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ANALYSIS OF DIFFERENT LEVELS OF APPROXIMATION OF THE REYNOLDS STRESS TENSOR USING TENSOR DECOMPOSITION THEOREMS

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Abstract. *In the so-called RANS approach, turbulent models provide closure equations that relate the Reynolds stress with kinematic tensors. In this study, we extend a methodology presented by Thompson* et al. *(2010) to quantify the dependence of the Reynolds stress tensor on mean kinematic tensor basis. The methodology is based upon tensor decomposition theorems which allows to extract from the anisotropic Reynolds stress tensor the parts that are or proportional or in-phase with the rate-of-strain tensor and the parts that are or proportional or in-phase to the persistence-of-straining tensor (which is orthogonal to the rate-of-strain tensor). The study was conducted using DNS (direct numerical simulation) data for square duct flow with, ($Re_\tau = 160$). Different sets of tensorial basis provide different levels of approximations. These levels are measured through normalized indexes that are essentially a ratio of the Euclidean norm of the model to the Euclidean norm of the Reynolds stress. We have performed 3 new tests in order to complete a previous analysis of 3 other approximation models done in Thompson* et al. *(2010). Interestingly, we are able to identify the regions of the domain better approximated by each model. With the proposed methodology, the scalar coefficients of nonlinear algebraic turbulent models can also be determined.*

Keywords: *RANS, Tensor decomposition, Turbulent closures*

1. INTRODUCTION

The Reynolds Averaged Navier-Stokes (RANS) is by far the most popular methodology used to predict turbulent flow in the day life of industrial environment. The advantages of this approach are: the low time consuming of simulations and the simplified set of equations, easy to interpret. This approach requires a closure equation that relates the Reynolds stress with the mean kinematic quantities. A significant number of closure schemes have been proposed ranging from the simple algebraic specification using turbulent velocity and length scales to Reynolds Stress models. At the intermediate level of closure complexity, there are models which retain some aspects of the algebraic linear model and of Reynolds stress models. This level of closure is known as *nonlinear eddy viscosity models* (NLEVM).

The present paper is an extension of the methodology developed by Thompson *et al.* (2010) to evaluate the Reynolds stress dependence upon mean kinematic tensors. The proposed methodology is based upon tensor decomposition theorems which allow determination of the orientation of the Reynolds stress tensor from DNS data. Through comparisons with DNS databases, this methodology allows to quantify how good the tensorial formulation of the Reynolds stress tensor is, and more importantly to compute the scalar coefficients appearing in the models.

2. METHODOLOGY

The following usual Reynolds decomposition notations will be used: $\overline{(.)}$ indicates the time average operation and a single quote ($'$) denotes the fluctuations with respect to the average. The Reynolds stress tensor, $\mathbf{R}$, is defined through

$$\mathbf{R} = -\overline{\mathbf{u}'\mathbf{u}'}, \tag{1}$$

where $\mathbf{u}'$ is the fluctuation velocity vector. and the traceless Reynolds stress tensor is defined as

$$\mathbf{B} = \mathbf{R} - \frac{1}{3}\mathrm{tr}(\mathbf{R}), \tag{2}$$

where $\mathrm{tr}(.)$ is the trace operator.

2.1 Tensor decompositions

We use two kinds of decompositions of a tensor with respect to a second one. These two kinds were shown by Thompson *et al.* (2010) to be the only ones that decouple the tensor into a part which is coaxial and another which is orthogonal to the second one and at the same time these two parts of the decomposition are orthogonal to each other. One decomposition is referred to as proportional-orthogonal and the other one in-phase-out-of-phase decompositions. The mathematical analysis of these two kinds can be found in Thompson (2008). These two tensor decompositions were applied to the Reynolds stress tensor with respect to two kinematic tensors: the symmetric part of the velocity gradient, $\mathbf{D}$ and the non-persistence-of-straining tensor $\mathbf{P}$, defined by

$$\mathbf{P} = \mathbf{D} \cdot \mathbf{W}^* - \mathbf{W}^* \cdot \mathbf{D}, \tag{3}$$

where $\mathbf{W}^*$ is the relative vorticity tensor, defined as the vorticity measured with respect to the rate of rotation of the eigenvectors of $\mathbf{D}$. Since $\mathbf{D}$ and $\mathbf{P}$ are orthogonal, we were able to produce 6 levels of representations of the Reynolds stress, depending on the combinations of the decompositions adopted. Besides that, we apply indices of adherence to quantify the ability of the particular model to capture the Reynolds tensor.

2.2 The models

The six models presented in the present work are

$$M_I : \mathbf{B}_I = \alpha \mathbf{D} \tag{4}$$

$$M_{II} : \mathbf{B}_{II} = \alpha_0 \mathbf{I} + \alpha_D \mathbf{D} + \alpha_{D2} \mathbf{D}^2 \tag{5}$$

$$M_{III} : \mathbf{B}_{III} = \alpha_0 \mathbf{I} + \alpha_D \mathbf{D} + \alpha_{D2} \mathbf{D}^2 + \beta \mathbf{P} \tag{6}$$

$$M_{IV} : \mathbf{B}_{IV} = \alpha \mathbf{D} + \beta \mathbf{P} \tag{7}$$

$$M_V : \mathbf{B}_V = \beta_0 \mathbf{I} + \alpha \mathbf{D} + \beta_P \mathbf{P} + \beta_{P2} \mathbf{P}^2 \tag{8}$$

$$M_{VI} : \mathbf{B}_{VI} = (\alpha_0 + \beta_0)\, \mathbf{I} + \alpha_D \mathbf{D} + \alpha_{D2} \mathbf{D}^2 + \beta_P \mathbf{P} + \beta_{P2} \mathbf{P}^2 \tag{9}$$

and the indexes, $R_i, i \in \{I, II, III, IV, V, VI\}$ that measure the quality of the approximations are given by

$$R_i = 1 - \frac{2}{\pi} \cos^{-1} \sqrt{\frac{\mathrm{tr}\, \mathbf{B}_\mathrm{i}^2}{\mathrm{tr}\, \mathbf{B}^2}} \tag{10}$$

The quantities R_i are local. It is straightforward to infer that, at any point of the domain, the following inequalities hold:

$$0 \le R_I \le R_{II} \le R_{III} \le R_{VI} \le 1 \tag{11}$$

$$0 \le R_I \le R_{IV} \le R_V \le R_{VI} \le 1 \tag{12}$$

$$0 \le R_I \le R_{IV} \le R_{III} \le R_{VI} \le 1 \tag{13}$$

Models M_I, M_{II} and M_{III} were already presented in Thompson *et al.* (2010) and are repeated here for convenience and comparison. From the inequalities above, we can establish the different complexity levels of approximation. One important issue is to address the differences between models of the same level of complexity such as the pair M_{II} and M_{IV} and the pair M_{III} and M_V.

3. RESULTS

These models are applied to the DNS data base of the flow through a square duct, Gavrilakis (1993).

Figures 1 and 2 show that adding a quadratic term on the rate of strain tensor does not alter significantly the main result. This conclusion does not necessarily hold for other problems besides the square duct.

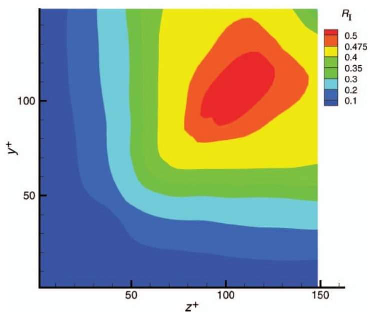

Figure 1. Index R_I. A normalized measure of the stress intensity associated with the Boussinesq hypothesis.

The addition of the persistence of straining tensor as a tensor basis for the explanation of the anisotropic Reynolds stress tensor enhances significantly the ability to capture this entity, as illustrated by Fig. 3. A direct comparison between Figs. 2 and 4, shows that, indeed, adding the persistence-of-strainning tensor to the set of basis functions lead to a better

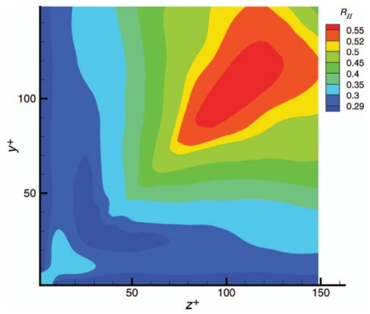

Figure 2. Index R_{II}. A normalized measure of the hypothesis that the Reynolds stress depends solely on the mean rate of strain.

description of the Reynolds stress tensor than adding a quadratic term on the rate of strain.

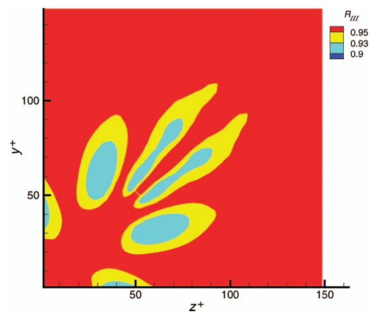

Figure 3. Index R_{III}. A normalized measure of the hypothesis that the Reynolds stress depends non-linearly with the rate of strain and linearly with the persistence-of-strain tensor.

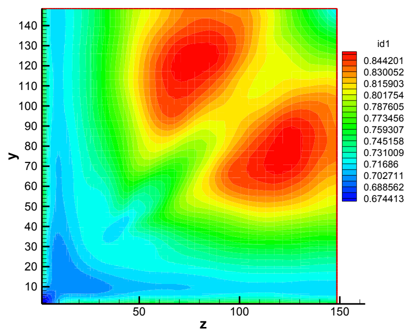

Figure 4. Index R_{IV}. A normalized measure of the hypothesis that the Reynolds stress depends linearly with the rate of strain and linearly with the persistence-of-strain tensor.

On the other hand, a direct comparison between Figs. 3 and 5, shows that a non-linearity on the rate of strain lead to better explanation capability than the non-linearity on the persistence of straining. It is important to notice that the persistence of straining tensor does not a good job near the wall, specially near the wall corner, when compared to the non-linear role played by rate of strain tensor.

Interestingly, however is the result depicted in Fig. (6). The total Reynolds stress tensor is predicted by the full model M_{VI}. This result shows that the different regions of the domain are captured by different tensors of the set of basis tensors.

4. CONCLUSIONS

The present methodology presents 6 (six) different models for representing the anisotropic Reynolds stress tensor. They are based on linear and non-linear descriptions as functions of the rate of strain and persistence of straining tensors. The use of the rate of strain tensor is costumary in the literature. There are examples of using the non-objective version of

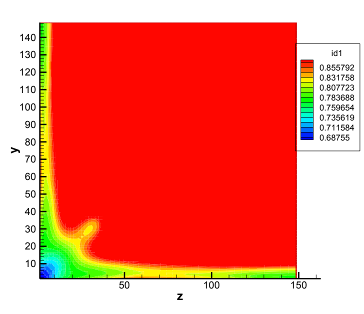

Figure 5. Index R_V. A normalized measure of the hypothesis that the Reynolds stress depends linearly with the rate of strain and non-linearly with the persistence-of-strain tensor.

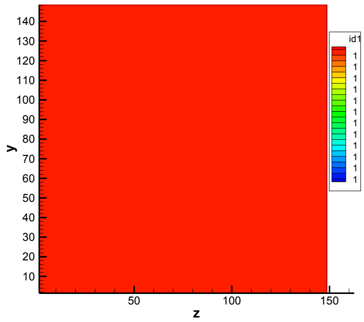

Figure 6. Index R_{IV}. A normalized measure of the hypothesis that the Reynolds stress depends non-linearly with the rate of strain and non-linearly with the persistence-of-strain tensor.

the persistence of straining tensor, with the vorticity in the place of relative-vorticity tensor. This tensor has the property of being orthogonal to the rate of strain tensor and, therefore, is able to explain parts of the Reynolds stress tensor which are impossible for the rate of strain. The different levels of complexity were able to capture different regions of the domain.

The results of the present work can be used to build models for the Reynolds stress tensor, by constructing the dependence of the coefficients (not shown here) as functions of the relevant parameters.

5. REFERENCES

Gavrilakis, S., 1993. “Turbulent velocity structures derived from pod analyses.” *Institut de Machines Hydrauliques et de Mécanique des Fluides, École Polytechnique Fédérale de Lausanne.*, Vol. Report T-93-30, pp. C134–C156.

Thompson, R.L., 2008. “Some perspectives on the dynamic history of a material element”. *Int. J. Engng. Sci*, Vol. 46, pp. 524–549.

Thompson, R.L., Mompean, G. and Thais, L., 2010. “A methodology to quantify the non-linearity of the reynolds stress tensor.” *J. Turb.*, Vol. 11, pp. 1–27.