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INVESTIGATION OF CLOSURE RELATIONS FOR 1D TWO-FLUID MODEL IN VERTICAL PIPES

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Abstract. *The 1D Two-Fluid Model is based on the time and pipe cross-section area averages of the transport equations governing the flow, requiring the inclusion of closure parameters for the gas and liquid phases. The closure relations must be able to realistically represent the physical phenomenon of interest, and the resulting set of governing equations must also be well-posed. At the present paper, two closure relations are considered to render a well posed set of conservation equations to predict the upward slug flow in vertical pipes. These parameters are: pressure jump due to the curvature of the interface, and a momentum flux parameter. Main characteristics for statistically steady flow regime are compared with experimental data presenting good agreement. It is shown that the inclusion of a constant momentum flux parameter for the liquid phase plays a more significant role than the pressure jump to improve well-posedness in vertical flows.*

Keywords: *Two-phase flow, vertical pipe, 1D, two-fluid model, closure parameter*

1. INTRODUCTION

Slug flow is a complex flow characterized by a succession of liquid plugs and Taylor bubbles, and it can be found in a variety of industrial applications, such as chemical process, nuclear and petroleum industries. The prediction of this type of flow is very important to correctly design pipelines and separation tanks. However, especially due to its intermittent nature, the correct slugging behavior is very difficult to predict numerically. Slug length, frequency, and translational velocities are the most important parameters. Most models have a semi-empirical nature, requiring the incorporation of experimental correlations for slug parameters (Dukler and Hubbard, 1975, Taitel and Barnea, 1990). However, due to the intrinsically unsteady and irregular character of slug flow, statistical evolution of slug variables is many times required for its proper description (Carneiro et al., 2011).

The Two-Fluid Model (Ishii and Hibiki, 2006) is one of the most popular approaches for the simulation of multiphase flows. This model is based on the solution of a set of conservation equations for each phase. Although two phase pipe flow has intrinsically a three-dimensional nature, due to phase distribution and especially in the presence of turbulence, it is common to assume that the main variations occur only in the axial direction and therefore the one – dimensional model is a realistic approximation. To attain the 1D flow equations, it is necessary to perform an area average of the conservation equations. This process results in loss of information along the cross section, which has to be compensated with constitutive closure relationships. Furthermore, for vertical flow situations, the traditional set of conservation equations is ill-posed. Holmâs et al (2008) suggested including artificial diffusion terms in the momentum equations, while Montini and Issa (2010) evaluated the inclusion artificial diffusion terms in both mass and momentum equations. Song and Ishii (2000) explored in detail the issue of accounting for effect of cross stream profiles. They suggested the introduction of a coefficient in the convective terms of the equations which they called "the momentum flux parameter" (MFP) and subsequently analysed its effect on the nature of equations. Issa and Montini (2010) showed that presence of momentum flux parameter renders the equation conditionally well – posed by analyzing the numerical solution of the Two-Fluid Model for horizontal slug flows under very fine meshes. The influence of surface tension was also investigated in horizontal situations (Montini and Issa, 2010).

At the present work, two closure relations are investigated aiming at obtaining a well-posed system of equations, in order to predict vertical slug flows. The first one is the momentum flux parameter and the second one is the pressure jump due to the interface curvature. The model is applied to test cases and the predictions are compared with available experimental data.

2. MATHEMATICAL MODEL

Isothermal two-phase flow in a vertical pipe is considered. The liquid is incompressible and the gas follows the ideal gas law. Additionally, viscous diffusion is neglected and no mass transfer takes place between phases. The one-dimensional continuity and momentum equations (comprising the Two-Fluid Model) can be written for each phase *k* as

$$\frac{\partial(\alpha_k \rho_k)}{\partial t}+\frac{\partial(\alpha_k \rho_k U_k)}{\partial x}=0 \quad \frac{\partial(\alpha_k \rho_k U_k)}{\partial t}+\frac{\partial(C_k \ \alpha_k \rho_k U_k^2)}{\partial x}=-\alpha_k \frac{\partial P_{ik}}{\partial x}-\alpha_k \rho_k g-\frac{\tau_{wk} S_k}{A}\pm\frac{\tau_i S_i}{A} \tag{1}$$

where α represents the volume fraction (with $\alpha_g + \alpha_l = 1$), U is the average velocity weighted by the volume fraction, ρ is density, t is time and x the axial coordinate. P_{ik} is the pressure at the interface at the k phase side, g is the gravity acceleration, τ_{wk} is phase k shear stress at the wall, τ_i is the gas-liquid interfacial shear stress, S_k and S_l are the phase wet perimeter and the interface perimeter. It should be mentioned that annular flow is considered as the base flow configuration, i.e., geometrical parameter are determined based on this assumption. Therefore, S_g =0, and $S_l = \pi D$, where D is the pipe diameter. The shear stress can be obtained from the friction factor f as

$$\tau_{wl} = f_l \rho_l |U_l| U_l / 2 \quad , \quad \tau_i = f_i \ \rho_g \, |U_g - U_l| \, (U_g - U_l)/2 \ , \tag{2}$$

Empirical correlations must be employed to obtain f_l and f_i as function of Reynolds number (Carneiro et al. 2011; Wallis, 1969).

2.1 Closure parameters

To help stabilize the conservations equations, in order to obtain a well posed set of equation, the momentum flux parameter C_k can be employed. It accounts for the variation of velocity and void fraction over cross – section, containing information on the flow structure in the flow area normal to the main direction. It is defined as

$$C_k = \int \alpha_k \, U_k^2 \, dA \ / \ (\alpha_k \, U_k^2) \tag{3}$$

For fully developed single phase flow, it is possible to establish the values of the shape parameter C_k by integrating the analytical velocity profiles. For laminar flow (Poiseuille flow), C_k =1.33 is a classical result of fluid mechanics. Turbulent flow profiles are flatter, so there is a tendency to give lower C_k values than for laminar flow.

Another parameter that can render the set equations as well posed is the pressure jump at the interface. It can be determined by the Young – Laplace equation, as a function of interface curvature κ, which is based on the principal curvature radii at the surface. For an annular geometry, two principal curvatures are involved, one in the axial direction R_1 and other in the radial direction R_2

$$P_{ig} - P_{il} = \sigma \ \kappa \quad ; \quad \kappa = \frac{1}{R_1}+\frac{1}{R_2} \quad \text{where} \quad \frac{1}{R_1}=\frac{\partial^2 h_l}{\partial x^2} \quad ; \quad \frac{1}{R_2}=\frac{1}{(D/2)-h_l}=\frac{2}{D\sqrt{1-\alpha_l}} \tag{4}$$

To solve the governing equations, the finite-volume method (Patankar, 1980) was employed, using the totally implicit scheme for time integration and upwind scheme for the convective terms. The conservation equations are solved sequentially and the system of algebraic equations for each variable is solved with the TDMA algorithm.

3. ANALYSIS OF WELL-POSEDNESS

A set of equations is well posed if a solution for the initial value problem exists, is unique and depend continuously on the data (i.e. small variations in the initial data correspond to small variations of the solution). When these three conditions do not hold, the problem is ill posed although it may still give a solution.

The governing equations of the 1D Two-Fluid Model are known to be mathematically ill-posed under certain conditions, depending strongly on closure models incorporated, as mentioned above.

To investigate the well-posedness of the Two-Fluid Model, a characteristic analysis of the governing differential equation is performed. To this end, it is convenient to rewrite the governing differential equations in a more compact form as:

$$\mathbf{A}\frac{\partial \phi}{\partial t}+\mathbf{B}\frac{\partial \phi}{\partial x}=C \quad \text{where} \quad \phi=\begin{bmatrix}\alpha_g & U_g & U_l & P_{il}\end{bmatrix}^{\mathrm{T}} \tag{5}$$

$$\mathbf{A}=\begin{bmatrix}\rho_g & 0 & 0 & 0\\ -\rho_l & 0 & 0 & 0\\ \rho_g U_g & \rho_g \alpha_g & 0 & 0\\ -\rho_l U_l & 0 & -\rho_l \alpha_l & 0\end{bmatrix} \quad \mathbf{B}=\begin{bmatrix}\rho_g U_g & \rho_g \alpha_g & 0 & 0\\ -\rho_l U_l & 0 & \rho_l \alpha_l & 0\\ \rho_g C_g U_g^2 & 2\rho_g \alpha_g C_g U_g & 0 & \alpha_g\\ -\rho_l C_l U_l^2+\alpha_l \sigma(1/(D\,\alpha_g^{3/2}-\partial\kappa/\partial\alpha_l) & 0 & 2\rho_l \alpha_l C_l U_l & \alpha_l\end{bmatrix} \tag{6}$$

where **A** and **B** are the Jacobian matrices, ϕ is the solution vector and C is the column vector with algebraic source terms. The initial value problem under consideration is to find a solution of system in some region $0 \le x \le L$, where L is length of the pipe, and $t \ge 0$; subject to initial condition $\phi(0,x)=G(x)$. To verify if the system is well posed, the characteristics λ_n of the set of equations must be analyzed. The characteristics λ_n are determined when det [**B**- λ **A**]=0.

If all *n* roots are real, the set of differential equations is hyperbolic and the problem is well-posed (Song and Ishii 2000). On the other hand, in the case of complex roots, the system of equations is elliptic and the problem is ill – posed. In the latter case, it is not guaranteed that the system has a unique solution and the results obtained by solving numerically the system are unreliable. Usually, in such cases, the numerical calculations do not converge to the same values regardless of mesh density.

To estimate the term related to the variation of the local curvature of the interface with the liquid level, it is useful the introduction of a disturbance in accordance with the following expression: $h_l = h_{l,eq} + h_{l,0}\, e^{i(wt - \chi_p x)}$, where $h_{l,eq}$ is the liquid level of equilibrium. The second term represents the fluctuation in time of the liquid h'_l, , where $h_{l,0}$, w and χ_p correspond to the amplitude disturbance, frequency and angular wave number. Assuming for simplicity that χ_p is constant along the axial coordinate, it follows that $\partial\kappa/\partial\alpha_g = \partial\kappa/\partial h_l\, \partial h_l/\partial\alpha_g$ and $\partial\kappa/\partial h_l = -\chi_p^2$. To guarantee that the characteristics are real, the following criteria must be respected

$$(\rho_g\, C_g\, U_g\, \alpha_l + \rho_l\, C_l\, U_l\, \alpha_g)^2 - (\rho_l\; \alpha_g + \rho_g\, \alpha_l)\left[(\rho_g\, C_g\, U_g^2\, \alpha_l + \rho_l\, C_l\, U_l^2\, \alpha_g) + \frac{\sigma\,\alpha_l}{4\,D\sqrt{\alpha_g}}(4 - \chi_p^2\, D^2 \alpha_g)\right] \geq 0 \quad (7)$$

It can be easily shown that if $C_g = C_l = 1$ and $\sigma = 0$ (no pressure jump), the above condition will never be attained, unless $U_g = U_l$.

4. RESULTS

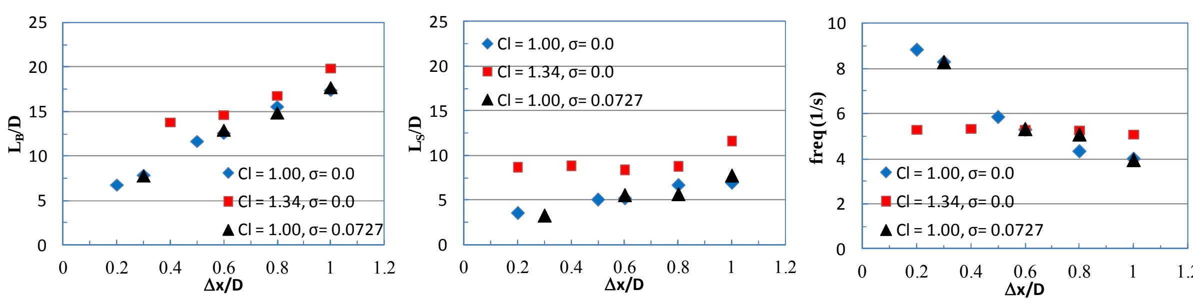
At the present paper, one case was selected to be examined. Table 1 presents the pipe and physical properties of the fluids. This case was investigated experimentally by Rodrigues et al (2010), who examined the slug flow in horizontal and vertical pipes.

By applying the criteria given by Eq.(7) it can be shown that the pressure jump due to the interface curvature does not help to transform the problem to well posed if $\chi_p < 2/D$, but the inclusion of the coefficients C_l or C_g, even close to one, are more effective to improve well-posedness. Issa and Montini (2010) showed that for a horizontal pipe, the liquid momentum flux coefficient can extend the well posedness region of the system, but not the gas parameter. Therefore, the gas momentum flux parameter was kept constant and equal to one. The liquid momentum flux parameter was defined as C_l =1.00 and 1.34, as employed by Issa and Montini (2010).

Table 1.-Physical properties vertical pipe (Rodrigues et al., 2010)

Fluids	Density (kg/m^3)	Viscosity (Pa s)	Superficial velocity (m/s)	Superficial shear (N/m)	Pipe length (m)	Pipe Diameter (m)	Pressure out (Pa)
Air	1.243	1.81×10^{-5}	0.828				
water	998.2	8.55×10^{-4}	0.880	0.0727	5.811	0.026	105 114

As mentioned above, ill-posed problems usually fail to present mesh-independent solutions. Thus, to verify the behavior of the system of equations, this case was simulated with fine meshes and with different combinations of the flux parameter and pressure jump. Figure 1 illustrates the bubble length, L_B/D, slug length L_S/D and slug frequency as a function of the mesh size $\Delta x/D$ with and without the liquid momentum flux coefficient C_l and pressure jump (σ =0 implies in no pressure jump). Note that as the mesh is refined, the inclusion of the pressure jump does not stabilize the system and the slug parameters do not become mesh independent. However, the introduction of C_l=1.34 results in a mesh independent solution. This result is in agreement with the stability analysis performed with the aid of Eq. (7).

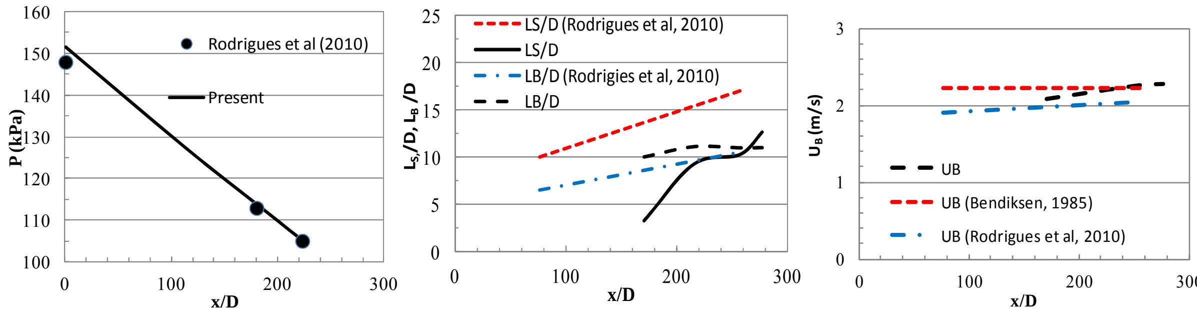


(a) Bubble length (b) Slug length (c) frequency

Figure 1. – Momentum flux coefficient and pressure jump influence on slug parameters with varying meshes.

A comparison of the present numerical predictions with the experimental data of Rodrigues et al (2010) is shown in Fig. 2. The results were obtained with the flux distribution parameter C_l=1.34, without pressure jump, and with mesh size Δx/D=0.4. Fig. 2a shows the comparison of the pressure distribution along the pipe, where an excellent agreement was obtained. The statistically steady slug parameters are shown in Figs. 2b and 2c. Good agreement with experimental data was obtained for the bubble length distribution along the pipe. The slug length was underpredicted, specially near the entrance. It is very difficult to faithfully represent the experimental inlet conditions, therefore, there seems to be a longer "transition-length" in the numerical predictions. Simulations with longer pipe lengths are currently being investigated.

Figure 2c compares the bubble velocity measured by Rodrigues et al (2010), with the present results and Bendiksen (1985) correlation. It can be seen that excellent agreement was obtained.



(a) Pressure (b) Slug and bubble length (c) bubble velocity

Figure 2.– Distribution along the pipe of pressure, slug and bubble length and bubble velocity

5. CONCLUSION

In this work the momentum flux parameter and pressure jump were incorporated in the 1D Two-Fluid Model to investigate the well-posedness of the equation system, for the simulation of slug flow in a vertical pipe. By examining the ability to obtain mesh independent solutions, it was shown that the momentum flux parameter was more effective than the pressure jump term. The results were compared with available experimental data and good agreement was obtained. Nevertheless, investigations on a wider range of cases and the model performance for different values of the momentum flux parameter in slug flows are still necessary.

6. ACKNOWLEDGEMENTS

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