Modeling initial fracture patterns in human long bones

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Introduction

Long bones can be fractured in wide number of different patterns. Mechanical behavior of a long bone submitted to a static loading is modeled with the aid of well established stress analysis approach. Experimental results of referred literature are used to support several initial hypothesis of the presented model, like the selection of a failure criterion that contributes to keep the model simple.

Stress Analysis

The stress analysis expressions are furnished with the support of Figure 1, where are shown geometrical variables and loadings on a section of a human femur.

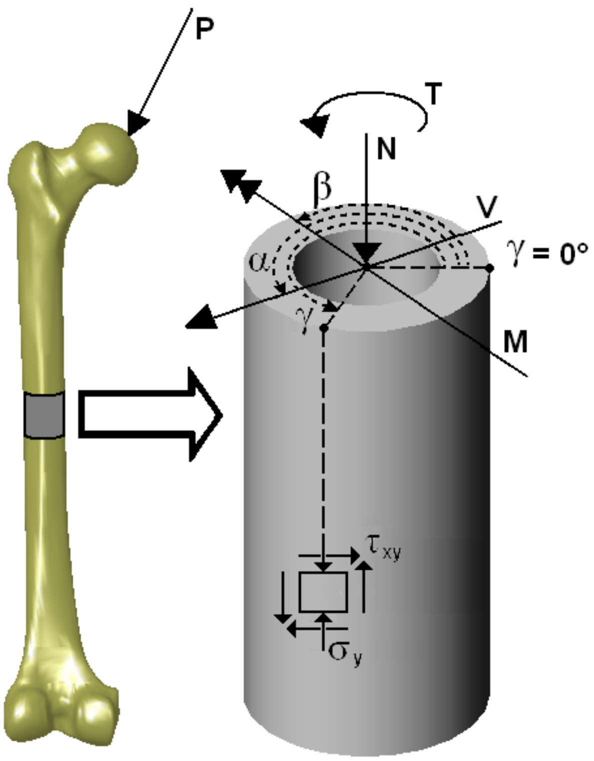

Figure 1: Geometrical variables and loadings on a section of a human femur.

where, *P* is the force that act at the head of a human femur, α is the angle of direction of shear force *V*, β is the angle of neutral axis of bending moment *M* and γ is the angle that determine the point of interest, all of them referred to *X* axis (positive at γ = 0°). *Y* axis is positive at γ = 90°.

In this introductory model the transversal section of bone is modeled as a hollow circle. The axial stress σ_N expression is:

$$\sigma_N = \frac{1}{\pi}\frac{1}{\left(r_e^{\,2} - r_i^{\,2}\right)} N$$

where, r_e is the outer radius, r_i is the inner radius, *N* is axial force.

The bending stress σ_F expression is:

$$\sigma_F = \frac{4}{\pi}\frac{r_e \sin(\gamma - \beta)}{\left(r_e^{\,4} - r_i^{\,4}\right)} M$$

The transverse shear stress expression τ_V is:

$$\tau_V = \frac{VQ}{It}$$

where,

$$I = \frac{\pi}{4}\left(r_e^{\,4} - r_i^{\,4}\right)$$

$$t = \left|2 r_e \sin(\gamma - \alpha)\right|$$

for $0 < y_c \le r_i$:

$$Q = \frac{2}{3}\left[\left(r_e \sin(\gamma - \alpha)\right)^3 - \left(r_i^{\,2} - \left(r_e \cos(\gamma - \alpha)\right)^2\right)^{3/2}\right]$$

for $r_i < y_c \le r_e$:

$$Q = \frac{2}{3}\left[\left(r_e \sin(\gamma - \alpha)\right)^3\right]$$

where, Q is the first moment of area, I is the second moment of area, t is a width at a distance y_c from hollow circle centre.

The torsional stress expression τ_T is:

$$\tau_T = \frac{2}{\pi}\frac{r_e}{\left(r_e^{\,4} - r_i^{\,4}\right)} T$$

where, T is the torsional moment or torque.

To access principal and maximum shear stresses as well as its respective planes the Mohr circle is used, as shown at Figure 2.

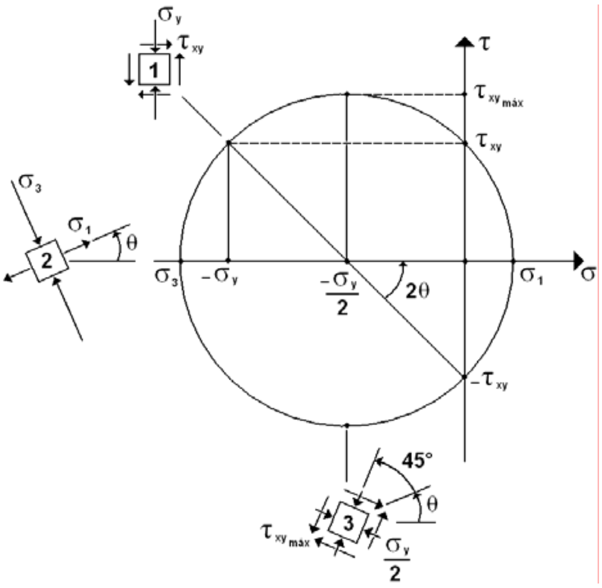

Figure 2: Mohr circle.

The principal stresses σ_1 and σ_3 for this Mohr circle are:

$$\sigma_1, \sigma_3 = \frac{-\sigma_y}{2} \pm \sqrt{\frac{\sigma_y}{4}^2 + \tau_{xy}^{\;2}}$$

where, σ_y is the sum of σ_N and σ_F, it's signal is positive for traction and negative for compression resultants. τ_{xy} is also the sum of τ_V and τ_T.

At fig.2, point 1 correspond to θ = 0°, point 2 correspond to the angle θ of σ_1:

$$\theta = \frac{1}{2}\arctan\left(\left|\frac{2\tau_{xy}}{\sigma_y}\right|\right)$$

Point 3 corresponds to the angle of $\tau_{\max}$, $\theta^* = \theta + 45°$.

Failure Criteria

Although many adaptations of well established failure criteria were suggested to model long bones behavior, until now no model truly describe it. (Keyak et al., 2000) obtained experimental evidences that failure theories based in Distortion energy, as von Mises Theory and based in $\tau_{\max}$ theories, as Tresca, furnish reasonable performance, even not recognizing the existence of differences between tensile and compressive proprieties and the anisotropy of bone material. The expression of Mises Theory for biaxial stress is:

$$\sigma_e = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

where, σ_x is $\perp$ to σ_y.

Example

Figure 3 shows the angle of initial fracture by shear and Mises stresses at outer surface of a transversal section of a human femur, calculated by utilization of former expressions with experimental data obtained from referred literature (Bergmann et al., 2001).

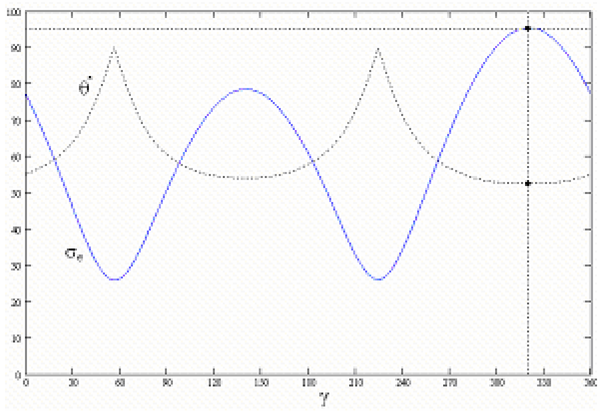

Figure 3: Initial fracture angle and Mises stress.

In this case σ_e is maximum at γ = 320º, that correspond to initial fracture angle of θ^*= 52º.

Conclusions

A simple model has been constructed using well establish stress analysis and failure criterion, to model the angle of initial failure of human long bones submitted to static loading. Numerical simulations will be implemented as this research develops.

References

G. Bergmann, G. Deuretzbacher, M. Heller, F. Graichen, A. Rohlmann, J. Strauss & G. N. Duda, "Hip contact forces and gait patterns from routine activities", J. Biomech., Vol. 34, pp.859-871, 2001

Joyce H. Keyak and Atephen A. Rossi, "Prediction of femoral fracture load using finite element models: an examination of stress-and-strain-based failures theories", J. Biomech., Vol. 33, pp. 209-214, 2000

M. Doblarél, J. M. García and M. J. Gómez, "Modeling bone tissue fracture and healing: a review", Engineering Fracture Mechanics, Vol. 71, pp. 1809-1840, 2004