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THE IMPORTANCE OF THREE-DIMENSIONAL ASPECTS FOR SIMULATIONS OF FLOWS OVER A BACKWARD FACING STEP

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Abstract. *Flows over a backward-facing step is a classic Fluid Dynamic problem. It consists of a suddenly expansion duct, where the flow develops downstream of the expansion. The geometry is simple (Cartesian) and the detachement point is known, at every time. The flow is complex, where many physical structures are generated, like recirculation zone, Kelvin-Helmholtz instabilities, reattachment of boundary layer, etc. The aim of the present article is to study the influence of tree-dimensional simulations in flow over backward-facing step, for this purpose the IMERSPEC methodology is used to solve the Navier-Stokes and continuity equations. The IMERSPEC consist of to use the Pseudo-Spectral Fourier Method (PSFM). This method provides an excellent numerical accuracy and low computational cost in comparison with another high-accuracy methods. Another important issue is that in Fourier space, the solution of Navier-Stokes equations does not require to solve a Poisson equation, which is usually the most computational onerous part in classic methodologies. In order to model the boundaries conditions of the problem Immersed Boundary Method (IBM) is used. This approach is necessary due to the requirement of periodic boundary conditions to use FFT. The results show that the dependence of flow at high Reynolds numbers of three-dimensional simulations.*

Keywords: *Backward-facing step, Fourier Pseudo-Spectral Method, Immersed Boundary Method*

1. INTRODUCTION

The backward-facing step is widely used to validate new methodologies due to its geometrical simplicity and the high control of detachment point of boundary layer of expansion. On the other hand, the flow developed downstream the expansion is very complex. A large number of experimental (Lee and Mateescu, 1998; Armaly *et al.*, 1983; Eaton and Johnston, 1980) and numerical (Gartling, 1990; Silveira-Neto *et al.*, 1993; Le *et al.*, 1997) studies has been carried out.

The lower boundary layer detachs on the expansion point. Independent of upstream flow the detachment point does not change its position. After this point, a shear layer arises. It is characterized by inflectional mean velocity field, appearing Kelvin-Helmholtz instabilities $(K - H)$. This instabilities are carried and collisions occurs over inferior wall in a particular point, named reattachment point.

Normally this flow is unsteady and therefore the reattachment point must be statistically determined. The $K - H$ instabilities that make collisions against the bottom wall can be transported to the outlet or can be retained into recirculation, left of the reattachment point. After the reattachment point there are strong interactions among $K - H$ eddies and the walls. On the other hand, in the flow through an expansion there is a pressure drop, which is recovered through the outlet channel. So there is an adverse gradient pressure and the superior boundary layer detaches. This fact yields instabilities in the upper wall vicinity. The interactions of instabilities with the walls yield counter-clockwise vortex which travel through the channel. Vortex created in one wall interacts with the opposite wall, as sketched in Fig. 1.

The main characteristic of this problem is that the geometry is very simple, however the flow inside it is very complex, where one can found physical instabilities of several natures, like boundary layer, Kelvin-Helmholtz, collision of instabilities with the walls, interaction between them, boundary layer detachments, boundary layer reattachments and boundary layer and Kelvin-Helmholtz intabilities interactions. The interaction of Kelvin-Helmholtz instabilities with walls creates counter-rotating pairs that can cross the entire channel, going from a wall to the opposite one. Therefore, this geometry results a very complex and interesting benchmark to validate a new methodology.

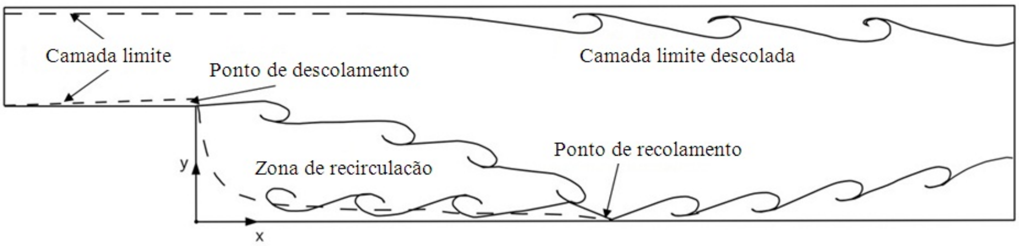

Figure 1. Schematic flow in backward-facing step flow.

2. MATHEMATICAL MODELING

The flow is modeled by momentum equation (Eq. 1) and the continuity equation (Eq. 2). These equations are solved in Eulerian domain, Ω. The information of the fluid/solid interface (domain Γ) is passed to eulerian domain (Ω) for addition of the source term to Navier-Stokes equations. The source term represents the boundary conditions of the immersed geometry as a body force (Goldstein *et al.*, 1993). The equations that govern the problem are presented in tensorial form:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i, \tag{1}$$

$$\frac{\partial u_j}{\partial x_j} = 0, \tag{2}$$

where $\frac{\partial p}{\partial x_i} = \frac{1}{\rho}\frac{\partial p*}{\partial x_l}$; $p*$ is the static pressure in $[N/m^2]$; u_i is the velocity in i direction in $[m/s]$; $f_i = \frac{f_i^*}{\rho}$; f_i is the term source in $[N/m^3]$; ρ is the density; ν is the cinematic viscosity in $[m^2/s]$; x_i is the spatial component (x, y) in $[m]$ and t is the time in $[s]$. The initial condition is any velocity field that satisfies the continuity equation.

The source term is defined in all Eulerian domain, but has values different from zeros only over Lagrangean points coincidents with the Eulerian collocation points. Equation 3 enables the eulerian field perceives the presence of a solid interface (Enriques-Remigio and Silveira-Neto, 2007).

$$f_i(\vec{x}, t) = \begin{cases} F_i\left(\vec{X}, t\right) & \text{if} \quad \vec{x} = \vec{X} \\ 0 & \text{if} \quad \vec{x} \neq \vec{X} \end{cases}, \tag{3}$$

where $\vec{x}$ is the position of a fluid particle and $\vec{X}$ is the position of a fluid particle that is placed besides of the solid interface.

The Lagrangian force field is calculated by direct forcing methodology, (Uhlmann, 2005). It allows the modeling of no-slip boundary conditions on immersed interface. The lagrangian force $F_i\left(\vec{X}, t\right)$ is given solving the momentum equation, Eq. 1, over the fluid-solid interface.

To transform the Eq. 1 to the Fourier spectral space, we apply the Fourier transformed. For instance, Fourier transformed of continuity Eq. 2, gives:

$$\imath k_j \widehat{u}_j = 0, \tag{4}$$

where "$\widehat{\ }$" means that variable is in Fourier spectral space. The Fourier transformation is performed using the FFT algorithm implemented by Takahashi (2006).

Equation 4 defines the wave number vector k_i is orthogonal to transform velocity, $\widehat{u}_i(\vec{k},t)$ and the gradient pressure field is orthogonal to the plane of divergency velocity-free. So, the pressure and velocities fields at Fourier space are not coupled anymore (Canuto *et al.*, 2007). Therefore the momentum equation in the Fourier space, using the projection method, assumes the following form (Mariano *et al.*, 2010):

$$\frac{\partial \hat{u}_i\left(\vec{k},t\right)}{\partial t} + \nu k^2 \hat{u}_i^*\left(\vec{k},t\right) = \hat{f}_i\left(\vec{k},t\right) - \imath k_j \wp_{im} \int_{\vec{k}=\vec{r}+\vec{s}} \hat{u}_m^*\left(\vec{r},t\right) \hat{u}_i^*\left(\vec{k}-\vec{r},t\right) d\vec{r}. \tag{5}$$

The non-linear term is solved using the pseudo-spectral method (Canuto *et al.*, 2007). The velocity product is calculated at physical space and transformed to the spectral space.

3. Results

A simulation of backward-facing step flow at $Re_h = 1000$ was performed in domain shown in Fig. 2 with $h = 0.5$ $[m]$, $L_x/h = 54.86$, $L_y/h = 2.29$ and $L_z/h = 4.30$, divided in $N_x = 768$, $N_y = 32$ and $N_z = 32$ collocation points, respectively. The aspect ratio is $W/h = 2.0$; $L_{BZ}/h = 3.60$ and $L_{FZ}/h = 0.70$.

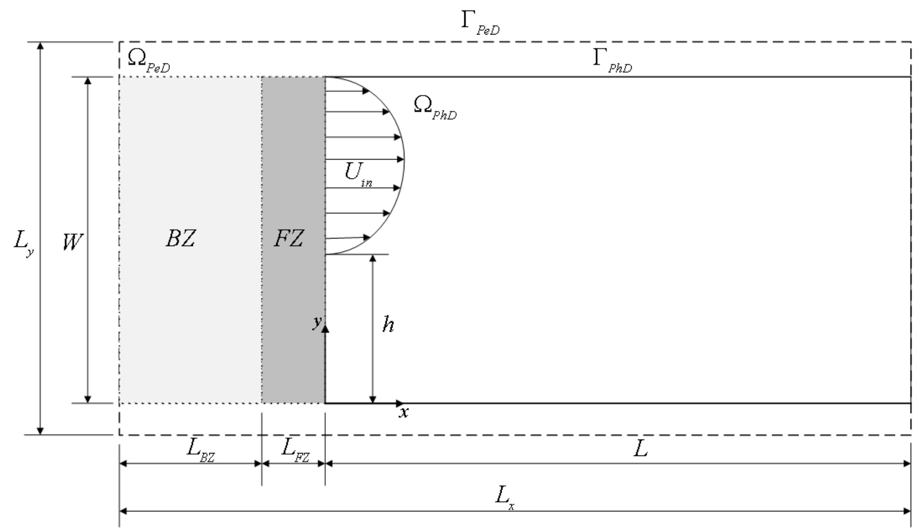

Figure 2. Domain of backward-facing step flow.

In Fig. 3 the position of reattach points are defined as in Lee and Mateescu (1998) and Tab. 1 is the comparison between the position of inferior reattachment point, X_r, for different Reynolds numbers, Eq. 6, based in step high and inlet mean velocity:

$$Re_h = \frac{U_\infty h}{\nu}. \tag{6}$$

The position of inferior reattachment point is no more close of experimental results as from $Re_h = 400$, Tab. 1. The three-dimensional instabilities became important and in two-dimensional simulations results increase the recirculations. To obtain numerical results close the experiment is necessary solve the spanwise direction of flow.

Fig. 4 shown the z vorticity component in different planes. In Fig. 4 (a) we can see the Kelvin-Helmoltz instabilities that are generated after reattachment point, x_r/h. In Fig. 4 (b) is possible to note the oscillations in spanwise direction, show the three-dimension features of backward-facing step flow at $Re_h = 1000$ and the transition to turbulence. We

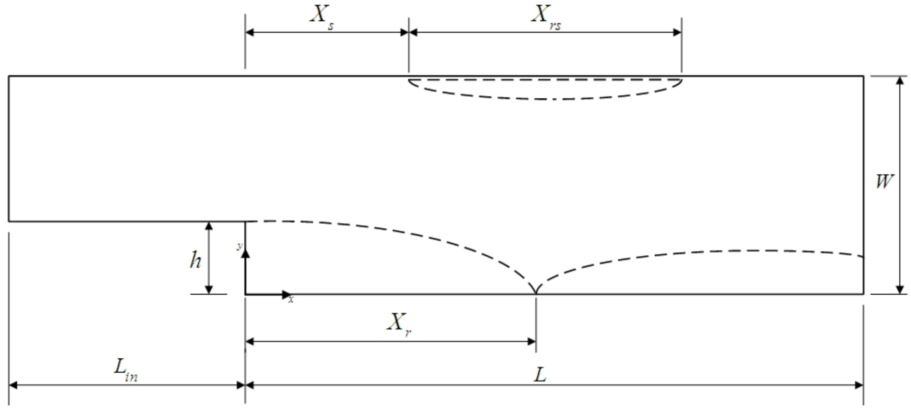

Figure 3. Definition of reattachment points of backward-facing step flow.

Table 1. Comparison between the inferior reattachment point to different Reynolds numbers, Re_h.

Re_h	Lee and Mateescu (1998)	Two-dimensional model
200	8,30	8,50
250	9,10	9,71
300	10,30	10,64
350	11,10	11,39
400	12,90	12,18
450	13,20	12,61
500	15,50	13,50

observed the recirculation zone generated by adverse gradient of pressure in Fig. 4 (c).

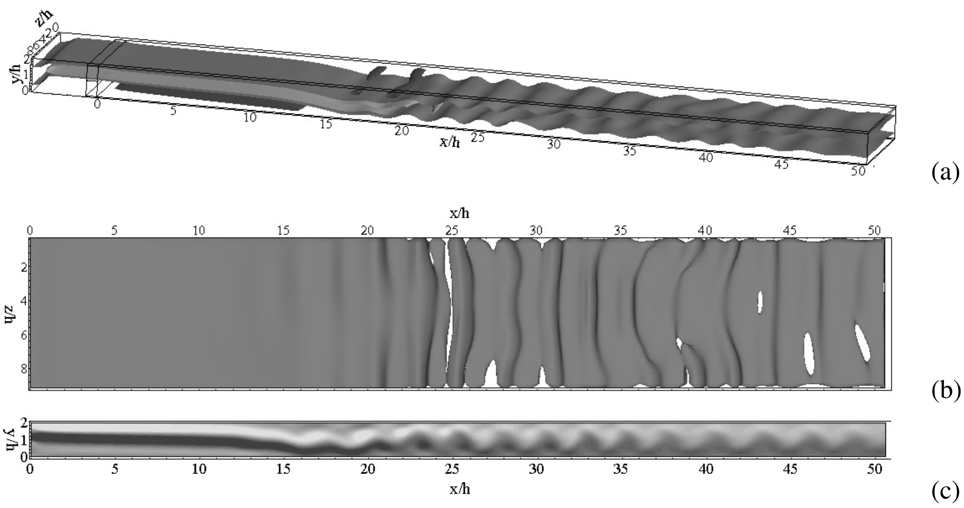

Figure 4. Backward-facing step flow at $Re_h = 1000$ and $tU_\infty/h = 100.0$. (a) Iso-surface of vorticity spanwise component $\omega_z = -1.0$ (black) and $\omega_z = 1.0$ (white); (b) Iso-surface of vorticity spanwise component $\omega_z = -1.0$; (c) Vorticity spanwise component at z center plane $-1.0 < \omega_z < 1.0$.

In Tab. 2 a comparison of numerical mean positions of reattachment point of inferior wall (x_r), detached point of superior wall (x_s) and the reattached point of superior wall (x_{rs}) with experimental results of Lee and Mateescu (1998) and the two-dimensional results of Mariano *et al.* (2010) is given.

The results of two-dimensional simulations (IMERSPEC 2D) of backward-facing step flow at $Re_h = 1000$ presented in Tab. 2 are not in good agreement with experimental data. On the other hand, the results of IMERSPEC 3D are very closed of Lee and Mateescu (1998) results, shown the importance of the three-dimensional effects.

Table 2. Comparison of position of reattachment point on the inferior wall x_r/h; the detachment point on the superior wall x_s/h; and detachment point on the superior wall, x_{rs}/h, for the backward-facing step flow at $Re_h = 1000$.

Works	x_r/h	x_s/h	x_{rs}/h
IMERSPEC 2D	18.10	15.15	36.09
IMERSPEC 3D	12.42	16.31	19.36
Experimental Lee and Mateescu (1998)	12.80	9.70	18.40

4. CONCLUSIONS

In the present article the simulation of three-dimensional backward-facing step flows, at $Re_h = 1000$ are performed and compared with simulations two-dimensional and experimental data. It is important to highlight that results are close to experimental data (Lee and Mateescu, 1998) only with three-dimensional model, because the instabilities in spawnwise direction became important. Specifically, in backward-facing step flow, this behavior is noted as from $Re_h = 500$.

5. ACKNOWLEDGEMENTS

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