Proceedings of ENCIT 2012
14th Brazilian Congress of Thermal Sciences and Engineering
November 18-22, 2012, Rio de Janeiro, RJ, Brazil

SIMPLIFIED MATHEMATICAL MODEL FOR MIGRATION OF GAS BUBBLES IN VISCOPLASTIC FLUIDS

William Lopez, wlopez@aluno.puc-rio.br

Aline Abdu, abdu@puc-rio.br

Mônica Naccache, naccache@puc-rio.br

Department of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro RJ, Gávea, Rua Marquês de São Vicente, 225, CEP 22451-900.

Abstract. *In this work, a study of the rising movement of single gas bubble in vicoplastic fluid is performed, using the Ansley and Smith equations. The solution is obtained numerically and the numerical results are compared with experimental results from the literature. The calculations are performed for spherical bubbles, low Reynolds number (<1), constant temperature, and neglecting wall effects. The effects of bubble mass and surface tension are analyzed. In addition, a bi-dimensional numerical study is performed, also for a single bubble motion. The numerical solution of the governing conservation equations of mass and momentum is obtained with the FLUENT software, using the finite volume technique and the volume of fluid (VOF) method. The results obtained with the simplified model are in a fair agreement with the literature. Moreover, comparisons will be evaluated between the numerical 2-D model and the simplified one.*

Keywords: *gas bubble, viscoplastic fluid*

1. INTRODUCTION

The study of gas bubble behavior in viscoplastic fluids is of great interest to the industry. In the oil industry, gas bubbles may invade the well during the cementing and cannonade processes. Knowledge of the behavior of gas bubble dynamics inside the cement paste allows a better planning and control of these processes. Therefore, a simplified model providing reliable and fast results, can be very useful to optimize the processes.

2. SIMPLIFIED MATHEMATICAL MODEL

The kinematics of the bubble inside a fluid, driven by buoyancy, is obtained with a force balance at the bubble (Fig. 1), following the procedure described in Pinto et al. (2011):

$$\sum F = -F_e + F_a = m\frac{\partial^2 h}{\partial t^2} = ma \tag{1}$$

Where F_e [N] is the buoyancy force, F_a [N] is the drag force, a [m/s^2] is the bubble acceleration, m [kg] is the bubble mass, and t is time.

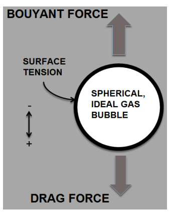

Fig 1. Force balance at the bubble

To calculate the buoyancy force F_e [N], it was used the Archimedes equation, where ρ_p [kg/m^3] is the bubble density:

$$F_e = \frac{1}{6}(\rho_f - \rho_p)g\pi d^3 \tag{2}$$

The bubble diameter is obtained as a function of its displacement, considering that it is a spherical ideal gas bubble in a liquid fluid with constant temperature. Therefore,

$$P\,V = n_{mm}RT \tag{3}$$

Where V [m^3] is the gas volume, P [Pa] is the gas pressure, n_{nm} is the gas molar weight [kg/mol], R [J K/mol] is the universal gas constant and T [k] is the gas temperature. By assuming constant temperature,

$$P_1V_1 = P_2V_2 \tag{4}$$

The pressure is related to the bubble displacement by:

$$P_x = P_o + \rho_f gh + \Delta p_{\tau s} \quad (5)$$

where P_0 [Pa] is the atmospheric pressure, P_x [Pa] is the hydrostatic pressure, $\Delta p_{\tau s} = 4\tau_s / d$ [Pa] is the pressure caused by the surface tension, τ_s , of the fluid in the gas-liquid interface of the bubble, h [m] is the depth, g [m/s^2] is the gravity and ρ_f [kg/m^3] is the fluid density.

Finally, the bubble diameter d [m] is obtained by:

$$d^3 + \frac{4\tau_s}{P_o + \rho_f gh} d^2 - \frac{(P_o d_o + \rho_f gh_o d_o + 4\tau_s)}{P_o + \rho_f gh} d_o^2 = 0 \quad (6)$$

where h_0 [m] is the initial depth of the bubble, and d_0 [m] is the initial diameter.

The viscoplastic fluid behavior is modeled by the Generalized Newtonian Fluid constitutive equation, and the viscosity function is given by the Herschel-Bulkley model

$$\eta = \begin{cases} \frac{\tau_o}{\dot{\gamma}} + k\dot{\gamma}^{n-1} & if \quad \tau \geq \tau_o \\ \infty & if \quad \tau < \tau_o \end{cases} \quad (7)$$

In the equation above, τ_o [Pa] is the yield stress, n is the Power-Law index, k [Pa.s^n] is the consistency index, η is the viscosity and $\dot{\gamma}$ [s^{-1}] is the shear rate fluid

To calculate the drag force F_a [N] we use the model postulated by Ansley and Smith (1967) for Newtonian, Power-Law, Bingham and Herschel-Bulkley fluids where $\mathbf{Re}_g$ is the Generalized Reynolds number, Bi_g is the Bingham number, v [m/s] is the bubble velocity, and x is the correction factor for Power-Law fluids.

$$C_D = \frac{24x}{\mathbf{Re}_g}(1 + kBi_g) \qquad \text{for} \quad 9.6 * 10^{-5} \leq \mathbf{Re}_g \leq 0.36 \,;\, 0.25 \leq Bi_g \leq 280 \,;\, 0.43 \leq n \leq 0.84 \quad (8)$$

$$\mathbf{Re}_g = \frac{v^{2-n} d^n \rho_p}{k} \qquad Bi_g = \frac{\tau_o}{k(v/d)^n} \quad (9)$$

To calculate F_a [N] we use the Beaulne M. and Mitsoulis E. (1997) methodology. Therefore,

$$C_D = \frac{F_a}{8\rho_p v^2 \pi d^2} \qquad F_a = 3x\pi d^{2-n}(m_c v^n + kd^n \tau_O) \quad (10)$$

From the equations (2), (6), and (10) we calculate the bubble acceleration by:

$$\frac{\partial^2 h}{\partial t^2} = \frac{g(\rho_l - \rho_p)}{\rho_p} - \frac{18x}{\rho_p d}\left(\frac{k\left(\partial h/\partial t\right)^n}{d^n} + k\tau_o\right) \quad (11)$$

3. SIMPLIFIED NUMERICAL SOLUTION

The equation for the bubble acceleration is soved numerically using the second and thrid order Runge-Kutta method. We define the maximum acceptable time step in 0.1[sec/iteration] to provide stable results and calculate the effect of the mass of the bubble in the acceleration equation. The results show that for small displacements (<1 m) the bubble mass can be neglected. However, for larger displacements (> 50m), the results are affected considerably. Therefore, we take into account the mass of the bubble in the calculations.

Numerical calculations confirm that the effect of surface tension is considerable when the bubble is being formed, whereas it is not important when the bubble moves.

Figure 2 shows the comparison between the experiments of Raymond F. and J. Rosant (1999) for a gas bubble flowing in a Newtonian fluid and our numerical results. Table 1 shows the rheological properties of the fluid.

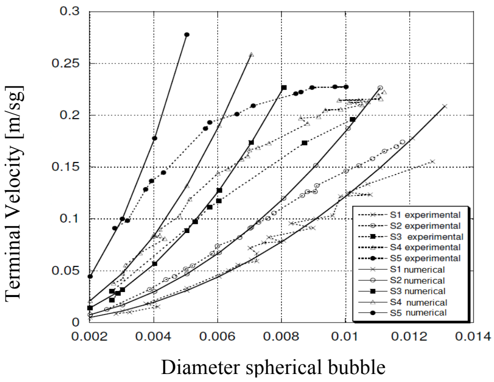

Fig 2. Terminal velocity VS Diameter spherical bubble in Newtonian fluid comparison with experiments (Raymond F. and J. Rosant, 1999)

Table 1. Rheological properties for Newtonian fluid (at 22°C).

Name	Viscosity [Pa.s]	Density [kg/m^3]	Surface tension [N/m]
S1	0.7	1250	0.063
S2	0.46	1245	0.063
S3	0.24	1230	0.063
S4	0.16	1222	0.063
S5	0.075	1205	0.063

Figure 3 shows the comparison between the experiments conducted by Tabuteau et al. (2006) and our numerical results. These results are obtained for a Herschel–Bulkley fluid and rigid spheres, with different densities, b1=1866 kg/m^3, b2=1801 kg/m^3, b3=1736 kg/m^3, b4=1932 kg/m^3 , and a constant sphere diameter, equal to 39.6 mm. Table 2 shows the rheological properties of the fluid.

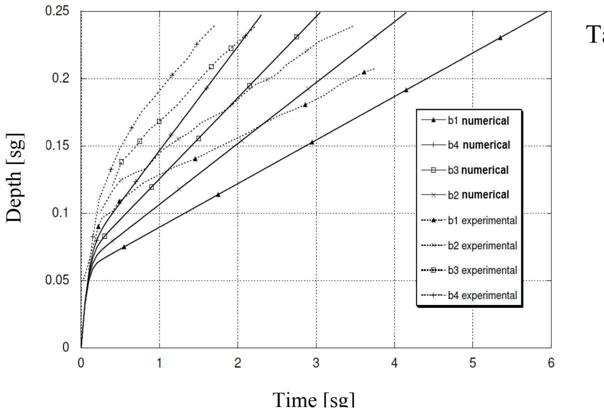

Fig 3: Depth vs. Time in a viscoplastic fluid: comparison with experiments (Tabuteau et al., 2006)

Table 2. Rheological properties for viscoplastic fluid (at 22°C).

k [Pa.s^n]	ρ_f [kg/m^3]	n	τ_o [Pa]
0.7	1000	0.5	7

It is observed that the results compare well for gas bubbles and rigid spheres in Newtonian and viscoplastic fluids, for lower Reynolds numbers. However, for higher Reynolds it is necessary to use different mathematical models, as expected.

4. BI-DIMENSIONAL NUMERICAL SOLUTIONS

The numerical solution of a single bubble motion in Newtonian and viscoplastic incompressible fluids is also analyzed using an axisymmetric bi-dimensional geometry. The governing conservation equations of mass and momentum are discretized via the finite volume method described by Patankar (1980), using the SIMPLE algorithm to couple velocity and pressure. The numerical results are obtained using the commercial software FLUENT (ANSYS). The volume of fluid method (VOF) (Fluent User's Guide, 2010) is used to take into account the multiphase flow. The VOF method solves a set of mass conservation equations and obtains the volume fraction of each phase α_i through the domain, which should sum up unity inside each control volume. Therefore, if $\alpha_i = 0$, the cell is empty of phase i; if $\alpha_i = 1$, the cell is full of phase I and if $0< \alpha_i <1$, the cell contains the interface between the fluids.

In this study, there are only two phases, so that any variable is given by:

$$\varphi = \alpha_2\varphi_2 + (1-\alpha_2)\varphi_1 \tag{16}$$

The interface between phases is obtained by the solution of continuity equation for α_i for the 2 phases:

$$\frac{\partial \alpha_i}{\partial t} + u_i \frac{\partial \alpha_i}{\partial x_j} = 0 \tag{17}$$

The momentum conservation equation is presented below:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_k)}{\partial x_i} = -\frac{\partial P}{\partial x_k} + \frac{\partial}{\partial u_i}\left[\eta\left(\frac{\partial(u_i)}{\partial x_k} + \frac{\partial(u_i)}{\partial x_i}\right)\right] + \rho g_k$$

Figure 4 shows a sketched computational domain. A mesh test was performed, and the selected mesh was a non-uniform one with 120000 elements.

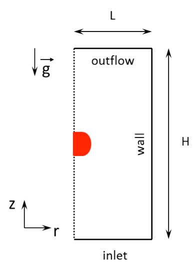

Fig 4. Schematic computational domain.

The objective of this numerical analyzes is to evaluate the assumptions and correlations used in the simplified solution modeling. A quantitative comparison between the different approaches will be performed, to indicate the validity range of the simplified model.

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