CONSTRUCTAL DESIGN OF T-SHAPED ASSEMBLIES OF FINS COOLING A CYLINDRICAL SOLID BODY

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Abstract. *This paper considers the constructal design of the shape of T-shaped assembly of fins cooling a cylindrical solid body. The objective is to minimize the maximal excess of temperature between the solid cylindrical body and the ambient. Internal heat generating is distributed uniformly throughout the solid body. The T-shaped assembly of fins are bathed by a steady stream with constant ambient temperature and convective heat transfer. The outer surfaces of the cylindrical body are adiabatic. The objective function is the maximal excess of temperature that is minimized subject to constraints: the total volume of the body and the total volume of the assembly of fins. The fraction of the area occupied by one fin, ψ, and the ratio between the total volume of the assembly of fins and the volume of the cylindrical body, ϕ, are design parameters. The optimized geometry and performance are reported graphically as functions of ϕ, and ψ.*

***Keywords**:* Constructal design, heat generation, cylindrical solid body, T-shaped assembly fins.

1. INTRODUCTION

Increasing numbers of people have been applying Constructal theory (Bejan and Lorente, 2008) to optimize the performance of flow systems by generating geometry, flow structure and to explain natural self organization. In Bejan et.al., 2008, and Bejan and Lorente, 2011, it is demonstrated that the most basic features of tree and forest architecture can be put on a unifying theoretical basis provided by the constructal law. Trees and forests are studied as integral components (along with river basins, atmospheric and oceanic circulation, etc) of the much greater global architecture that facilitates the flow of water in nature and the flow of stresses between wind and ground.

The same principle employed for the determination of natural configuration across the board (river basins, turbulence, animal design, vascularization, locomotion, cracks in solids, dentritic solidification, earth climate, droplet impact configuration, etc) is also used to yields new designs for electronics, fuel cells, and tree networks for transport of people, goods and information (Bejan and Lorente, 2006, Rocha et.al., 2010, Rocha et.al., 2005, Biserni et.al., 2007, Lorenzini et.al., 2011). The maximization of the rate of heat transfer in a given volume has become the basic principle of designing structures for heat and fluid flow. An important thermal design constraint is that temperatures have not to exceed a certain threshold. This approach is consistent to the constructal method in its original focus (Bejan, 1997 and Bejan, 2000), where design is the result of a "permanent struggle for better and better global system performance under global constraints".

This paper documents numerically the relation between the maximization of global performance and the morphing architecture of a flow system. The present numerical study aims to discover, by means of the constructal method, the optimal geometric configuration of a T-shaped assembly of fins cooling a cylindrical solid body with uniform internal heat generation. The assembly of fins is bathed by a steady stream with constant ambient temperature and convective heat transfer, while the solid body has adiabatic conditions on the outer surface.

The total volume of the body and the total volume of the fins are fixed, but the fins lengths are free to vary. The fraction of the area occupied by one fin, ψ, and the ratio between the total volume of the assembly of fins and the volume of the cylindrical body, ϕ, are design parameters. The optimized geometry and performance are reported graphically as functions of ϕ, and ψ. Volume-to-point (or area-to-point) heat conduction problem was initially defined in Ref. (Bejan, 1997) as follows: "consider a finite volume heated uniformly, with a finite amount of high conductivity material. Determine its optimal distribution through the given volume such that the highest temperature, i.e. the hot spot, is minimized."

The problem statement here treated is not conceptually dissimilar from the above mentioned Bejan's conduction problem: in this paper we attach fins to remove heat by means of the convection mechanism instead of the insertion of the high conductivity material, i.e. the conduction path.

2. METHODOLOGY

Consider the domain shown in Fig. 1. There is an adiabatic cylindrical body with internal constant heat generation per unit of volume q''' and constant thermal conductivity k. Attached to the cylinder is a T-shaped assembly of fins. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with the diameter of the cylinder. The heat transfer coefficient h is uniform over all the exposed surfaces of the T-shaped assembly of fins and the temperature of the fluid (T_∞) is known. The maximum temperature (T_{max}) occurs into the cylinder and varies with the geometry of the T-shaped assembly of fins.

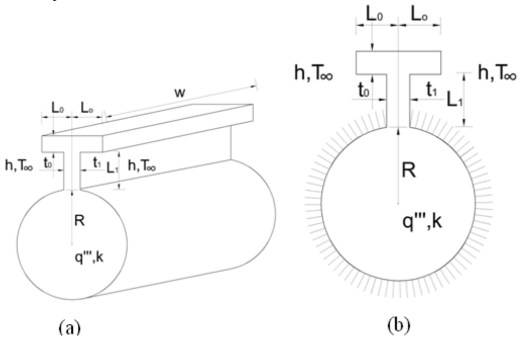


Figure 1. T-shaped assembly of fins cooling a cylindrical solid body

The objective of the analysis is to determine the optimal geometry (L_1/L_0, t_1/t_0) that is characterized by the maximal excess of temperature $(T_{max} - T_\infty)/q'''A/k_b$, where k_b is the thermal conductivity of the body. According to constructal design (Bejan and Lorente, 2008), this optimization can be subjected to constraints, namely, area of the cylindrical body constraint,

$$A = \pi R^2 \tag{1}$$

the fin-material area constraint,

$$A_f = N(2L_0t_0 + L_1t_1) \tag{2}$$

where N is defined as the number of T-shaped assemblies of fins, and the approximate area occupied by one fin,

$$A_c = 2L_0L_1 \tag{3}$$

Equations (2) and (3) can be expressed as the fin area fraction

$$\phi = \frac{A_f}{A} \tag{4}$$

and the fraction of the area occupied by one fin

$$\psi = \frac{A_c}{A} \tag{5}$$

The analysis that delivers the maximal excess of temperature as a function of the e assembly geometry consists to solve numerically the heat conduction equation along the entire domain. The solid body is governed by the steady heat conduction equation with heat generation

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \tag{6}$$

while the heat conduction equation without heat generation is applied in the T-assembly of fins

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \tag{7}$$

where the dimensionless variables are

$$\theta = \frac{T - T_\infty}{q''' A / k_b} \tag{8}$$

and

$$\tilde{x}, \tilde{y}, \tilde{L}_0, \tilde{L}_1, \tilde{t}_0, \tilde{t}_1, \tilde{R} = \frac{x, y, L_0, L_1, t_0, t_1, R}{A^{1/2}} \tag{9}$$

The outer surfaces of the cylindrical solid body are insulated and the boundary conditions are given by

$$\frac{\partial \theta}{\partial \tilde{n}} = 0 \tag{10}$$

while the boundary conditions on the fin surfaces are

$$-\frac{\partial \theta}{\partial \tilde{y}} = \lambda\theta \qquad \text{or} \qquad -\frac{\partial \theta}{\partial \tilde{x}} = \lambda\theta \tag{11}$$

where the parameter λ is defined by

$$\lambda = \frac{hA^{1/2}}{k_f}$$

and k_f is the thermal conductivity of the fin.

The dimensionless form of equations (1) and (4) and (5) are

$$1 = \pi \tilde{R}^2 \tag{13}$$

$$\phi = 2\tilde{L}_0\tilde{t}_0 + \tilde{L}_1\tilde{t}_1 \tag{14}$$

$$\psi = 2\tilde{L}_0\tilde{L}_1 \tag{15}$$

The dimensionless maximal excess of temperature, θ_{max}, according to equation (8) is given by

$$\theta_{max} = \frac{T - T_\infty}{\frac{q''' A^{1/2}}{k_b}} \tag{16}$$

3. RESULTS

It is not possible to express the global objective function in analytical form, in terms of the geometric parameters of the solid cylindrical body and the T-shaped assembly of fins. This function can be determined numerically, by solving for the temperature field in every assumed configuration to see whether the dimensionless excess of temperature can be minimized by varying the configuration.

Equation (6 - 7) were solved using a finite elements code, based on triangular elements, developed in MATLAB environment, precisely the PDE (partial-differential-equations) toolbox (MATLAB, 2000). Details about the optimization method, numerical method, mesh refinement and validation can be found in Rocha et al., 2010.

The structure shown in Fig. 1 has two degrees of freedom: L_1/L_0, t_1/t_0. We start the simulations varying the ratio t_1/t_0 and keeping constant the other degree of freedom. Figure 2a shows that there is an optimal ratio $(t_1/t_0)_o$ that minimizes the dimensionless maximal excess of temperature, θ_{max}, when the ratio L_1/L_0 is fixed as well the parameters ϕ, ψ, and λ. It is a shallow minimum for small values of the ratio L_1/L_0 and becomes more pronounced when the ratio L_1/L_0 increases. The results of Fig. 2a were summarized in Fig.2b, which presents the once minimized excess of temperature, $(\theta_{max})_m$, and the once optimized ratio $(t_1/t_0)_o$, as function of the ratio L_1/L_0. The results show that $(\theta_{max})_m$, increase and $(t_1/t_0)_o$ decreases when the ratio L_1/L_0 increases. Some of the best shapes calculated in Fig. 2a and 2b are shown in Fig. 3.

The procedure used in Figs. 2a and 2b is repeated now for several values of the area fraction ψ. Figure 4a shows that the once minimized dimensionless maximal excess of temperature, $(\theta_{max})_m$, decreases as the of the area occupied by one fin ψ increases. The result makes sense because increasing ψ means increasing the freedom that the configuration has to morph leading to a better performance. Note that $(\theta_{max})_m$, decreases as the ratio L_1/L_0 decreases.

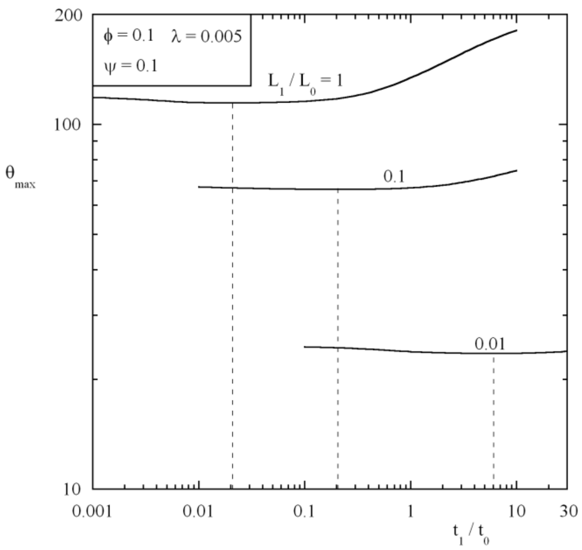


Figure 2a. The minimization of the dimensionless maximal excess of temperature as a function of the ratio t_1/t_0 for several values of the ratio L_1/L_0.

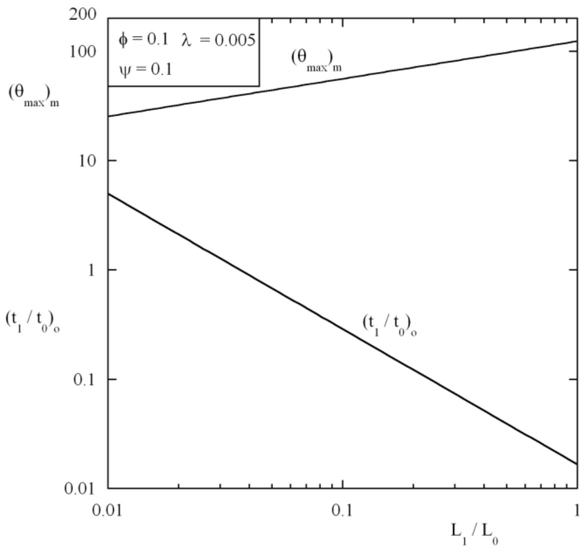


Figure 2b. The once minimized dimensionless maximal excess of temperature and its corresponding optimal ratio $(t_1/t_0)_o$ as a function of the ratio L_1/L_0.

Figure 4b shows that once minimized ratio $(t_1/t_0)_o$ increases as the fraction of the area occupied by one fin, ψ, also increases and the ratio L_1/L_0 decreases . Some of the best shapes calculated in Figs. 4a and 4b are shown in Fig. 5 when $\psi = 0.5$.

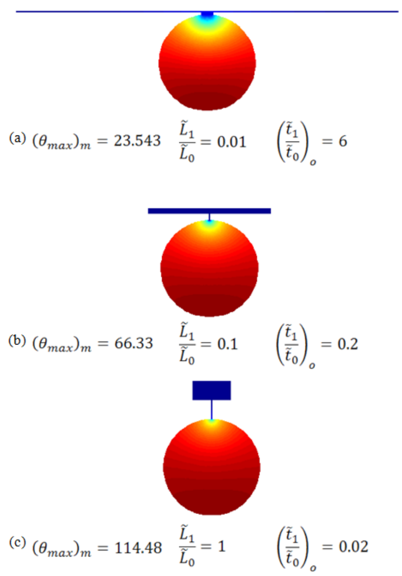


Figure 3: The best shapes calculated in Fig. 2a and 2b.

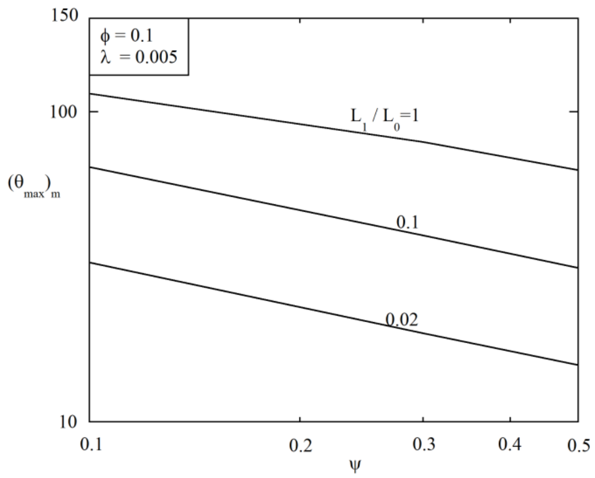


Figure 4a. The once minimized dimensionless maximal excess of temperature as a function of the fraction ψ for several values of the ratio L_1/L_0 ($\psi = 0.5$).

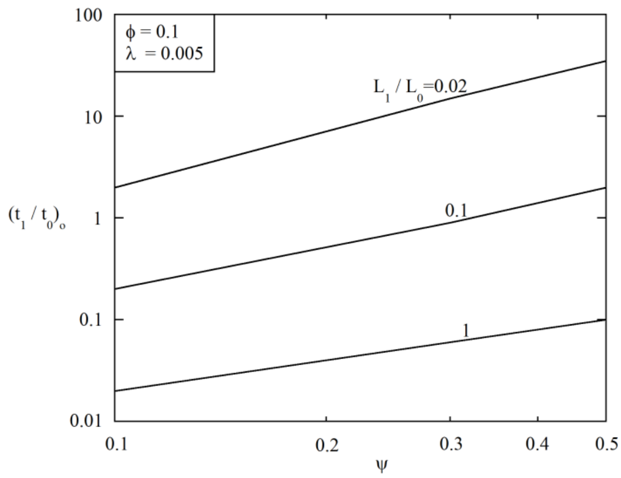
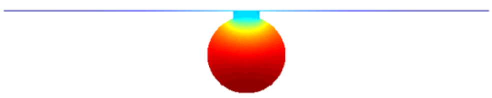
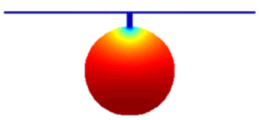


Figure 4b. The once minimized ratio $(t_1/t_0)_o$ as a function of the fraction ψ for several values of the ratio L_1/L_0 ($\psi = 0.5$).

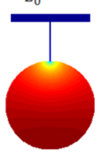
In Figure 6a shows that the once minimized dimensionless maximal excess of temperature decreases as the area fraction ϕ increases. However, this effect is not very sensitive for small values of the ratio L_1/L_0. For example, $(\theta_{max})_m$ is approximately constant when $L_1/L_0 = 0.01$. In the other side, the optimal ratio $(t_1/t_0)_o$ decreases as the area fraction ϕ increases and the ratio L_1/L_0 increases. Figure 7 shows the best shapes calculated in Figs. 6a and 6a when ϕ = 0.2. Note that the optimal shapes shown in Figures 3, 5 and 7 illustrate how the hot spot, the points with highest temperatures, are distributed into the cylindrical body to achieve better performance. In sum, the best performance is achieved for smaller L_1/L_0, larger $(t_1/t_0)_o$, and larger area fractions ϕ and ψ.



(a) $(\theta_{max})_m = 15.2444$ $\frac{\tilde{L}_1}{\tilde{L}_0} = 0.02$ $\left(\frac{\tilde{t}_1}{\tilde{t}_0}\right)_o = 35$



(b) $(\theta_{max})_m = 31.3774$ $\frac{\tilde{L}_1}{\tilde{L}_0} = 0.1$ $\left(\frac{\tilde{t}_1}{\tilde{t}_0}\right)_o = 2$



(c) $(\theta_{max})_m = 64.8241$ $\frac{\tilde{L}_1}{\tilde{L}_0} = 1$ $\left(\frac{\tilde{t}_1}{\tilde{t}_0}\right)_o = 0.1$

Figure 5: The best shapes calculated in Fig. 4a and 4b ($\psi = 0.5$).

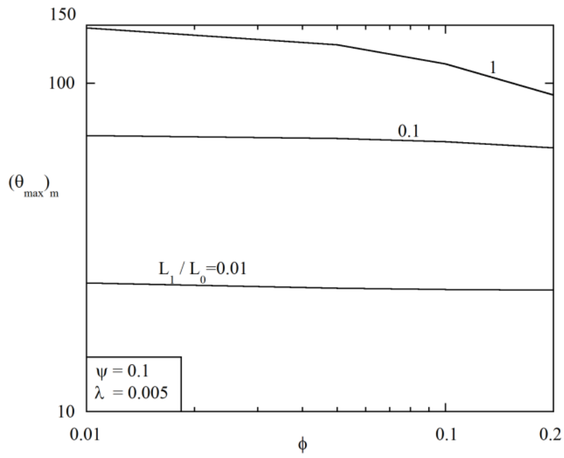


Figure 6a. The once minimized dimensionless maximal excess of temperature as a function of the area fraction ϕ for several values of the ratio L_1/L_0.

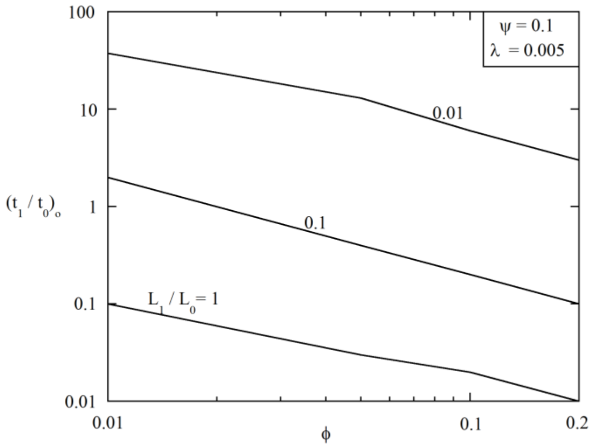


Figure 6b. The once minimized ratio $(t_1/t_0)_o$ as a function of the area fraction ϕ for several values of the ratio L_1/L_0.

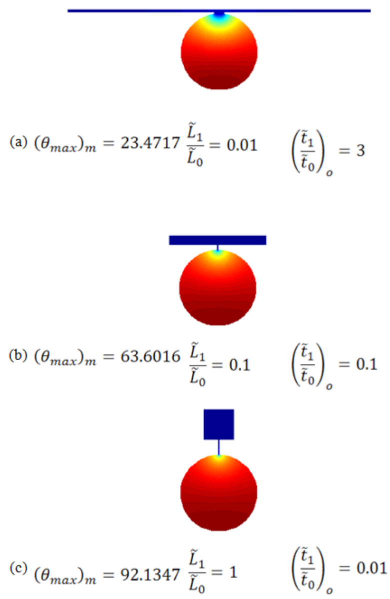


Figure 7: The best shapes calculated in Fig. 6a and 6b ($\phi = 0.2$).

4. CONCLUSIONS

This work applies constructal method to obtain the optimal geometric architecture of a T-shaped assembly of fins cooling a cylindrical solid body, with uniform internal heat generation. The assembly of fins is bathed by a steady stream with constant ambient temperature and convective heat transfer, while the solid body has adiabatic conditions on the outer surface.

The results indicate that there is an optimal ratio $(t_1/t_0)_o$ that minimizes the dimensionless maximal excess of temperature, θ_{max}, when the ratio L_1/L_0 is fixed as well the parameters ϕ, ψ, and λ. It is also important to notice that in general the once minimized maximal excess of temperature, $(\theta_{max})_m$, decreases as the ratio L_1/L_0 decreases and the ratio t_1/t_0 increases, and the optimal ratio $(t_1/t_0)_o$ decreases as the ratio L_1/L_0 increases. Better performance is also achieved larger area fractions ϕ and ψ. The best shapes also are the ones that distribute better the imperfections, i. e. the hot spots into the cylindrical body.

Future work will include the study of the effect of increasing the number of T assembly of fins and λ on the behavior of the dimensionless maximal excess of temperature.

5. ACKNOWLEDGEMENTS

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