MHD FLOW IN A TILTED CAVITY USING RADIAL BASIS FUNCTIONS

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Abstract. *Meshless methods have been increasingly applied to the solution of partial diferencial equations. In the present work, a meshless technique, based on the Radial Basis Functions (RBF), was used to the solution of a magnetohydrodynamic (MHD) problem. In this problem, a tilted cavity was considered. Such cavity was filled with an electrically conducting fluid and it was permeated by an external magnetic field. The mass and momentum equations were combined in a biharmonic equation, written in terms of stream functions. The results obtained with RBF solution were in good agreement with other results found in the literature.*

Keywords*: Radial Basis Functions, Magnetohydrodynamic, Meshless methods.*

1. NOMENCLATURE

B	Magnetic field vector
B_0	Externally applied magnetic field of reference
c	shape parameter used in the Radial Basis Functions
f	Exact value of the functions at the interpolation points
g	Acceleration of gravity vector
Gr	Grashof number
Ha	Hartmann number
J	Electric current density vector
L	Length of the cavity
M, N	Number of collocation points in the x and y direction, used in the RBF approximation
P	Pressure
Pr	Prandtl number
r	Euclidian norm between any two points
RBF	Radial Basis Function
s	Interpolated functions
T	Temperature
T_h	Hot temperature
T_c	Cold temperature
T_0	Reference temperature
V	Velocity field vector
x, y	Coordinate axis

Greek symbols

α, β	Unknown coefficients of the RBF expansion
γ	Inclination of the cavity
α_T	Thermal diffusivity
β_T	Thermal expansion coefficient
ϕ	Electric potential
η	Unknown coefficients of the RBF representing the temperature and stream-function
μ_0	Magnetic permeability of the vacuum
σ	Electrical conductivity
ν	Kinematic viscosity
ρ	Viscosity
ξ	Base functions of the RBF representing the temperature and stream-function
ψ	Stream-function

Superscript

‘	Dimensionless quantities

2. INTRODUCTION

Traditional numerical approaches for the approximate solution of partial differential equations require the use of a mesh, as in the finite element, finite difference, and finite volume methods, among others. The main difficulty associated with the use of a mesh is the definition of the mesh itself (Leitão, 2004). Meshless methods have thus some advantages over traditional methods, due to the fact that they do not require a mesh generation. The geometry discretization is updated by simply adding or deleting points in the domain of interest (Demirkaya et al., 2008). The first use of the RBF interpolation method for solving differential equations was made by the physicist Edward Kansa in 1990. Once the method was first used to solve PDEs, its popularity continued to grow rapidly and a large number of applications of the method appeared (Sarra e Kansa, 2009).

Meshless methods have been used to solve different problems. Colaço et al. (2006) used Radial Basis Functions, based on the multiquadrics and Wendland expansions, to solve convective-diffusive problems. Divo and Kassab (2007) developed a localized meshless collocation using the Hardy’s Multiquadrics method for coupled viscous fluid flow and convective heat transfer problems. Leitão (2001) presented a meshless method for the analysis of bending of thin homogeneous plates, based on the use of radial basis functions to build an approximation of the general solution of the partial differential equations governing the Kirchho plate-bending problem. Chinchapatnam et al. (2006) used RBFs to solve a lid-driven cavity problem. Colaço et al. (2009) used the radial basis function formulation to solve a magnetohydrodynamic problem in two dimensions in an incompressible, steady-state and laminar flow-field with constant magnetic field applied. Wang et al. (2005) presented a meshless method developed by combining the virtual boundary collocation method with RBF approximation to solve isotropic and anisotropic heat conduction problems.

The effect of magnetic field on free convection in cavities has received great attention, being studied by several authors. These studies mainly analyze the effect of magnetic force in the flow field and the heat transfer rate. This increasing attention is due to the large number of applications.

Applications range from the process of manufacturing crystals such as crystal silicon to the use of liquid metals as coolant medium in a variety of nuclear reactors involving the presence of strong magnetic fields (Alchaar et al.,1995).

Al-Najem et al. (1997) examined the influence of the magnetic field on the heat transfer process inside a tilted enclosures for a wide range of inclination angles at moderate and high Grashof numbers. They concluded that the heat transfer mechanisms and the flow characteristics inside the tilted enclosures depend strongly upon both the strength of the magnetic field and the inclination angle. Ece et al. (2006) studied natural-convection flow in the presence of a magnetic field in a tilted square/rectangular enclosure. The results showed that the flow characteristics and, therefore, the convection heat transfer inside the tilted enclosure, depend strongly upon the strength and direction of the magnetic field, the aspect ratio and the inclination of the enclosure.

In this work a magnetohydrodynamic problem in a tilted square cavity was studied using a RBF approximation. Different inclination angles as well as different Hartmann and Grashof numbers were analyzed. Several tests were conducted by changing the number of collocation points. The equations were written in the biharmonic form, thus eliminating the pressure gradient and also the need of a pressure-velocity coupling scheme.

3. RADIAL BASIS FUNCTION

Radial basis functions are essential ingredients of the techniques generally known as "meshless methods". In one way or another all meshless techniques require some sort of radial function to measure the influence of a given location on another part of the domain. Kansa's method (or asymmetric collocation) starts by building an approximation to the field of interest (normally displacement components) from the superposition of radial basis functions (globally or compactly supported) conveniently placed at points in the domain (and, or, at the boundary).

Radial Basis Function (RBF) is a function that depends only on the distance between the center x_j and another any point x. The unknowns (which are the coefficients of each RBF) are obtained from the (approximate) enforcement of the boundary conditions as well as the governing equations by means of collocation. Usually, this approximation only considers regular radial basis functions, such as the globally supported multiquadrics or the compactly supported Wendland (Wendland,1998) functions.

Radial basis functions (RBFs) may be classified into two main groups:

1. the globally supported ones namely the multiquadric (MQ, $\sqrt{\left(x-x_j\right)^2+c_j^2}$, where c_j is a shape parameter), the inverse multiquadric, thin plate splines, Gaussians, etc;
2. the compactly supported ones such as the Wendland (Wendland,1998) family (for example, $(1-r)_+^n+p\left(r\right)$ where $p(r)$ is a polynomial and $(1-r)_+^n$ is 0 for r greater than the support).

In a very brief manner, interpolation with RBFs may take the form:

$$f(x_i)=\sum_{j=1}^{N}\alpha_j\phi(|x_i-x_j|) \tag{1}$$

or

$$\begin{aligned} f(x_1)&=\alpha_1\phi(|x_1-x_1|)+\alpha_2\phi(|x_1-x_2|)+\cdots+\alpha_N\phi(|x_1-x_N|)\\ f(x_2)&=\alpha_1\phi(|x_2-x_1|)+\alpha_2\phi(|x_2-x_2|)+\cdots+\alpha_N\phi(|x_2-x_N|)\\ &\vdots\\ f(x_N)&=\alpha_1\phi(|x_N-x_1|)+\alpha_2\phi(|x_N-x_2|)+\cdots+\alpha_N\phi(|x_N-x_N|) \end{aligned} \tag{2}$$

or in a system

$$\boldsymbol{\phi\alpha}=\boldsymbol{f} \tag{3}$$

where $\boldsymbol{\phi}$ is the matrix of known RBFs and their derivatives, $\boldsymbol{f}$ is the vector of independent terms and $\boldsymbol{\alpha}$ is the vector of unknown parameters.

4. PHYSICAL PROBLEM

The physical problem analyzed here involves the laminar, steady and incompressible fluid flow of an electrically conducting fluid within a tilted square cavity whose left and right walls are subjected to different and constant temperatures and the top and bottom walls are kept thermally insulated. The fluid properties are considered constants and the buoyancy force is approximated using the Boussinesq’s hypothesis. The fluid is permeated by a constant magnetic field which will create an additional buoyancy force. All boundaries are subjected to no-slip boundary. Figure1 shows the geometry and boundaries for this problem.

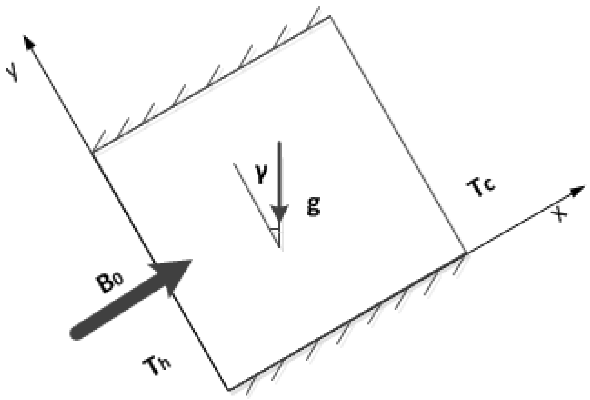


Figure 1 - Geometry and coordinate system

4.1 Mathematical Equations

The mathematical formulation of the problem studied in this paper is given by Eqs. (4), which represent the mass, momentum, and energy conservation, along with the Maxwell equations:

$$\nabla.\mathbf{V} = 0 \tag{4.a}$$
$$(\mathbf{V}.\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla P + \frac{\mathbf{J}}{\rho}\times\mathbf{B} + \nu\nabla^2\mathbf{V} - \beta_T\mathbf{g}(T - T_0) \tag{4.b}$$
$$(\mathbf{V}.\nabla)T = \alpha_T\nabla^2 T + \frac{\mathbf{J}}{\rho C_p}\cdot(-\nabla\phi + \mathbf{V}\times\mathbf{B}) \tag{4.c}$$
$$\nabla\cdot\mathbf{J} = 0 \tag{4.d}$$
$$\mathbf{J} = \sigma(-\nabla\phi + \mathbf{V}\times\mathbf{B}) \tag{4.e}$$
$$\nabla\times\frac{\mathbf{B}}{\mu_0} = \mathbf{J} \tag{4.f}$$

where μ_0 is the magnetic permeability of the vacuum.

Defining the stream-function

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \tag{5.a,b}$$

and the following dimensionless quantities

$$x' = \frac{x}{L} \quad y' = \frac{y}{L} \quad \psi' = \frac{\psi}{L} \quad T' = \frac{(T-T_c)}{(T_h-T_c)} \tag{6.a-d}$$

where L is the length of the cavity and T_c and T_h are the cold the hot temperatures of the container walls, respectively, we can combine Eqs. (4.a), (4.b), and (5.a, b), in order to obtain a bi-harmonic equation:

$$\frac{\partial\psi'}{\partial y'}\left(\frac{\partial^3\psi'}{\partial x'\,\partial y'^2} + \frac{\partial^3\psi'}{\partial x'^3}\right) - \frac{\partial\psi'}{\partial x'}\left(\frac{\partial^3\psi'}{\partial y'\,\partial x'^2} + \frac{\partial^3\psi'}{\partial y'^3}\right)$$
$$= \frac{\partial^4\psi'}{\partial x'^4} + 2\frac{\partial^4\psi'}{\partial x'^2\,\partial y'^2} + \frac{\partial^4\psi'}{\partial y'^4} - \mathrm{Ha}^2\frac{\partial^2\psi'}{\partial x'^2} - \mathrm{Gr}\left(\frac{\partial T'}{\partial x'}\cos\gamma - \frac{\partial T'}{\partial y'}\sin\gamma\right) \tag{7.a}$$

Also, substituting (5.a, b) into (4.c) we obtain

$$\frac{\partial\psi'}{\partial y'}\left(\frac{\partial T'}{\partial x'}\right) - \frac{\partial\psi'}{\partial x'}\left(\frac{\partial T'}{\partial y'}\right) = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \tag{7.b}$$

Also, substituting (5.a, b) in the boundary conditions, we have

$$\psi' = \frac{\partial\psi'}{\partial x'} = 0; \qquad T' = 1 \qquad \text{at}\ \ x = 0 \tag{7.c}$$
$$\psi' = \frac{\partial\psi'}{\partial x'} = 0; \qquad T' = 0 \qquad \text{at}\ \ x = 1 \tag{7.d}$$

$$\psi' = \frac{\partial \psi'}{\partial y'} = 0; \qquad \frac{\partial T'}{\partial y'} = 0 \qquad \text{at } \ y = 0 \tag{7.e}$$

$$\psi' = \frac{\partial \psi'}{\partial y'} = 0; \qquad \frac{\partial T'}{\partial y'} = 0 \qquad \text{at } \ y = 1 \tag{7.f}$$

where Ha, Gr and Pr are the Hartmann, Grashof and Prandtl numbers, respectively. They are defined as

$$\mathrm{Ha} = \mathrm{B_0 L}\sqrt{\frac{\sigma}{\mu}}; \qquad \mathrm{Gr} = \frac{g\beta_T (T_h - T_c) L^3}{\nu^2}; \qquad \mathrm{Pr} = \frac{\nu}{\alpha_T} \tag{8.a - c}$$

where B_0 is the steady externally applied magnetic field of reference.

5. SOLUTION TECHINIQUE

Classical numerical methods, such as the finite volume and the finite difference methods, need to use some kind of pressure-velocity coupling scheme. In the present work, the use of the bi-harmonic form given by Eq. (7.a), eliminates the pressure gradient. In this paper, we used a RBF formulation to solve Eqs. (7.a.-b) as well as the boundary conditions given by Eqs. (7.c-f). There are several types of RBF functions. In this paper we used the multiquadrics, which is given as

$$\xi(\boldsymbol{r}_i) = \xi_i(x,y) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + c^2} \tag{9}$$

where c is a shape parameter which directly influences the quality of the solution. In this paper, we defined c to be proportional to the minimum distance between two points over the entire domain, a procedure suggested by Chinchapatnam (2006). Therefore, c is increased until the residual of the solution of the equations. (7.a-f) is minimum.

The variables appearing in Eqs. (7.a-f) were expanded by using

$$\psi'(x', y') = \sum_{i=1}^{M} \eta_i \xi(r_i) \tag{10.a}$$

$$T'(x', y') = \sum_{j=1}^{N} \lambda_j \xi(r_j) \tag{10.b}$$

where the RBFs ξ are the same for the two expansions, but the parameters η and λ are different for each one. In Eqs. (10.a, b), M and N are the number of centers used in the two RBF approximations. Substituting Eqs. (10.a, b) into Eqs. (7.a-f) we can obtain

$$\sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial y'}\right]\left\{\sum_{i=1}^{M}\left[\eta_i \frac{\partial^3 \xi(r_i)}{\partial x' \partial y'^2}\right] + \sum_{i=1}^{M}\left[\eta_i \frac{\partial^3 \xi(r_i)}{\partial x'^3}\right]\right\} - \sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial x'}\right]\left\{\sum_{i=1}^{M}\left[\eta_i \frac{\partial^3 \xi(r_i)}{\partial y' \partial x'^2}\right] + \sum_{i=1}^{M}\left[\eta_i \frac{\partial^3 \xi(r_i)}{\partial y'^3}\right]\right\} =$$
$$= \sum_{i=1}^{M}\left[\eta_i \frac{\partial^4 \xi(r_i)}{\partial x'^4}\right] + 2\sum_{i=1}^{M}\left[\eta_i \frac{\partial^4 \xi(r_i)}{\partial x'^2 \partial y'^2}\right] + \sum_{i=1}^{M}\left[\eta_i \frac{\partial^4 \xi(r_i)}{\partial y'^4}\right] - Ha^2 \sum_{i=1}^{M}\left[\eta_i \frac{\partial^2 \xi(r_i)}{\partial x'^2}\right] -$$
$$-Gr\left\{\sum_{j=1}^{N}\left[\lambda_j \frac{\partial \xi(r_j)}{\partial x'}\right]\cos\gamma - \sum_{j=1}^{N}\left[\lambda_j \frac{\partial \xi(r_j)}{\partial y'}\right]\sin\gamma\right\} \tag{11.a}$$

$$\sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial y'}\right]\sum_{j=1}^{N}\left[\lambda_j \frac{\partial \xi(r_j)}{\partial x'}\right] - \sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial x'}\right]\sum_{j=1}^{N}\left[\lambda_j \frac{\partial \xi(r_j)}{\partial y'}\right] = \frac{1}{Pr}\left\{\sum_{j=1}^{N}\left[\lambda_j \frac{\partial^2 \xi(r_j)}{\partial x'^2}\right] + \sum_{j=1}^{N}\left[\lambda_j \frac{\partial^2 \xi(r_j)}{\partial y'^2}\right]\right\} \tag{11.b}$$

$$\sum_{i=1}^{M} \eta_i \xi(r_i) = \sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial x'}\right] = 0; \qquad \sum_{j=1}^{N} \lambda_j \xi(r_j) = 1 \qquad \text{at } \ x = 0 \tag{11.c}$$

$$\sum_{i=1}^{M} \eta_i \xi(r_i) = \sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial x'}\right] = 0; \qquad \sum_{j=1}^{N} \lambda_j \xi(r_j) = 0 \qquad \text{at } \ x = 1 \tag{11.d}$$

$$\sum_{i=1}^{M} \eta_i \xi(r_i) = \sum_{i=1}^{M}\left[\eta_i \frac{\partial \xi(r_i)}{\partial y'}\right] = 0; \qquad \sum_{j=1}^{N}\left[\lambda_j \frac{\partial \xi(r_j)}{\partial y'}\right] = 0 \qquad \text{at } \ y = 0 \tag{11.e}$$

$$\sum_{\mathrm{i}=1}^{\mathrm{M}} \eta_{\mathrm{i}} \xi(\mathrm{r_i}) = \sum_{i=1}^{M} \left[\eta_i \frac{\partial \xi(r_i)}{\partial y'} \right] = 0; \qquad \sum_{j=1}^{N} \left[\lambda_j \frac{\partial \xi(r_j)}{\partial y'} \right] = 0 \qquad \text{at} \;\; \mathrm{y} = 1 \qquad (11.\mathrm{f})$$

Equations (11.a-f) result in a system of nonlinear algebraic equations that was solved by the Broyden's quasi-Newton method (Press et al., 1992).

6. RESULTS AND DISCUSSION

The magnetohydrodynamic problem in tilted square cavity was qualitatively analyzed to the following angles of inclination of the cavity: -30^o and -60^o. The analysis of the average Nusselt number was performed for the following angles of inclination of the cavity: -10^o to -90^o. We used Grashof numbers equal to $\mathrm{Gr} = 10^4$ and $\mathrm{Gr} = 10^6$, where several Hartmann numbers were utilized. In all test cases, the Prandtl number was taken as 0.71. Different tests were analyzed, changing the number of collocation points in the RBF expansion. A uniform distribution of points was used. The numerical results were compared with the previous results of Al-Najem et al. (1998) where the authors used the control volume method on a uniform grid of 41 x 41 grid cells.

Figures 2 and 3 show qualitatively the isotherms and stream functions for $\mathrm{Gr} = 10^4$, and $\mathrm{Ha} = 0$ and 25, with an inclination of the cavity $\gamma = -30^o$ and $\gamma = -60^o$, respectively. We used for this test 36 and 225 collocation points. We can see that even with a few collocation points, the results obtained with RBF formulation are in good agreement with the reference. Increasing the number of points, there is an improvement in the cases: $\gamma = -30^o$ with $\mathrm{Ha} = 0$, and $\gamma = -60^o$ with $\mathrm{Ha} = 25$

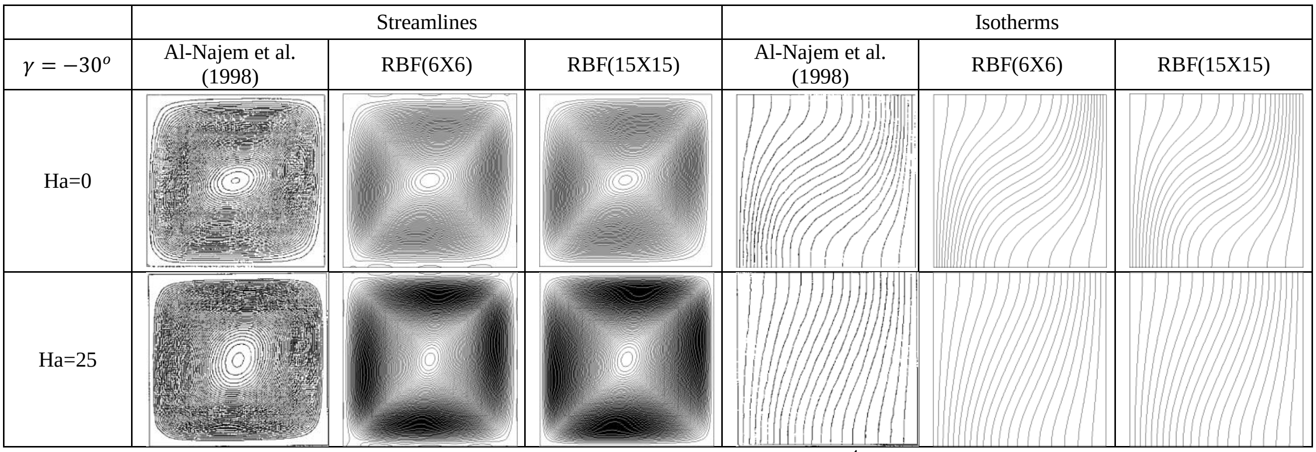


Figure 2 - Streamlines and isotherms for Gr=10^4 and γ=-30^o

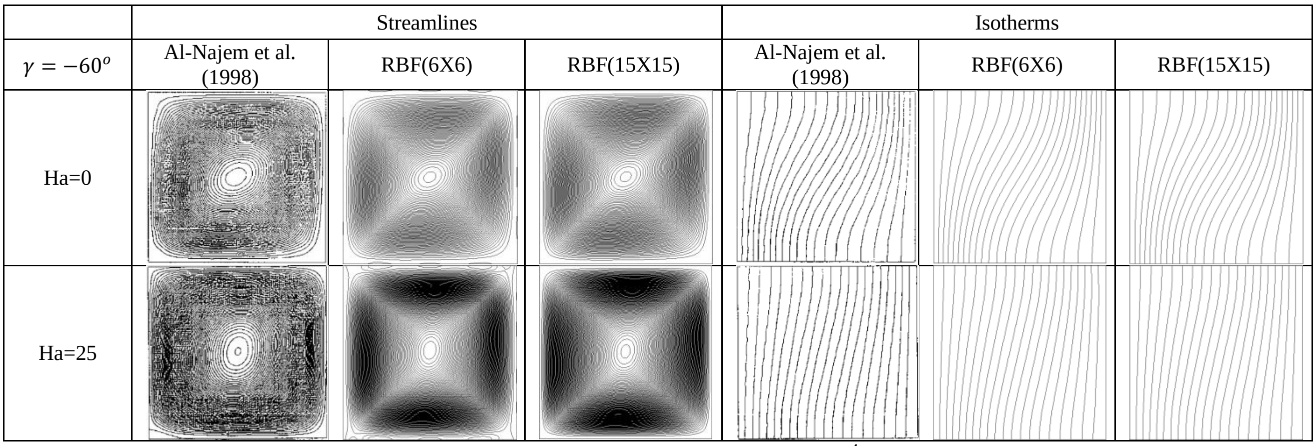


Figure 3 - Streamlines and isotherms for Gr=10^4 and γ=-60^o

Figures 4 and 5 show qualitatively the isotherms and stream functions for $\mathrm{Gr} = 10^6$, $\mathrm{Ha} = 25$, with an inclination of the cavity $\gamma = -30^o$ and $\gamma = -60^o$, respectively. We used for this test 225 and 625 collocation points. For $\gamma = -30^o$ we can see that results improved when we increase the number of collocation points. For $\gamma = -60^o$ the results did not improve with the number of collocation points.

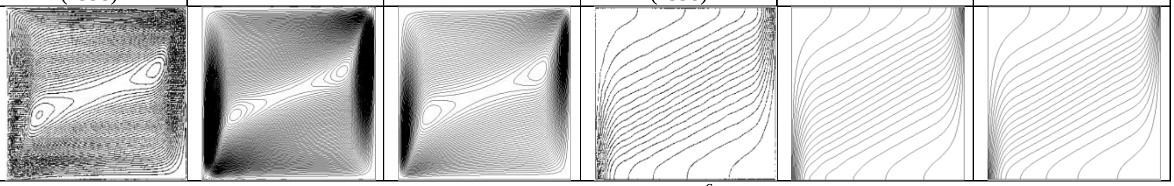
	Streamlines			Isotherms		
$\gamma = -30^o$	Al-Najem et al. (1998)	RBF(15X15)	RBF(25X25)	Al-Najem et al. (1998)	RBF(15X15)	RBF(25X25)
Ha=25						

Figure 4 - Streamlines and isotherms for Gr=10^6 and γ=-30^o

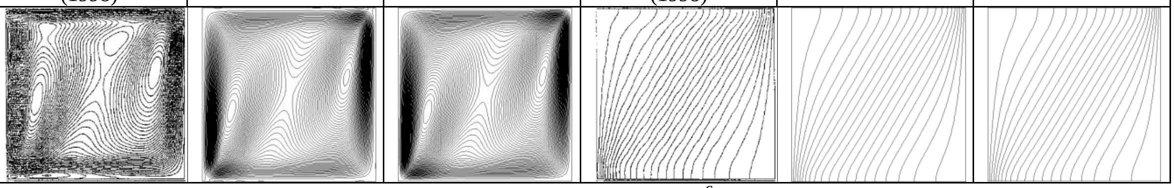
	Streamlines			Isotherms		
$\gamma = -60^o$	Al-Najem et al. (1998)	RBF(15X15)	RBF(25X25)	Al-Najem et al. (1998)	RBF(15X15)	RBF(25X25)
Ha=25						

Figure 5 - Streamlines and isotherms for Gr=10^6 and γ=-60^o

Table 1 shows the comparison of the results obtained in the present work against the ones of Al-Najem et al. (1998) for $\mathrm{Gr} = 10^4$, with $\gamma = -20^o$. The results show that the relative error decreases when we increase the number of collocation points, except for $\mathrm{Ha} = 25$ which presents the best result with a distribution of 15x15 points. Errors less than 2% were found, indicating that the results are in good agreement with those found in the literature.

Table 1 - Relative errors of Nusselt numbers for Gr=10^4 and γ=-20^o

Average Nusselt number ($\gamma = -20^o$)							
$\mathbf{Gr = 10^4}$	Al-Najem et al. (1998)	RBF(6x6)	Relative error (%)	RBF(15x15)	Relative error (%)	RBF(25x25)	Relative error (%)
$\mathbf{Ha = 0}$	1.71	2.00	16.96	1.74	1.75	1.73	1.17
$\mathbf{Ha = 10}$	1.48	1.60	8.11	1.49	0.68	1.47	0.68
$\mathbf{Ha = 25}$	1.11	1.16	4.50	1.11	0	1.12	0.89
$\mathbf{Ha = 50}$	0.99	0.89	10.10	0.96	3.03	1.01	1.98

Table 2 shows the comparison against the results of Al-Najem et al. (1998) for $\mathrm{Gr} = 10^4$, with $\gamma = -50^o$. The results show that the relative error decreases when we increase the number of collocation points, except for $\mathrm{Ha} = 0$ which presents the best result with a distribution of 15x15 collocation points. In this case we found relative errors smaller than 1%.

Table 2 - Relative errors of Nusselt numbers for Gr=10^4 and γ=-50^o

Average Nusselt number ($\gamma = -50^o$)							
$\mathbf{Gr = 10^4}$	Al-Najem et al. (1998)	RBF(6x6)	Relative error (%)	RBF(15x15)	Relative error (%)	RBF(25x25)	Relative error (%)
$\boldsymbol{Ha = 0}$	1.25	1.33	6.40	1.25	0	1.26	0.8
$\boldsymbol{Ha = 10}$	1.16	1.22	5.17	1.17	0.86	1.15	0.86
$\boldsymbol{Ha = 25}$	1.02	1.04	1.96	1.04	1.96	1.03	0.98
$\boldsymbol{Ha = 50}$	0.98	0.94	4.08	1.00	2.04	0.98	0

Table 3 shows the comparison against the results of Al-Najem et al.(1998) for $\mathrm{Gr} = 10^6$, with $\gamma = -20^o$. The results shows that for $\mathrm{Ha} = 100$ the best result was found with a distribution of 15x15 collocation points. We found errors less than 1% for $\mathrm{Ha} = 0$, 10 and 25. The results for $\mathrm{Ha} = \infty$ were not good, showing high relative errors.

Table 3 - Relative errors of Nusselt numbers for Gr=10^6 and γ=-20°

Average Nusselt number ($\gamma = -20^o$)							
$\mathbf{Gr = 10^6}$	Al-Najem et al. (1998)	RBF(15x15)	Relative error (%)	RBF(25x25)	Relative error (%)	RBF(30x30)	Relative error (%)
$\mathbf{Ha = 0}$	6.92	8.32	20.23	7.24	4.62	6.92	0
$\mathbf{Ha = 10}$	6.76	8.12	20.11	7.20	6.11	6.76	0
$\mathbf{Ha = 25}$	6.11	7.23	18.33	6.43	4.98	6.06	0.82
$\mathbf{Ha = 100}$	3.01	2.92	2.99	2.72	9.63	2.65	11.96
$\mathbf{Ha = \infty}$	0.99	0.88	11.11	0.90	9.09	0.90	9.09

Table 4 shows the comparison against the results of Al-Najem et al.(1998) for $\text{Gr} = 10^6$, with $\gamma = -50^o$. The results show that the case with 25x25 collocation points was the best for this case. Again, for $\text{Ha} = 100$, the best result was obtained with a distribution of 15x15 collocation points. This was observed at all angles of inclination and the results for $\text{Ha} = \infty$ were not good, showing high relative errors.

Table 4 - Relative errors of Nusselt numbers for Gr=10^6 and γ=-50°

Average Nusselt number ($\gamma = -50^o$)							
$\mathbf{Gr = 10^6}$	Al-Najem et al. (1998)	RBF(15x15)	Relative error (%)	RBF(25x25)	Relative error (%)	RBF(30x30)	Relative error (%)
$\mathbf{Ha = 0}$	2.47	2.66	7.69	2.36	4.45	2.27	8.10
$\mathbf{Ha = 10}$	2.47	2.78	12.55	2.33	5.67	2.25	8.91
$\mathbf{Ha = 25}$	2.33	2.57	9.34	2.21	5.15	2.14	8.15
$\mathbf{Ha = 100}$	1.55	1.53	1.29	1.50	3.23	1.49	3.87
$\mathbf{Ha = \infty}$	0.97	0.88	9.28	0.90	7.22	0.90	7.22

Figure 6 and 7 present the variaton of the average Nusselt number with the inclination of the cavity, for $\text{Gr} = 10^4$ and $\text{Gr} = 10^6$, respectively. These figures also present a comparison against the previous results of Al-Najem et al. (1998). All results were obtained with a distribution of 25x25 collocation points for $\text{Gr} = 10^4$, and a distribution of 30x30 collocation points for $\text{Gr} = 10^6$. As we can observe, the agreement between the solutions is better when the Grashof number is smaller. As we saw above in Tables 3 and 4, for $\text{Ha} = 100$, the use of 30x30 collocation points presented a large error. In this case it is best to use fewer collocation points.

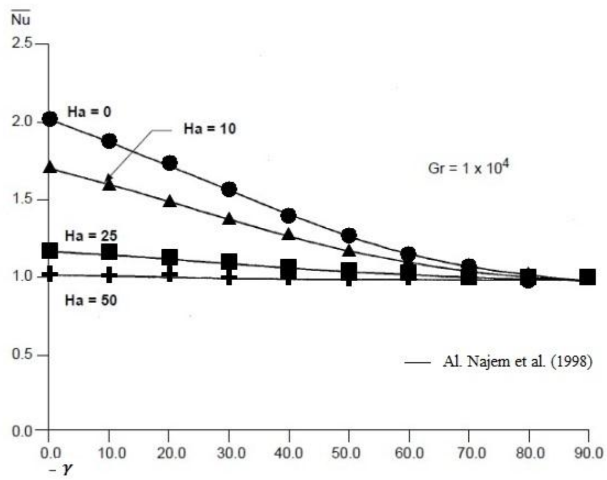


Figure 6 - Nu vs. γ for Gr=10^4

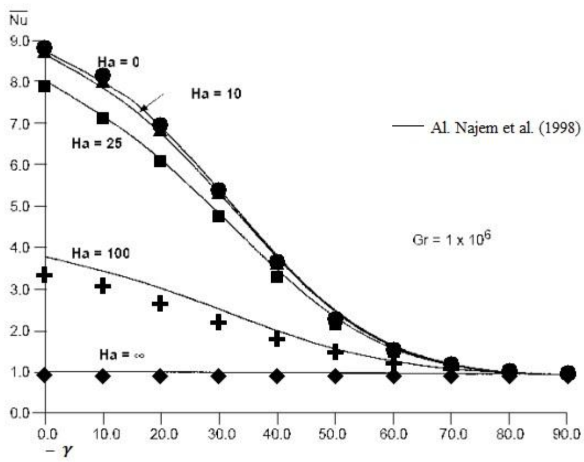


Figure 7- Nu vs. γ for Gr=10^6

7. CONCLUSIONS

In this work we used the Radial Basis Function method to solve a magnetohydrodynamic problem in a tilted square cavity in an incompressible, steady-state and laminar flow-field, with a constant magnetic field applied. Different tilt angles as well as different Hartmann and Grashof numbers were studied. We used a uniform distribution of points and changed the number of collocation points in the RBF expansion. The results showed that the RBF found good estimates for the stream function and isotherms, as well as the Nusselt number, when compared with another work found in the literature. Some tests must be performed in the future, regarding the use of a non-uniform distribution. The shape parameter required in the multiquadrics formulation needs further investigation, since in the literature studied there is no rule or mathematical methodology to find an optimum value of it.

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