USE OF THE GITT FOR SOLVING EQUATIONS IN AGRICULTURE PRODUCTS

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Abstract. *This paper deals with the numerical simulation of heat and moisture transfer in capillary-porous bodies. The mathematical model is based on a set of coupled, heat and mass transfer equations proposed by Luikov subject to specified initial and boundary conditions. The Generalized Integral transform Techinique (GITT) is employed to obtain numerical-analytical solutions for mathematical model that pretict the transient distributions of temperature and moistue in a slab of agricultural products during drying. Was shown that the method have an application to problems of heat and mass transfer in agricultural products. It is shown that the governing dimensionless parameters have a considerable influence on the kinetics of the heat and moisture transfer.*

Keywords*: Mathematical modeling, Luikov's equations, Heat and mass transfer, GITT.*

Nomenclature

a	thermal diffusion coefficient of the porous medium (m²/s)
a_m	moisture diffusion coefficient of the porous medium (m²/s)
B_{im}	dimensionless mass transfer Biot number
B_{iq}	dimensionless heat transfer Biot number
c	specific heat of the porous medium (J/kg K)
h_c	convective heat transfer coefficient (W/m² K)
h_m	convective mass transfer coefficient (kg/m² s°M)
k	thermal conductivity of the porous medium (w/m K)
k_m	moisture conductivity of the porous medium (kg/m s°M)
Ko	Kossovitch number
L	thickness of porous medium (m)
Lu	Luikov number
Pn	Posnov number
T	temperature (ºC)
t	time variable (s)
u	moisture content d.b. (kg/kg)
u^*	equilibrium moisture content d.b. (kg/kg)
X	dimensionless space variable
x	space variable

Greek symbols

δ	thermogradient coefficient (kg/kg° C)
ε	phase conversion factor
θ	dimensionless temperature
λ	latent heat of evaporation (J/kg)
τ	dimensionless time variable
ϕ	dimensionless moisture content

1. INTRODUCTION

The operation of drying has been widely used to preserve food product, and fruits in particular, since the reduction of their water content to certain levels inhibits microbial growth and enzymatic modifications. Dried fruit and vegetables have gained commercial importance and their growth on a commercial scale has given rise to an important sector of the agriculture industry (Karim and Hawlader, 2005). Therefore, over the last few years, numerous researchers have focused on food drying processes (Giri and Prasad, 2007; Jambrak et. al, 2007; Lecorvaisier et. al, 2010; Silva et. al, 2009a; Wald et. al, 2006).

The interrelation between heat and mass transfer and transfer and their gradients was first proposed by Luikov (1966). Thus, for the mathematical modeling of heat and mass transfer in capillary porous media, Luikov (1966) has proposed his widely known formulation, based on a system of coupled partial differential equations, which takes into account the effect of moisture gradient on temperature transfer and temperature gradient on moisture migration. Recently, several articles dealing Luikov equations for different porous media appeared in the literature (Abalone et. al, 2005; Dantas et. al, 2003; Kulasiri and Woodhead, 2005; Younsi et. al, 2006).

A hybrid numerical-analytical solution is obtained by making use of the so-called Generalized Integral Transform Technique (GITT). Its hybrid numerical-analytical structure permits the automatic control of the global error in the simulation, which avoids the need for several computer program runs to inspect for the convergence on the final results, and therefore yields codes that automatically work towards user prescribed accuracy targets (Barros et al, 2006). A number of applications were considered within the last year in different areas, in relation to hybrid approach discussed here, as a representative those listed below (Almeida et. al, 2008; Dantas et. al, 2007; Monteiro et. al, 2009; Naveira et. al, 2007; Silva, et. al, 2010; Venezuela et. al, 2009). However, the application of the GITT to solve heat and mass transfer problems in agriculture products has not been widely explored. Thus, the model of this work aims the study of the food drying, specifically the determination of profiles of temperature and moisture content within the product. For such, this work's methodology employs an analytical-numerical solution technique of many convection-diffusion problems, called Generalized Integral Transform Technique (GITT).

2. PHYSICAL PROBLEM AND MATHEMATICAL MODELLING

The physical problem considered here involves a process of indirect solar drying of a one-dimensional capillary porous medium, initially at uniform temperature and uniform moisture content. Inside the drying chamber the two sides of the product are exposed to a flow of dry air at a temperature T_0, as shown in Fig. 1.

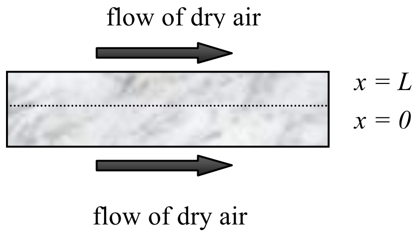

Figure 1. Geometry for the drying of a moist porous medium.

The linear system of equations proposed by Luikov (1966) with associated initial and boundary conditions, for the modeling of such physical problem involving heat and mass transfer in a capillary porous media, can be written in dimensionless form as:

$$\frac{\partial \theta(X,\tau)}{\partial \tau} = (1+\varepsilon LuKoPn)\frac{\partial^2 \theta(X,\tau)}{\partial X^2} - \varepsilon LuKo\frac{\partial^2 \phi(X,\tau)}{\partial X^2}, \qquad 0<X<1, \quad \tau>0 \tag{1}$$

$$\frac{\partial \phi(X,\tau)}{\partial \tau} = -LuPn\frac{\partial^2 \theta(X,\tau)}{\partial X^2} + Lu\frac{\partial^2 \phi(X,\tau)}{\partial X^2}, \qquad 0<X<1, \quad \tau>0 \tag{2}$$

$$\theta(X,0)=0, \qquad \phi(X,0)=0, \qquad \tau=0, \quad 0<X<1 \tag{3,4}$$

$$\frac{\partial \theta(0,\tau)}{\partial X}=0, \qquad \frac{\partial \phi(0,\tau)}{\partial X}=0, \qquad \tau>0\,,\ X=0 \tag{5,6}$$

$$\frac{\partial \theta(1,\tau)}{\partial X} - Biq[1-\theta(1,\tau)] + (1-\varepsilon)BimKoLu[1-\phi(1,\tau)] = 0, \qquad \tau>0\,,\ X=1 \tag{7}$$

$$-\frac{\partial \phi(1,\tau)}{\partial X} + Pn\frac{\partial \theta(1,\tau)}{\partial X} + Bim[1-\phi(1,\tau)] = 0, \qquad \tau>0\,,\ X=0 \tag{8}$$

where the following dimensionless groups were defined:

$$\theta(X,\tau)=\frac{T(x,t)-T_0}{T_{ar}-T_0},\quad \phi(X,\tau)=\frac{u_0-u(x,t)}{u_0-u*}$$
$$\tau=\frac{at}{L^2},\qquad X=\frac{x}{L}$$
$$Biq=\frac{h_cL}{k},\qquad Bim=\frac{h_mL}{k_m},\qquad Lu=\frac{a_m}{a}$$
$$Ko=\frac{\lambda}{c}\frac{u_0-u*}{T_{ar}-T_0},\quad Pn=\delta\frac{T_{ar}-T_0}{u_0-u*}$$

The objective, now, is to determine the temperature and moisture content fields in the food.

2.1 Method of Solution For The Mathematical Model

The mathematical model is solved here applying the Generalized Integral Transform Technique (GITT). The GITT is a powerful hybrid numerical-analytical approach, which has been successfully applied to obtain *benchmark* solutions. Performing some lengthy but straightforward manipulations, we obtain the following coupled system of ordinary differential equations for the transformed variables:

$$\frac{d\bar{\theta}_i(\tau)}{d\tau}+K_{11}\mu_i^2\bar{\theta}_i(\tau)+K_{12}\mu_i^2\sum_{j=1}^{\infty}a_{ij}\,\bar{\phi}_j(\tau)=K_{11}Bim^{**}\tilde{\psi}_i(1)\phi_h(1,\tau)$$
$$+K_{12}\left[(Biq-Bim^{*})\tilde{\psi}_i(1)\phi_h(1,\tau)-BiqPn\tilde{\psi}_i(1)\theta_h(1,\tau)\right] \tag{9}$$
$$\frac{d\bar{\phi}_i(\tau)}{d\tau}+K_{22}\lambda_i^2\bar{\phi}_i(\tau)+K_{21}\lambda_i^2\sum_{j=1}^{\infty}b_{ij}\bar{\theta}_i(\tau)=-K_{22}BiqPn\tilde{\varphi}_i(1)\theta_h(1,\tau)$$
$$+K_{21}\left[(Bim^{*}-Biq)\tilde{\varphi}_i(1)\theta_h(1,\tau)+Bim^{**}\tilde{\varphi}_i(1)\phi_h(1,\tau)\right] \tag{10}$$

3. RESULTS AND DISCUSSION

The solution to the heat and mass transfer problem was obtained by experimental results given by Karim and Hawlader (2005), where a tunnel type solar dryer was used for drying banana. The necessary parameters for the solution of this problem are presented in Tab. 1.

Table 1. Experimental drying conditions and product properties (banana).

Properties	Test 1
L (m)	2.0×10^{-3}
ρ (kg/m³)	980
h_c (W/m² K)	31.45
h_m (kg/m² s°M)	4.97×10^{-7}
k (w/m K)	0.5424
K_m (kg/m s°M)	4.73×10^{-10}
c (J/kg K)	3350
c_m (kg/kg °M)	3.0×10^{-3}
λ (J/Kg)	2.3830×10^{6}
T_0 (°C)	24
T_{ar} (°C)	50
v_{ar} (m/s)	0.50
u_0 d.b.(kg/kg)	4
u^* d.b. (kg/kg)	0.22
δ (kg/kg°C)	0.015
Bi_q	**0.116**
Bi_m	**2.10**
Lu	**0.0010**
Pn	**0.103**
ko	**103.42**
ε	**0.3**

3.1 Comparison of Luikov Model and Fick Model

The distribution of moisture content for the positions of the center (X = 0.0) and surface (X = 1.0) of the food are shown in Fig. 2. A justification for the discrepancies found between the curves can be attributed to the fact that Fick's law assumes that moisture migration in the product is only due to a moisture gradient. While Luikov's equations for the model presented in this article refers to the simultaneous heating and mass transference and assume that the movement of humidity is conducted by gradients of temperature and humidity, which, by thermogradient effect offers resistance to outflow of humidity.

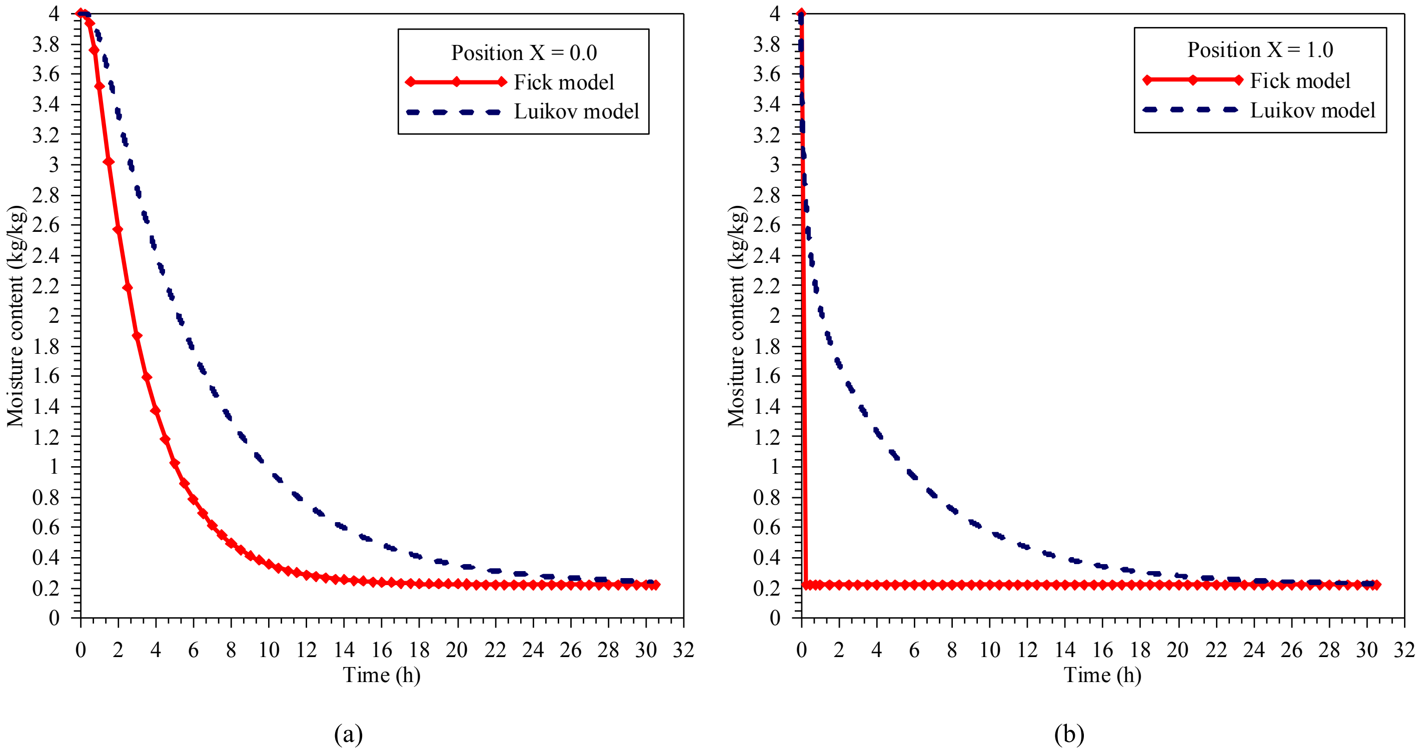

Figure 2. Comparison between the distributions of moisture content with the of Fick and Luikov models.

The curves shown in Fig. 2 are characteristic of the decreasing period of drying. In this period the vaporization of water, still present in the surface, will be responsible for generating a moisture gradient, starting the movement of water from the interior to the surface and a significant amount of heat is conducted from the surface into the center of the food. Thus, temperature gradients in the dry zone may not be negligible, and its effect on the drying should be considered.

In more severe drying conditions it becomes important the knowledge of the temperature in the product, since higher temperatures the product can provide the possibility of alteration its color (accelerating the browning effect), texture and flavor, thereby reducing its quality. In these cases, there is a limitation on Fick's law and the use of Luikov model is more appropriate.

3.2 Heat and Mass Transfer

The analysis here is directed to study the behavior of the simultaneous process of heat and mass transfer. Figures 3 and 4 show the variations of temperature and moisture content for different times and positions within the product.

It is verified that for short periods of drying there was a strong decrease in moisture content, u, in the region near the surface, $X = 1$, due to intense convective mass transfer that occurs when the potential differences of moisture on the border are more significant. This caused a rapid drop in temperature T, reaching values below T_0. However, this phenomenon is not always observable in the foods drying due to the short space of time in which subsists, where in the case studied was approximately 0.1 h of drying. After this initial time, part of the energy received by the surface temperature rise caused in the region, initiating the transfer of heat to the center of the food.

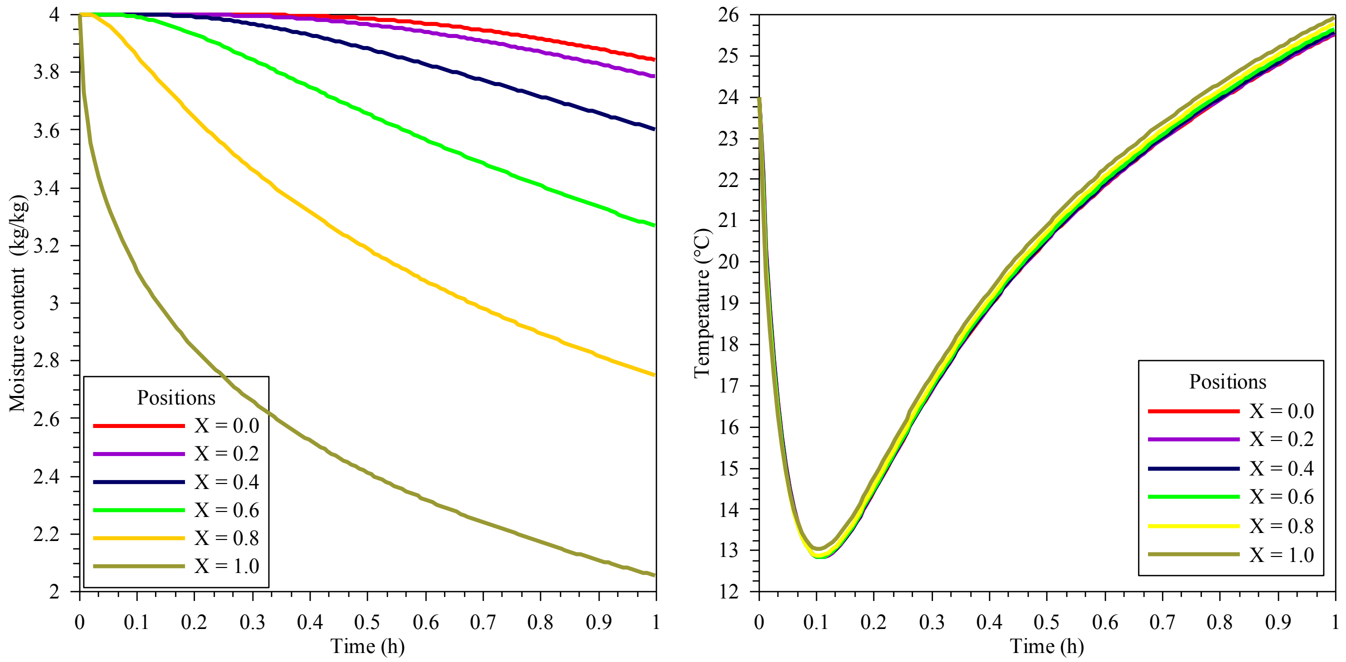

Figure 3. Profiles: (a) moisture content versus time in the first hour of drying, (b) temperature versus time in the first hour of drying

As the process has developed, the small temperature gradient was established on the surface and its dependence on mass transfer decreased. Therefore, during the latter stages of drying was mainly due to the presence of the gradient of moisture content. It was stressed drying the inner layers of the food and moisture content of the sample, in all positions, approached an equilibrium state.

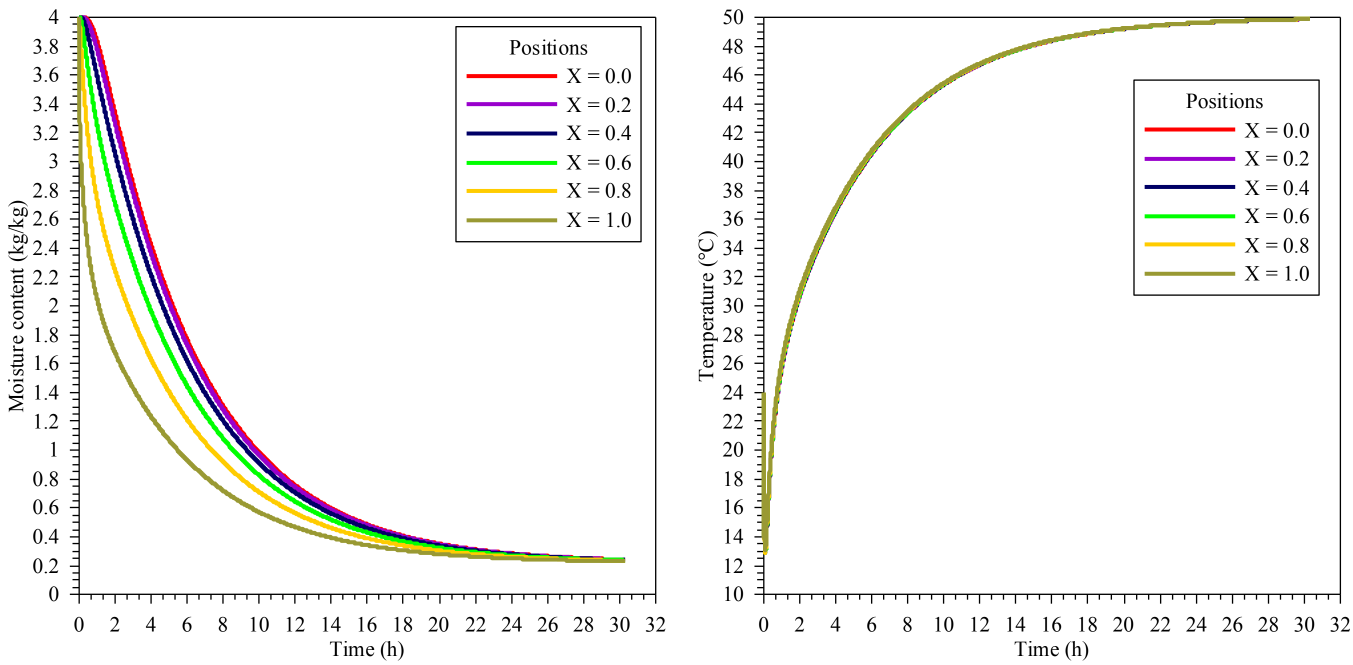

Figure 4. Profiles: (a) moisture content versus time along the whole domain, (b) temperature versus time along the whole domain.

3.3 Analysis of the Luikov Number

The Luikov dimensionless number correlates the ratio of the mass diffusivity and thermal diffusivity. In Figs. 5 and 6 shows the variations of the distributions of temperature and moisture content during the drying process for different values of *Lu*. It can be seen that, as Lu grew characteristic speed in the transport of moisture content was increased significantly, with improvements in the drying process and, consequently, caused a decrease in temperature.

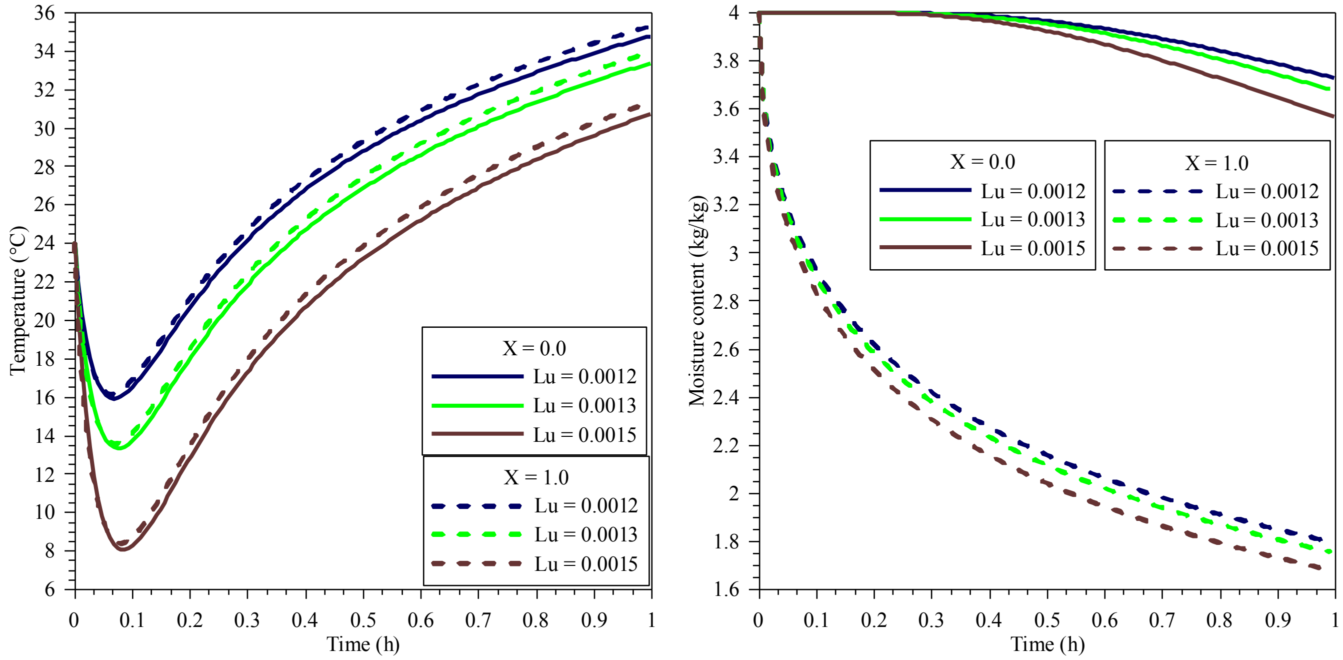

Figure 5. Profiles for the values *Biq* = 0.135, *Bim* = 2.43, *Pn* = 0.142, *Ko* = 74.32 e ε = 0.3, varying *Lu*: (a) temperature versus time in the first hour of drying, (b) moisture content versus time in the first hour of drying.

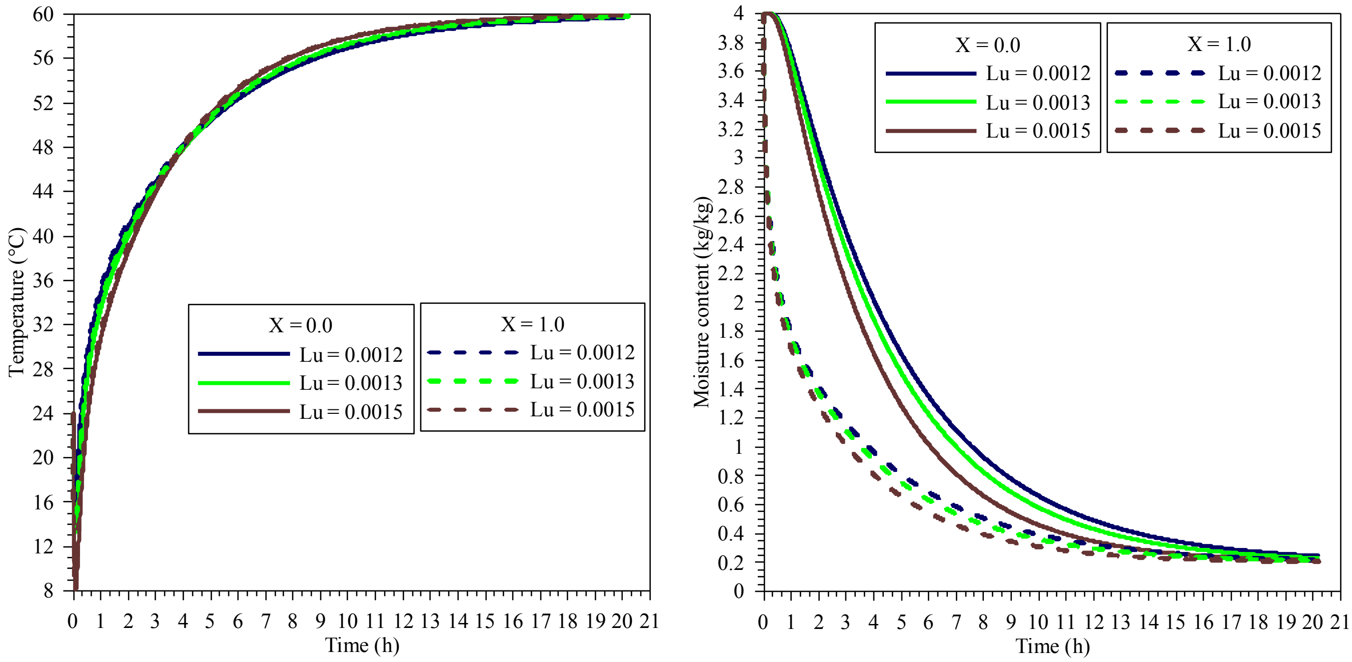

Figure 6. Profiles for the values *Biq* = 0.135, *Bim* = 2.43, *Pn* = 0.142, *Ko* = 74.32 e ε = 0.3, varying *Lu*: (a) temperature versus time along the whole domain, (b) moisture content versus time along the whole domain.

It can understand that for cut fruits in slices of small thickness, when *Lu* <<1, i. e, resistance to mass diffusion is much greater than the resistance to heat diffusion, it means that mass transfer dominates the simultaneous process heat and mass transfer, because this would be a much slower stage than the of heat transfer. This analysis is in agreement with Costa (2008), that studied the banana's drying in a static column dryers and with the drying process of banana examined by Lima (1999) which was also dominated by mass diffusion compared to heat diffusion.

4. CONCLUSIONS

This paper presents a hybrid-analytical method, GITT, for solving the Luikov equations for heat and mass transfer to predict the temperature and moisture content distributions of banana during drying. The results obtained within the framework of the model considered exhibit realistic physical behavior. Therefore, whenever the transport coefficients are available and exist an interest in knowing the food temperature, Luikov model should be used. The importance of the effect of the Luikov number for the heating treatment is demonstrated. The Luikov number affects both the heat and mass transfer.

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