INVESTIGATION OF HEAT FLUX AT THE CUTTING INTERFACE: COMPARISON BETWEEN SEQUENTIAL FUNCTION SPECIFICATION E DYNAMIC OBSERVERS BASED ON GREEN'S FUNCTION

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Abstract. *This study presents an investigation of heat flux generated at the cutting interface during an orthogonal cutting process. The main idea is to reconstruct the tool heat flux boundary using inverse technique. Two inverse techniques were evaluated, Sequential Function Specification and Dynamic Observers based on Green's Function Inverse Techniques. The thermal model was obtained by numerical solution of the transient three-dimensional heat diffusion equation considering a known heat flux. Finite volumes method was used to solve the heat diffusion equation. Some numerical simulations were performed in order to evaluate the techniques. Using synthetic temperatures the heat flux was estimated. Both techniques presented a good accordance with the imposed heat flux.*

keywords: *Inverse Problem, Dynamic Observer , Sequential Method, Orthogonal cutting, Heat Flux Estimation.*

1. INTRODUCTION

Inverse problems can be found in all areas of science. The advantage of inverse techniques consists in the possibility of indirectly obtaining the solution of a physical problem with partially unavailable information. For example, the determination of a surface temperature without direct access or the diagnosis of diseases using computerized tomography. In both cases, the boundaries are unknown and inaccessible. These problems can be solved using information obtained from sensors located in accessible positions.

Different techniques can be found in literature in the solution of inverse heat conduction problem (IHCP). For instance, the mollification method (Murio D. A., 1993),the conjugate gradient technique (Alifanov O. M., 1978), the sequential function specification (Beck J. V *et al*, 1985) or the use of optimization techniques such as genetic algorithms (Raundensky M. *et al,* 1995), simulated annealing (Gonçalves C. V. *et al* 2006) or golden section method (Carvalho S. *et al, 2006*).Technique based on filter such as the Kalman filter (Tuan P-C *et al*, 1996) or dynamic observers (Blum J. W. and Marquardt W., 1997) (Sousa, P.F.B. *et al, 2008*)have also been employed for the solution of the IHCP.

The Sequential Function Specification developed by Beck, J. V. *et al* (1985), is a sequential method stepping forward in time, based on least squares method and Duhamel theorem, well established and has been used successfully for about 30 years.

The Dynamic Observers Based on Green's Function inverse technique, proposed by Sousa (2008), is based on the concept that the IHCP solution algorithms are interpreted as filters passing low-frequency components of the true boundary heat flux signal while rejecting high-frequency components in order to avoid excessive amplification of measurement noise (Blum J. W. and Marquardt W., 1997), the proposal of obtaining numerically the heat f transfer unction based on Green functions gives a great flexibility to the technique allowing the technique to deal with 3D transient problems both simulated and experimental.

In order to estimate the heat flux boundary of a machining process both techniques, specified Function Method and Dynamic Observers Based on Green's Function Inverse Technique, were tested and presented suitable results for simulated cases.

3. FUNDAMENTALS

3.1 Thermal model

The 3D-transient thermal problem shown in Fig. 1 represents a cemented carbide tool and can be described by diffusion equation as

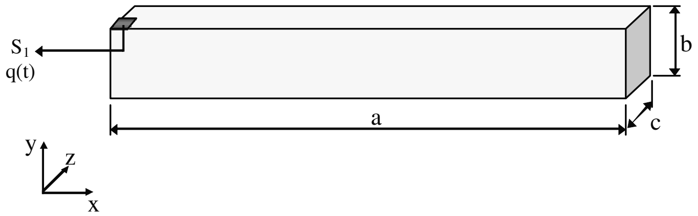


Figure 1. 3D transient thermal model

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \tag{1a}$$

In the region R (0 < x < a, 0 < y < b, 0 < z < c) and t> 0, subjected to the boundary conditions, where a= 0.1016 m and c=b=0.0127 m,

$$\begin{aligned} & -k\left.\frac{\partial \theta}{\partial y}\right|_{y=b} = q(t) \text{ on } S_1 \ (0 \le x \le x_H,\ 0 \le z \le z_H) \\ & -k\left.\frac{\partial \theta}{\partial y}\right|_{y=b} = 0 \text{ on } S_2 \ (x, z \in \left. S\right|_{(x,z)} \notin S_1) \end{aligned} \tag{1b}$$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = \left.\frac{\partial \theta}{\partial x}\right|_{x=a} = \left.\frac{\partial \theta}{\partial z}\right|_{z=o} = \left.\frac{\partial \theta}{\partial z}\right|_{z=c} = \left.\frac{\partial \theta}{\partial y}\right|_{y=b} = 0 \tag{1c}$$

and initial condition

$$\theta(x, y, z, 0) = T(x, y, z, 0) - T_0 \tag{1d}$$

where S is defined by $(0 \le x \le a, 0 \le z \le c)$ and x_H and z_H are the boundary of S_1 where the heat flux is applied.

3.2 Sequential Method

The inverse technique proposed by Beck J. V. *et al* (1985), called sequential function specification, involves numerical inversion of a convolution integral and the use of future steps in time. Based on the least squares method the technique can be described by the Eq. (2), that relates measured temperature with temperature calculated by the numerical thermal model

$$S = \sum_{i=1}^{r}\left(Y(L,t)_{M+i-1} - T(L,t)_{M+i-1}\right)^2 \tag{2}$$

Thus, Eq. (2) is minimized with respect to heat flow component to be estimated, where the index M represents the current step in time and r, the future steps. In this procedure the components of estimated heat flow, q_1, $q_2, \cdots$, q_{M-1}, are assumed to be known and the goal is the estimation of q_M. The values of $q_{M+1}, q_{M+2}, ..., q_{M+r-1}$ are assumed temporarily constant, i.e., $q_M = q_{M+1} = q_{M+2} = q_{M+r-1} = q_M^*$,

$$q_M = q_M^* + \Delta q_M \tag{3}$$

$$\Delta q_M = \frac{\sum_{i=1}^{r}\left(Y_{M+i-1} - T^*_{M+i-1}\right)\cdot X_{M+i-1}}{\sum_{i=1}^{r} X^2_{M+i-1}} \quad \text{where,} \quad X(x,t)_{M+i-1} = \frac{\partial T(x,t)_{M+i-1}}{\partial q_M} \tag{4}$$

3.2 Dynamic Observers Based on Green's Function inverse Technique

The unknown heat flux ***q(s)*** applied to the conductor (reference model), G_H, results in a measurement signal θ_M corrupted by noise *N*, as shown in Fig. 2.

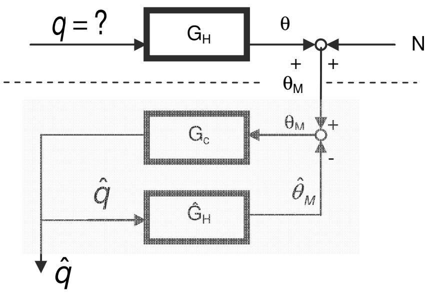


Figure 2. Frequency-domain block diagram proposed by Blum, J. W. and Marquardt, W (1997).

$$\theta_M = \theta + N = G_H\, q + N \tag{5}$$

The estimated value $\hat{q}$ can be computed from the output data θ_M. Thus, the estimator proposed by Blum, J. W. and Marquardt, W (1997).can be represented in a closed-loop transfer function of the feedback loop (Fig. 2) as

$$\hat{q} = G_Q\, q + G_N\, N \tag{6}$$

where the transfer function G_Q is chosen to have the behavior of type I Chebychev filter and G_N is identified by $G_N = G_Q\, G_H^{-1}$ provided that G_H is obtained.

In Eq. (6) if the algorithm estimates the heat flux correctly, G_Q is equal to unity and the frequency ***w*** is within the pass band. In which case, the noise transfer function G_N is equal to the inverse transfer function of the heat conductor, G_H^{-1}.

The inverse algorithm represented by the closed-loop transfer function of the feedback loop (Fig. 2) can be resumed using two discrete-time difference equations

$$q(k) = \sum_{i=0}^{n_n} b_{N,i}\, Y_M(k-i) - \sum_{i=1}^{n_n} a_{N,i}\, q(k-i) \tag{7}$$

and

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_{Q,i}\, q(k-i) - \sum_{i=1}^{n_n} a_{Q,i}\, \hat{q}(k-i) \tag{8}$$

where the coefficients $a_{N,i}$ and $b_{N,i}$ in Eq.(7) are the coefficients of the noise transfer function G_N and the coefficients $a_{Q,i}$ and $b_{Q,i}$ in Eq. (8) are the coefficients of type I Chebychev filter, G_Q .In which case, the inverse procedure is concluded with the identification of $\overline{G}_H(r, y, s)$, Sousa P.F.B. *et al* (2008).

4. RESULTS

4.1 Simulated cases

The inverse techniques, Sequential Function Specification and Dynamic Observers Based on Green's Function, were applied to solve some simulated cases. Simulations are important to evaluate the inverse technique algorithm and to investigate possible problems of sensitivity. This section present one of the simulated test cases.

Simulated temperature data of the direct problem were generated using the solution of Eq. (1) considering a known heat flux evolution *q(t)*. Random errors were then added to these temperatures. The temperatures with error were then used in the inverse algorithm to reconstruct the imposed heat flux.

The synthetic temperatures were calculated from the following equation

$$Y(L,t) = T(L,t) + \varepsilon_j \tag{9}$$

where ε_j is a random number. The parameter ε_j assumed values within ± 1°C . The tests simulate a cemented carbide tool as shown in Fig (1). Table 1 describes simulated thermocouples position. Figure (3) shows synthetic temperatures obtained from the direct problem.

Table1.: Simulated thermocouples position

Thermocouples	x [10^{-3} m]	y [10^{-3} m]	z [10^{-3} m]
1	3.03	12.7	3.42
2	3.35	12.7	9.94
3	5.14	7.12	12.7
4	9.17	12.7	11.3
5	9.11	12.7	2.53
6	9.88	5.06	0.

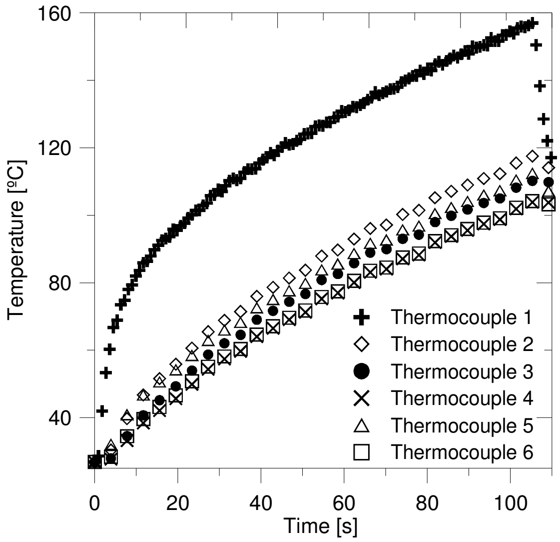
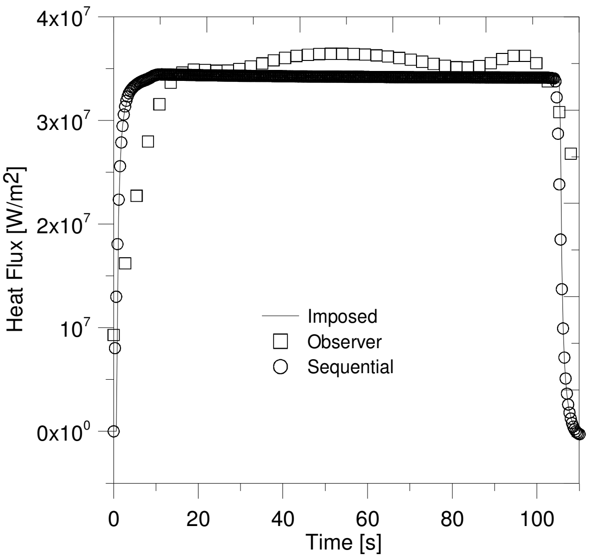


Figure 3. Synthetic temperatures

Using the inverse techniques and the synthetic temperatures heat flux was estimated, Fig. (4a e 4b).



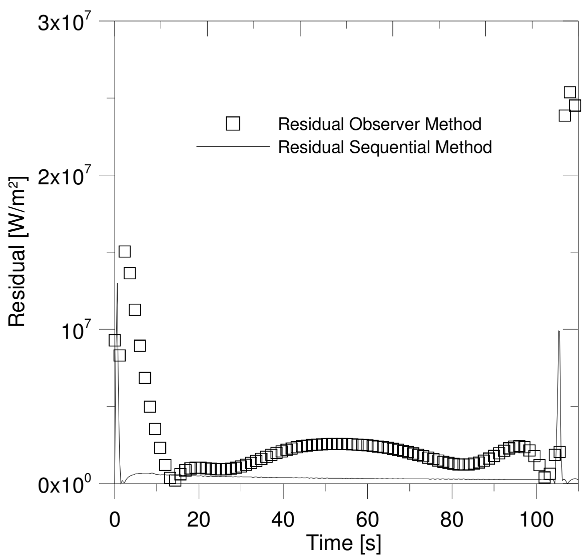


Figure 4. Comparison between the measured and estimated heat flux using $Y_1(t)$: a) Heat Flux b) Residual

5. CONCLUSION

Both techniques presented suitable results. Although the Sequential Function Specification has presented better approach than the inverse technique of Dynamic Observers Based on Green's Function despite of this the Dynamic Observers Based on Green's Function will be applied to the next step of the research. This technique has shown to be a robust method to deal with real machining process.

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