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NUMERICAL SIMULATION OF THE TWO-PHASE FLOW USING FOURIER-SPECTRAL METHOD

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Abstract. *To study two-phases flow numerically the high level of mesh refinement is required to model the physical discontinuities present in the flow. This approach has a high computational cost (memory storage and CPU time processing). On the other hand, high order methods are an alternative way to obtain the solution of Navier-Stokes equations. In the present work, a hybrid methodology Fourier pseudo-spectral method and Front tracking/Front capturing methods are presented for numerical simulation of two phase-flows with non periodic boundary condition. Results for the rise cylindrical bubble are presented.*

Keywords: *Two-phase flow; Front-Tracking/Front-Capturing method; Fourier pseudo-spectral method.*

1. INTRODUCTION

Bubbles flows have a important role in geophysical and industrial processes. The applications generate researches to understand the hydrodynamics moving of the bubbles at different conditions. A major issue is to understand how the bubble moves in flow and how the continuous phase is affected by the dispersed phase.

The numerical simulation of these flows requires high accuracy in order to model the thin interfaces. High accuracy can be obtained from mesh refinement, the drawback of this approach is the increase the computational cost (CPU time and memory storage). On the other hand, high order methods increase the accuracy of solution partial differential equations (EDP) using less points in domain. The disadvantage is the requirement of more mesh points to solve the EDP derivatives (large stencil).

In the present paper the Fourier pseudo-spectral method (FPSM) is adopted. It has high accuracy and high-order numerical convergence, furthermore, a low computational cost when compared with another high order methods (Canuto *et al.*, 2007). To solve Navier-Stokes equations for incompressible flows using FPSM we can use the projection method (Canuto *et al.*, 2007), this approach decouple the pressure field of the velocity field, thus it is not necessary to solve the Poison equation. The main drawback of FPSM is the imposition of boundary conditions which must be periodic (Briggs and Henson, 1995).

The objective of the present work is to simulate the rise of a bubble using the IMERSPEC methodology coupled with the Front-Tracking/Front-Capturing method.

2. MATHEMATICAL MODELING

The method Front-Tracking/Front-Capturing developed by Unverdi and Tryggvason (1991) is defined by a stationary regular mesh ($\Omega = \Omega_1 \cup \Omega_2$), Fig. 1, to solve the Navier-Stokes equations and an unstructured mesh to model the immersed interface, Γ, which can move on stationary mesh Ω, Fig. 1. The subdomains Ω_1 is the continuous phase and Ω_2 is the dispersed phase. The Eulerian formulation models the fluid dynamics in the field $\Omega = \Omega_1 \cup \Omega_2$ and the Lagrangian formulation describes the movement of interface Γ.

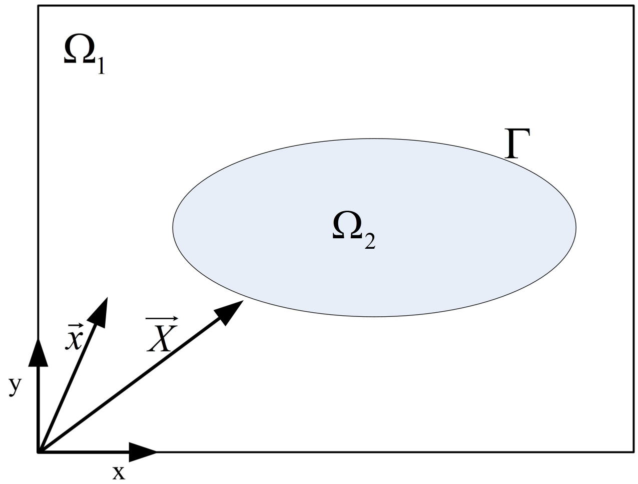


Figure 1. Schematic representation of domains: Eulerian, $\Omega = \Omega_1 \cup \Omega_2$ (continuos phase) and Lagrangian, Γ (dispersed phase).

The mathematical modelling to Eulerian field consider isothermal and incompressible flows of two immiscible Newtonian fluids with different physical properties, viscosity coefficient and density. Eqs. (1) and (2) represent the equations of continuity and balance of linear momentum, respectively.

$$\frac{\partial u_k}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial u_l}{\partial t} + \frac{\partial u_l u_k}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_l} + \frac{1}{\rho}\left\{\frac{\partial}{\partial x_k}\left[\mu\left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right)\right]\right\} + \left(\frac{\rho - \rho_0}{\rho}\right) g_l + \frac{1}{\rho} f_l, \tag{2}$$

where ρ e μ are the density and viscosity of the fluid, respectively, u_l is the component l the velocity vector, p is the fluid dynamic pressure, f_l is the component l the force vector field, g_l is the component l of gravitational acceleration and $l = 1, 2$, for two-dimensional problems, x_l the spatial coordinate and t the time.

The term $\frac{\rho_0}{\rho} g_l$, where $\rho_0 = (1-\alpha)\rho_c + \alpha\rho_d$, and α, the volume fraction between the continuous and dispersed phases, represents the fluid-static pressure gradient, which is a non-periodic pressure added to gravitational term. This is due to the fact that by using the Fourier pseudo-spectral method, the pressure-velocity decoupling is given by the projection method (Canuto *et al.*, 2007; Unverdi and Tryggvason, 1991).

The source term $f_l(\vec{x}, t) = \int_\Gamma F_l(\vec{X}, t)\delta(\vec{x} - \vec{X}) dx_l$ allows the communication between the Navier-Stokes equations, solved in the Eulerian domain , Eq. 2, and the equations for interface motion, solved in the Lagrangian domain. Thus, the Eulerian source term is nonzero on the interface and zero else where.

$F_l(\vec{X}, t) = \sigma\kappa\eta_l$, is the component l of Lagrangian force calculated on fluid particles which comprises the interface. σ is the surface tension coefficient, κ is the curvature and η_l the component l of the normal vector interface, the vector X_l is the position of a particle of fluid that is on the interface, as shown in Fig. 1.

From the definition of the Fourier transform and its properties (Canuto *et al.*, 2007) we obtain the continuity and Navier-Stokes equations (Eqs.(1) and (2)), transformed foward to Fourier space (Eqs. (3) and (4)).

$$ik_l \widehat{u_l} = 0, \tag{3}$$

$$\frac{\widehat{\partial u_l}}{\partial t} = \wp_{lm}\left[-\widehat{TNL}_m + \widehat{DIF}_m + \widehat{GRAV}_m + \frac{1}{\widehat{\rho}} * \widehat{f_m}\right], \tag{4}$$

where $\widehat{TNL}_l$ is the non-linear term $ik_k(\widehat{u_l u_k})$, $\widehat{DIF}_l$ is the diffusive term $\frac{1}{\widehat{\rho}} * ik_k * [\widehat{\mu} * (ik_k\widehat{u}_l + ik_l\widehat{u_k})]$ and $\widehat{GRAV}_l$ is the buoyancy term $\frac{1}{\rho} * \widehat{(\rho - \rho_0)g_l}$.

It is noteworthy in Eq. (4) the independence of the periodic pressure term, which has been replaced by projection tensor $\bar{\bar{\wp}}$ of the source terms, advective, diffusive and buoyancy. Compared with classical schemes, this approach is equivalent to replace the solution of Poisson equation for a matrix-vector product, in numerical terms, it is computationally cheaper. In physical terms, both have the same function, which is to ensure the conservation of mass. Although the periodic pressure field does not appear in the Navier-Stokes equations, it can be recovered using post-processing, as shown in Villela (2011).

3. NUMERICAL METHOD

3.1 Discrete Fourier Transform and Fast Fourier Transform

The discrete Fourier transform (DFT) given by Eq. 5, allow to work with Fourier Transform numerically.

$$\widehat{f}_k = \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} f_n e^{\frac{-2\pi ikn}{N}}, \tag{5}$$

where k is the wave number, N is the number of grid points, n provides the position x_n placement $(x_n = n\Delta x)$ and $i = \sqrt{-1}$.

The DFT transforms a function f, periodic (Villela, 2011), from physical space to Fourier spectral space (Eq. 5). The Inverse Discrete Fourier Transform (IDFT) is presented by Eq. 6:

$$f_k = \frac{1}{N} \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} \widehat{f}_n e^{\frac{2\pi ikn}{N}}. \tag{6}$$

Cooley and Tukey (1965) developed an algorithm named Fast Fourier Transform (FFT), which solves efficiently the DFT and the IDFT (Eq. 6), becoming it attractive to solve partial differential equations using the Fourier spectral method. The disadvantage of this technique is the restriction to problems with periodic boundary conditions.

The wave numbers k_l, used in the transformed equations, are calculated as follows (Eq. 7):

$$k_{l_\eta} = \begin{cases} \eta - 1 & 1 \leq \eta \leq \frac{N}{2} + 1, \\ \eta - 1 - N & \frac{N}{2} + 2 \leq \eta \leq N. \end{cases} \tag{7}$$

where η is the position vector in a direction of the domain.

This parameter should be fit for each subroutine FFT used. In the present paper, the subroutine FFTE was used and is given by Takahashi (2007), which can be found in www.ffte.jp, was used.

3.2 Treatment of non-linear term

When working with the Navier-Stokes equations using the Fourier pseudo-spectral method, the resolution of non-linear term is given by a convolution integral (Briggs and Henson, 1995). Solving this integral numerically becomes impractical, then Fourier pseudo-spectral method is applied and the basic algorithm can be found at Mariano *et al.* (2010a).

The non-linear term can be treated in different forms (Canuto *et al.*, 2007), and have different properties when it is discretized. In the present work, a skew-symmetric form (Eq.8) is used, because of their stability:

$$tnl = \frac{1}{2}\left(\frac{\partial(u_l u_j)}{\partial x_j} + u_j\frac{\partial(u_l)}{\partial x_j}\right) \quad \texttt{or} \quad \widehat{tnl} = \frac{1}{2}\left(\frac{\widehat{\partial(u_l u_j)}}{\partial x_j} + u_j\frac{\widehat{\partial(u_l)}}{\partial x_j}\right). \tag{8}$$

3.3 Discretization of source term

As seen in subsection 2., Lagrangian force density is depend on interface curvature, κ_l and the unit normal vector n_l. The discretizations of the vector n_l and the κ_l are made using the Lagrange polynomials (Villela, 2011).

3.4 Time discretization

It is necessary to use a time advanced methodology that is compatible with the high accuracy of the spectral methodology applied in the space. The best results are obtained with the of optimized Runge Kutta time advancement method (RK46). This method is an explicit method of fourth order and six stages. The algorithm is given by Allampalli *et al.* (2009).

3.5 Filtering Process

Another important procedure is the need to filter all the terms on the right hand side of the Navier-Stokes equations due to the discontinuities generated by the source term, which shows abrupt changes in physical properties. These discontinuities generate the Gibbs phenomenon (Villela, 2011), and with the filter process, tends to disappear leading to increased accuracy. The filtering process is given by Eq. 9:

$$\widehat{f}(\vec{k},t)_{filtrado} = \varphi(\theta)\widehat{f}(\vec{k},t), \tag{9}$$

where $\varphi(\theta)$ is the filter function.

In the present work the filter used is the "Raised Cosine". Its mathematical model can be found at Mariano *et al.* (2010b).

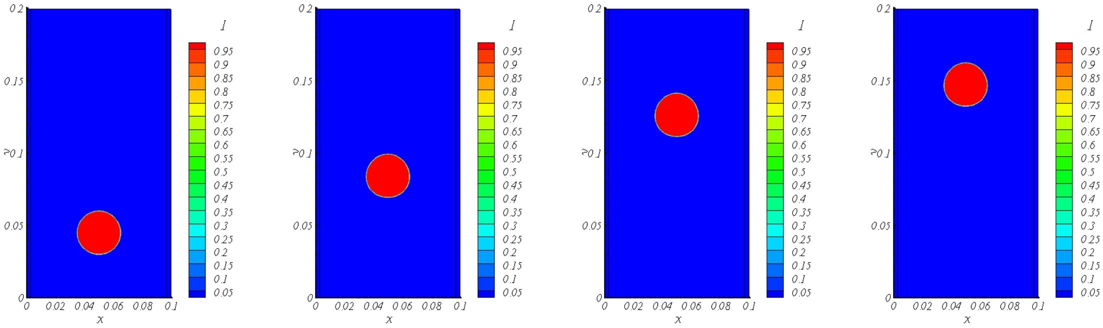
4. RESULTS

The verification of the algorithm proposed was done using the manufactured solution method, where it was found that the method reaches round-off errors. The results can be found in Villela (2011).

Figure 2 shows the time evolution of a rising non-deformable bubble. The computational domain is $\Omega = [0; 0,1] \times [0; 0,2]$, the surface tension $\sigma = 9,0\ [N/m]$, bubble diameter $d_d = 0,03\ [m]$ and the computational grid has $nxXny = 256 \times 512$ colocation points, in x and y directions, respectively. The boundary conditions are $u = 0$ and $v = 0$ at $y = 0,003125[m]$ and $y = 0,096875[m]$ and periodic in $y = 0[m]$ and $y = 0,2[m]$. To work with other boundary conditions, which is not provided periodicity, must couple the Fourier pseudo-spectral method with immersed boundary method (Mariano *et al.*, 2010a).

The bubble remains circular as expected for the parameters adopted in Clift *et al.* (1978). Furthermore, the pressure difference between continuos and dispersed phases, ΔP, is calculated. The numerical result is $\Delta P^N = 596,68[N/m^2]$ and analytical solution is $\Delta P^A = 600,00[N/m^2]$. The relative error is $0,55\%$.

In Fig. 2, we have dimensionless time is give by $t^* = \frac{t}{\sqrt{d_d/g}}$.



(a) (b) (c) (d)

Figure 2. Field of the indicator function in: (a) $t^* = 0$; (b) $t^* = 2,53$; (c) $t^* = 7,95$ e (d) $t^* = 13,38$.

5. CONCLUSIONS

The present work, two-dimensional numerical simulations of two-phase flows using the Fourier pseudo-spectral method coupled with the hybrid method Front-Tracking/Front-Capturing were presented. Flows with rising of cylindrical bubbles, are presented and the difference between numerical and analytical solutions of pressure difference is 0,55%. The results for rising bubbles are consistent experimental results of others authors.

6. ACKNOWLEDGEMENTS

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