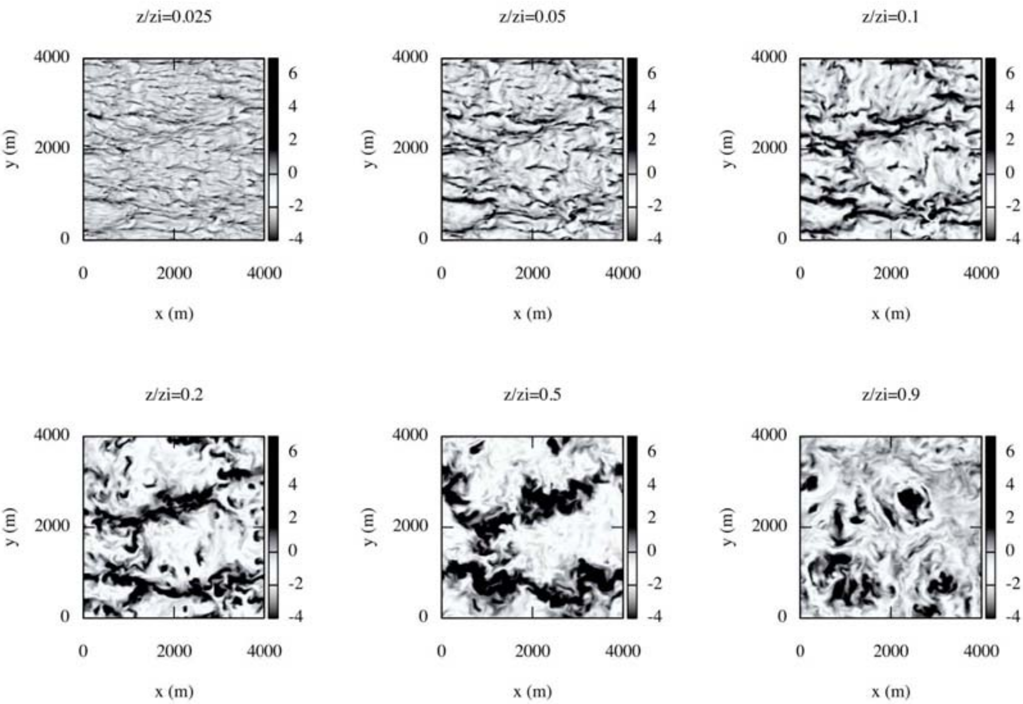

Figure 2. Contours of the vertical velocity on horizontal planes at different heights

5. CONCLUSIONS

In this paper we derived a new turbulent subfilter eddy viscosity for LES models based on the energy at cutoff. The methodology employed to obtain this parameterization describing the turbulent motions is based on Taylor statistical diffusion theory, considering Kolmogorov's turbulent energy spectrum for fully developed turbulence. Taylor subfilter turbulent viscosity is obtained as the product of a temporal scale times a velocity variance. The temporal scale is associated to the limiting frequency in the inertial subrange and the velocity variance yields from the integration Kolmogorov's spectrum in the inertial subrange. This new subfilter viscosity is expressed in terms of the cutoff wavenumber, showing a representation of the subfilter effects of the same kind as Heisenberg's turbulent viscosity (Muschinski & Roth, 1993; Degrazia *et al.* 2007). Therefore, the presence of the cutoff wavenumber in Eq (10) enables calculation of the ratio between the wavelength of the peak in the vertical velocity spectrum and the LES filter cut-off wavelength. Such knowledge imparts physical criterion for the choice of k_c that reproduces SFS fluxes in the proximity of the surface, that are a small fraction of the resolved turbulent fluxes.

The subfilter viscosity here derived containing explicitly the cutoff wavenumber k_c, suggests that, among other possible filtering procedures, the sharp Fourier filter is a natural choice for LES models.

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4. REFERENCES

Anfossi, D. and Physick, W., 2005. "Lagrangian Particle Models". Chapter 11: Of air quality modeling theories, methodologies, computational techniques, and available databases and software". *in*: Zannetti, P. (Ed.), Fundamentals, vol. II. The EnviroComp Institute and the Air Waste Management Association.

Armenio, V.; Piomelli, U., and Fiorotto, V.,1999,"Effect of the Subgrid Scales on Particle Motion", V. 11, pp.3030.

Arya, S., 1999, "Air Pollution Meteorology and Dispersion". Oxford University Press Oxford, 310 p.

Basu, S.; Porta-Agel, F.; Foufoula-Georgiou, E.; Vinuesa, J. and Pahlow, M., 2006, "Revisiting the Local Scaling Hypothesis in Stably Stratified Atmospheric Boundary-layer Turbulence: An Integration of Field and Laboratory Measurements with Large-Eddy Simulations". Boundary-Layer Meteorology 119, 3, pp. 473-500.

Degrazia, G.; Nunes, A.; Satyamurty, P.; Acevedo, O.; Campos Velho, H.; Rizza, U. and Carvalho, J.,2007,"Employing Heisenberg's Turbulent Spectral Transfer Theory to Parameterize Subfilter Scales in LES Models". Atmospheric environment, 41, 33, pp. 7059-7068.

Degrazia, G.; Rizza, U.; Puhales, F.; Goulart, A.; Carvalho, J.;Welter, G. and Marques Filho, E.,2009,"A variable Mesh Spacing for Large-Eddy Simulation Models in the Convective Boundary Layer". Boundary-Layer Meteorology, 131, pp. 277292.

Deardorff, J.W.,1973, "Three-Dimensional Numerical Modeling of the Planetary Boundary layer", *in:* Haugen, D.A. (Ed.) Workshop on Micrometeorology. Boston: American Meteorological Society, pp. 271-311.

Deardorff, J., 1980, "Stratocumulus-Capped Mixed Layers Derived from a Three Dimensional Model". Boundary Layer Meteorology 18, 4, pp. 495-527.

Fedorovich, E.; Kaiser, R.; Rau, M. and Plate, E., 1996, "Wind Tunnel Study of Turbulent Flow Structure in the convective boundary layer capped by a temperature inversion". Journal of the Atmospheric Sciences, 53, 9, pp. 1273-1273

Hanna, S., 1981, "Lagrangian and Eulerian Time-Scale Relations in the Daytime Boundary Layer", J. Appl. Meteorol 20, 3.

Hinze, J.O., 1975,"Turbulence", New York: McGraw-Hill, pp.790

Kolmogorov, A. N., 1941, "On Degeneration (decay) of Isotropic Turbulence in an Incompressible Viscous Liquid". Dolk Akad. Nauk. RSSS, v. 31, pp. 538-540.

Khanna, S. and Brasseur, J.G., 1998, "Three-Dimensional Buoyancy and Shear-Induced Local Structure of the Atmospheric Boundary Layer", Journal of the Atmospheric Sciences, 5, 55

Lamb, R.,1982,"Diffusion in the Convective Boundary Layer", Atmospheric and Air Pollution Modeling, edited by F.T.M. Nieuwstad and H. van Dop, pp. 159-206.

Leonard, A.: 1974, "Energy Cascade in Large Eddy Simulation of Turbulent Flow", Adv. Geophys. 18A, pp. 237-248.

Lesieur, M., and Metais, O., 1996, "New Trends in Large-Eddy Simulations of Turbulence". Annual Review of Fluid Mechanics, 28, 1, pp. 45-82

Lenschow, D.H.; Wyngaard, J.C., Pennell, W.T., 1980, "Mean Fields and Second-Moment Budgets in a Baroclinc, Convective Boundary Layer", J. Atmos. Sci. 37, pp.13131326.

Lilly, D., 1967, "The Representation of Small-Scale Turbulence in Numerical Simulation Experiments", Proc. IBM Scientific Computing Symposium on Environmental Sciences, 195.

Mason, P., 1994, "Large-eddy Simulation: a Critical Review of the Technique". Q.J.R. Meteorol. 120, pp. 1-26.

Moeng, C.-H.,1984, "A Large-Eddy-Simulation Model for the Study of Planetary Boundary Layer Turbulence". Journal of Atmospheric Science, 41, pp. 2052-2062.

Moeng, CH. and Rotunno, R., 1990, "Vertical-Velocity Skewness in the Buoyancy-Driven Boundary Layer". Journal of the Atmospheric Sciences 47, 1149-1162.

Moeng, C. and Wyngaard, J., 1988, "Spectral Analysis of Large-Eddy Simulations of the Convective Boundary Layer", Journal of the Atmospheric Sciences 45, pp. 3573-3587.

Muschinski A, 1996, "A Similarity Theory of Locally Homogeneous and Isotropic Turbulence Generated by Smagorinsky Type LES", Journal of Fluid Mechanics, 325, pp. 239260.

Muschinski, A and Roth, R., 1993, "A Local Interpretation of Heisenbergs Transfer Theory", Beitr. Phys. Atmos., 66, pp.335-346.

Pope, S., 2004, "Ten Questions Concerning the Large-Eddy Simulation of Turbulent flows", New Journal of Physics, 6, pp. 35.

Rotach, M.; Gryning, S. and Tassone, C., 1996. "A Two-Dimensional Lagrangian Stochastic Dispersion Modelfor Daytime Conditions". Quarterly Journal of the Royal Meteorological Society, Bracknell: Royal Meteorological Society, v. 122, n. 530, pp. 367-390.

Sagaut, P., 1998,"Large Eddy Simulation for Incompressible Flows",3.ed. Berlin: Springer, 560p.

Smagorinsky, J., 1963, "General Circulation Experiments with the Primitive Equations. I. The Basic Experiment", Mon. Wea. Rev., 91, 99164.

Sullivan, P.; Horst, T.; Lenschow, D.; Moeng, C. and Weil, J., 2003, "Structure of Subfilter-Scale Fluxes in the Atmospheric Surface Layer with Application to Large-Eddy Simulation Modeling". Journal of Fluid Mechanics, 482, pp. 101-139.

Sullivan, P.; McWilliams, J. and Moeng, C., 1994, "A Subgrid-Scale Model for Large-Eddy Simulation of Planetary Boundary-Layer Fows". Boundary Layer Meteorology, 71, 3, pp. 247-276.

Wandel, C. and Kofoed-Hansen, O., 1962, "On the Eulerian-Lagrangian Transform in the Statistical Theory of Turbulence", Journal of Geophysical Research 67, pp. 3089.

Weil, J., 1990, "A Diagnosis of the Asymmetry in Top-Downand Bottom-up Diffusion Using a Lagrangian Stochastic model". J. Atmos. Sci 47, 4, pp.501-515.

Weil, J.C., Sullivan, P.P. and Moeng, C.-H., 2004, "The Use of Large-Eddy Simulations in Lagrangian Particle Dispersion Models", J. Atmos. Sci., 61, 2877-2887.

Wyngaard, JC, 1982, "Boundary Layer Modeling", Atmospheric Turbulence and Air Pollution Modeling, pp. 69-106.

Wyngaard, J., 1988, "The Effects of Probe-Induced Flow Distortion on Atmospheric Turbulence Measurements: Extension to Scalars". Journal of the Atmospheric Sciences 45, 22 , pp. 3400-3412.

Wyngaard, J., 2010, "Turbulence in the Atmosphere", Cambridge, New York, 393 p.