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Numerical Simulation of Transient Pressure Data in Porous Media by Mixture Theory and Darcy Law

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Abstract. *This article compares two different theoretical formulations used in reservoir simulations. One is the hydraulic diffusivity equation and the other is the mixture theory equation applied to fluid flow in porous media. The hydraulic diffusivity equation is widely used in reservoir simulations. This equation is found when Darcy's law is applied in the continuity equation. However, there is another theoretical approach for fluid flow in porous media, the mixture theory. Mixture theory allows a local description of the flow in a porous medium, supported by a thermodynamically consistent theory which generalizes the classical continuum mechanics. This article compares the results obtained by the equation based on the mixture theory with the hydraulic diffusivity equation. This comparison is made by pressure transient data analysis. A practical application for these types of pressure data is in well tests. Transient well testing has been a core competency of the oil industry for a long time because it provides petroleum engineers with valuable information about reservoirs. The article presents a study of pressure transient data by applying both theories. The numerical method for the solution of both theories was the finite volume method.*

Keywords: *Mixture Theory, Finite Volumes Method, Well Test, Darcy's Law*

1. NOMENCLATURE

$\mathbf{x}$	Coordinate vector	c	Fluid compressibility
p	Pressure	c_f	Formation compressibility
$\mathbf{v}$	Superficial velocity vector	c_t	Total compressibility
$\mathbf{v}_i$	Constituent velocity vector	η	Hydraulic diffusivity constant
$\mathbf{v}_f$	Fluid velocity vector	ϕ	Porosity
μ	Viscosity	$\mathbf{T}_f$	Partial stress tensor for fluid
ρ	Mass Density	$\mathbf{K}$	Permeability Tensor
ρ_i	Constituent Mass Density	$\mathbf{g}$	Body force
$\mathbf{m}_f$	Interaction force	$\mathbf{I}$	Identity tensor
ρ_i	Constituent Mass Density	λ	Porous matrix microestruture
$\mathbf{D}_f$	Constituent symmetrical part velocity gradient	$\mathbf{m}_i$	Constituent interaction force
$\mathbf{T}_i$	Partial stress tensor for constituent		

2. INTRODUCTION

Well test, by definition, is the transient pressure measurement while the flow of the well is kept constant. So, when well test is mentioned, it invariably means transient pressure test. The constant-rate pressure-transient response depends on such reservoir and well properties as permeability, large-scale reservoir heterogeneities, and well damage (skin factor). Well test also determine, effective fracture lengths and conductivities and distances to boundaries. Information obtained from transient well testing was being used routinely by drilling, completion, production, and reservoir engineers.

When the well bottom hole flowing pressure is kept approximately constant, but measuring the flow rate change as a function of time, i.e., the transient rate, this type of test is referred to as decline curve analysis. It is increasingly used for the purpose of reservoir management.

Transient testing is facing a challenge to have the information it provides incorporated in numerical simulators used to predict reservoir performance (M.M. Kamal and J.L. Landa, 2005; Zheng and P. Corbett, 2005; Yao Jun, 2009). Most test wells simulators are based on Darcy's law and therefore the hydraulic diffusivity equation is used for single-phase flow. It is presented in this paper a theoretical formulation for fluid flow in porous media based on *continuum* theory of mixtures, a generalization of *continuum* mechanics.

A theoretical equation of flow in porous media can be found at least in two approaches. The local volume-averaging technique and mixture theory. Macroscopic equations obtained after volume integration over a Representative Elementary Volume (REV) were discussed in detail by Whitaker (1966, 1969, 1996); Quintard and Whitaker (1994a,b), to describe quantities such as temperature, pressure, concentration, and the velocity components. The local volume-averaging technique allows the use of the classical continuum mechanics approach.

Great strides have been made in developing a rational theoretical basis for studyning the mechanics of mixtures starting with the pioneering work of Truesdell (1957a,b) followed by those of Truesdell and Toupin (1960), Truesdell (1962), Adkins (1963), Green and Naghid (1965), Green and Naghid (1968), Sampaio (1976), Atkin and Craine (1976) and others. The use of mixture theory gives rise to new parameters in order to take into account the thermomechanical interaction among the constituents. Mixture theory allows a local description of the flow in a porous medium, supported by a thermodynamically consistent theory which generalizes the classical continuum mechanics (Martins-Costa and Saldanha da Gama, 1994).

The differences between the two theories for porous media flows studied in this article may be seen in table 1. This table shows the advantages and disadvantages of using each one:

Table 1. Two Theories Analysis

DARCY'S LAW	MIXTURE THEORY
Only newtonian fluid	Any fluid
Slow fluid flows	Any velocity
Based on an experiment	Extension of the classical *continuum* mechanics
Incompressible fluids	Any material
Approximate for compressible fluids	Any material
Linear iteration between fluid and rock	Fluid-rock iteration based in second law of thermodynamics
Viscous terms neglected	It takes into account viscous iteration

3. HYDRAULIC DIFFUSIVITY EQUATION

The Darcy's Law can be used together with the mass conservation law and constitutive laws to obtain an expression for the hydraulic diffusivity of the single phase flow of liquids in porous media (Craft and Hawkins, 1991). The mass conservation law for a porous medium and the Darcy'law are given by:

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \tag{1}$$

$$\mathbf{v} = -\frac{\mathbf{K}(x)}{\mu}\nabla p \tag{2}$$

where ρ is the fluid's density, ϕ is the porosity, **v** is the superficial velocity vector. Applying Darcy's law, the isothermal compressibility of liquids c and the formation compressibility c_f into eq. (1), one gets:

$$\phi(\mathbf{x})c_t\frac{\partial p(\mathbf{x},t)}{\partial t} = \nabla\cdot\left(\frac{\mathbf{K}(\mathbf{x})}{\mu}\nabla p(\mathbf{x},t)\right) \tag{3}$$

which is the hydraulic diffusivity equation for the single phase flow of liquids in an heterogeneous and anisotropic porous medium. In this equation, $\mathbf{K}(\mathbf{x})$ is the permeability tensor and **x** is the position vector according to any orthogonal coordinate system. The total compressibility is given by:

$$c_t = c + c_f \tag{4}$$

This paper will consider some assumptions to simplify eq. (3), such as homogeneous and isotropic porous media. According to these assumptions, and for a 1D single phase flow, eq. (3) becomes the diffusion or Fourier equation:

$$\frac{1}{\eta}\frac{\partial p(x,t)}{\partial t} = \frac{\partial^2 p(x,t)}{\partial x^2} \tag{5}$$

where η is the hydraulic diffusivity constant, given by:

$$\eta = \frac{K}{\phi c_t\mu} \tag{6}$$

4. MIXTURE THEORY

Darcy's law is amongst the most commonly used law to study the diffusion of one constituent through another. But there are a plethora of situations which cannot be adequately described by these law. Darcy's law within the context of mixture theory is an exceedingly simple approximation of the balance of linear momentum for mixtures.

To apply Darcy law, first consider a diffusion of a fluid through a rigid porous solid matrix. It assume that the flow of the fluid is slow so that we may ignore the inertial terms. There is an assumption of existence of a linear form in the velocity for the resistive force the fluid exerts upon the solid. The final assumption is that the partial stress for the fluid is given by:

$$\mathbf{T}_f = -p\mathbf{I} \tag{7}$$

It is imperative to recognize that the fluid under consideration is not ideal, for if it were inviscid there could be no drag. The interaction between the fluid and the solid is partitioned partially into the interaction term and the term for the partial stress, and we neglected the viscous effect (Rajagopal, 1995).

By the mixture theory, the mass balance may be written as:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0 \tag{8}$$

which ρ_i is the ith constituent mass density in mixture, and $\mathbf{v}_i$ is the ith constituent velocity in mixture. The field ρ_i is locally defined as the ratio between the ith constituent mass and the respective volume of the mixture. Since the mass of each constituent is preserved, the mass of the mixture as a whole is automatically conserved.

$$\frac{\partial (\phi\rho)}{\partial t} + \nabla \cdot (\rho\phi \mathbf{v}_f) = 0 \tag{9}$$

The balance of linear momentum is given by:

$$\frac{\partial (\rho_i \mathbf{v}_i)}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i \mathbf{v}_i) = \nabla \cdot (\mathbf{T}_i) + \rho_i \mathbf{g}_i + \mathbf{m}_i \tag{10}$$

which $\mathbf{T}_i$ is the partial stress tensor associated with the ith constituent, $\mathbf{g}_i$ is a body force per unit mass acting on the ith constituent, and $\mathbf{m}_i$ is an interaction force per unit volume acting on the ith constituent due to its interaction with the other constituents of the mixture. The balance of linear momentum for the fluid constituent:

$$\frac{\partial (\phi\rho \mathbf{v}_f)}{\partial t} + \nabla \cdot (\rho\phi \mathbf{v}_f \mathbf{v}_f) = \nabla \cdot (\mathbf{T}_f) + \phi\rho \mathbf{g} + \mathbf{m}_f \tag{11}$$

the following constitutive equations are employed (Williams 1978):

$$\mathbf{T}_f = -\phi p \mathbf{I} + 2\lambda\phi^2 \mu \mathbf{D}_f \tag{12}$$

$$\mathbf{m}_f = -\frac{\mu\phi^2}{K} \mathbf{v}_f \tag{13}$$

where λ is a scalar parameter depending on the porous matrix microstructure and for this paper $\lambda = 1$ and $\mathbf{D}_f$ is the symmetrical part of the fluid constituent velocity gradient.

The above set of equations may be treated as the general form of governing equations for all single phase flows. For instance, $\phi \to 1, \mathbf{K} \to \infty$, they are reduced to the standard governing equations for a pure fluid flow. When $\phi \to 1$, they become the widely used momentum resistance models in superficial velocity formulation (Wang CY and BY, 1999).

This physical velocity formulation has been implemented in an advanced, commercial, general-purpose CFD code, FLUENT (Li and Vasquez, 2007). For brevity, only the conservation equations for flow in an isotropic porous medium are described here.

5. PROBLEM FORMULATION

This paper compares the results obtained by the equation based on the mixture theory with the equation of hydraulic diffusivity. This comparison is made by pressure transient data analysis. The studied problem is unsteady compressible laminar flow within a porous media.

The hydraulic diffusivity equation is given by the following:

$$\frac{1}{\eta}\frac{\partial p\,(x,t)}{\partial t} = \frac{\partial^2 p(x,t)}{\partial x^2} \qquad \text{in} \qquad 0 \le \eta \le 1, \quad 0 \le \xi \le 1 \tag{14}$$

The data for simulation is showed in table 2. Four cases were solved with different boundary conditions:

Table 2. Data for Simulation

Properties	Values
K	1e-13 m^2
μ	0.01 $kg/m.s$
c	5e-6 $(1/Psi)$
c_f	5e-6 $(1/Psi)$
ϕ	0.2
L	50 (m)

- Pressure in $x = 0$ and sealed porous medium in $x = L$

$$p(0,t) = 0, \qquad \left(\frac{\partial p}{\partial x}\right)_{x=L} = 0, \qquad \text{in} \qquad t \geq 0, \tag{15}$$

- Flow rate in $x = 0$ and sealed porous medium in $x = L$

$$\left(\frac{\partial p}{\partial x}\right)_{x=0} = -2.0 \times 10^{-4}, \qquad \left(\frac{\partial p}{\partial x}\right)_{x=L} = 0 \qquad \text{in} \qquad t \geq 0, \tag{16}$$

- Pressure in $x = 0$ and pressure supply in $x = L$

$$p(0,t) = 0, \qquad p(L,t) = 1.0 \times 10^6 \qquad \text{in} \qquad t \geq 0, \tag{17}$$

- Flow rate in $x = 0$ and pressure supply in $x = L$

$$\left(\frac{\partial p}{\partial x}\right)_{x=0} = -2.0 \times 10^{-4}, \qquad p(L,t) = 1.0 \times 10^6 \qquad \text{in} \qquad t \geq 0 \tag{18}$$

The initial conditions for all of cases:

$$p(x,0) = 1.0 \times 10^6 \tag{19}$$

The mixture theory equations to be solved

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot (\rho\phi\mathbf{v}_f) = 0 \tag{20}$$

$$\frac{\partial \rho}{\partial p} = \rho c \tag{21}$$

$$\frac{\partial(\phi\rho\mathbf{v}_f)}{\partial t} + \nabla \cdot (\rho\phi\mathbf{v}_f\mathbf{v}_f) = -\phi\nabla p + \phi^2\mu\nabla^2\mathbf{v}_f - \frac{\mu\phi^2}{K}\mathbf{v}_f \tag{22}$$

The same physical conditions in the contours were defined for mixture theory. These cases were solved in FLUENT and the boundary conditions imposed in the program were:

- Pressure $p(0,t) = 0$ and wall boundary condition in $x = L$,
- Velocity $u_f(0,t) = 2.0 \times 10^{-6}$ and wall boundary condition in $x = L$,
- Pressure $p(0,t) = 0$ and pressure supply $p(L,t) = 1.0 \times 10^6$,
- Velocity $u_f(0,t) = 2.0 \times 10^{-6}$ and pressure supply $p(L,t) = 1.0 \times 10^6$.

The same mesh was used in all cases for both theories. A uniform mesh with fifty elements was used to solve the finite volume method. The time derivative was discretized using backward differences and the first-order implicit temporal discretization was used. The advantage of the fully implicit scheme is the unconditionally stable with respect to time step size.

The spatial discretization was computed by the Green-Gauss Cell-Based method for gradients and First-Order Upwind Scheme for interpolate faces values in convection terms. For the mixture theory, the SIMPLE was the pressure velocity coupling segregated algorithm chosen.

6. RESULTS AND DISCUSSION

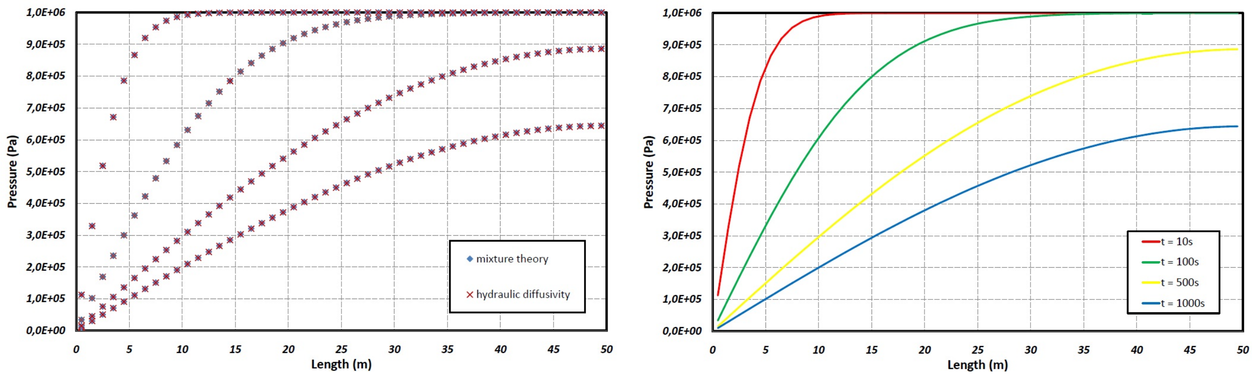
The behavior of the velocity and pressure field in porous media depend on the condition imposed on the reservoir contours. The output flow is $x = 0$, then the well will be represented in this contour. Different boundary conditions can be imposed at $x = L$. The first condition is pressure maintenance, this occurs when somehow the fluid produced is reinjected. The second possibility is when the porous medium is sealed, if the end of the reservoir is a wall.

For each case will be discussed and presented two graphs, the velocity field and pressure field along the porous medium.

6.1 Pressure and Sealed Porous Media

The porous medium has a stored energy. Due to the pressure imposed on the boundary be much smaller than the initial pressure, the imprisoned fluid begins to move. This case has as boundary conditions, pressure at x = 0 and wall at x = L, Fig. 1.

At the beginning of the flow occurs a velocity peak and not all of the fluid inside the domain moves. This velocity peak can be explained by the differential pressure that occurs only at the beginning of the domain. For this moment the pressure information has not yet arrived at x = L.

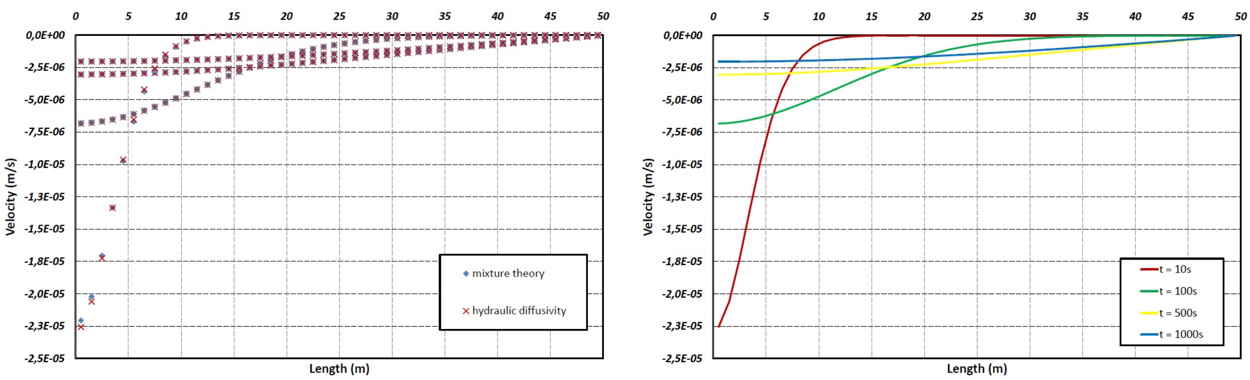


(a) Pressure Comparison between the two approaches. (b) Time analysis of pressure behavior.

Figure 1. Pressure Field for Pressure - Sealed Porous Media.

Over time, the fluid velocity at the beginning domain decrease while the fluid imprisoned in end of the domain starts to move,Fig. 2. After the differential pressure reaches to the end of the domain, the velocity across the porous medium decreases with time.

As the porous medium is sealed at x = L, the fluid flows until there is no pressure differential across the domain. At this moment the velocity field across the porous medium is equal to zero.

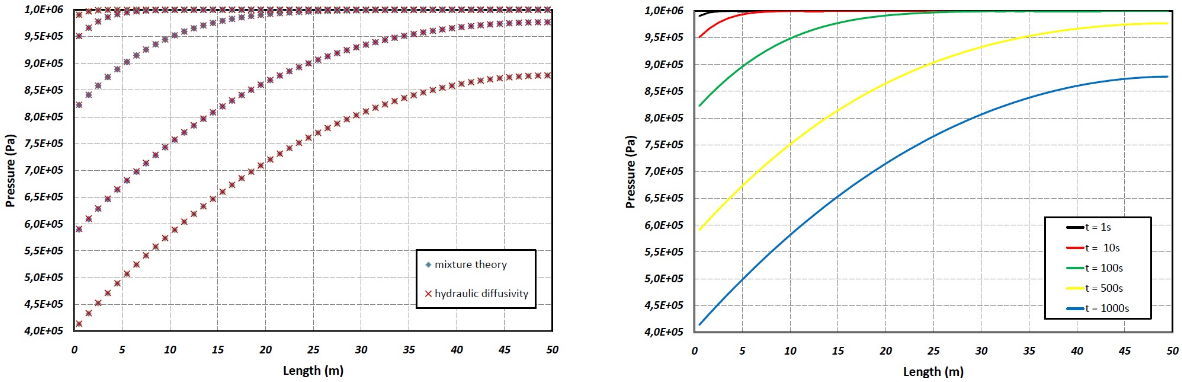


(a) Velocity Comparison between the two approaches. (b) Time analysis of velocity behavior.

Figure 2. Velocity Field for Pressure - Sealed Porous Media.

6.2 Velocity and Sealed Porous Media

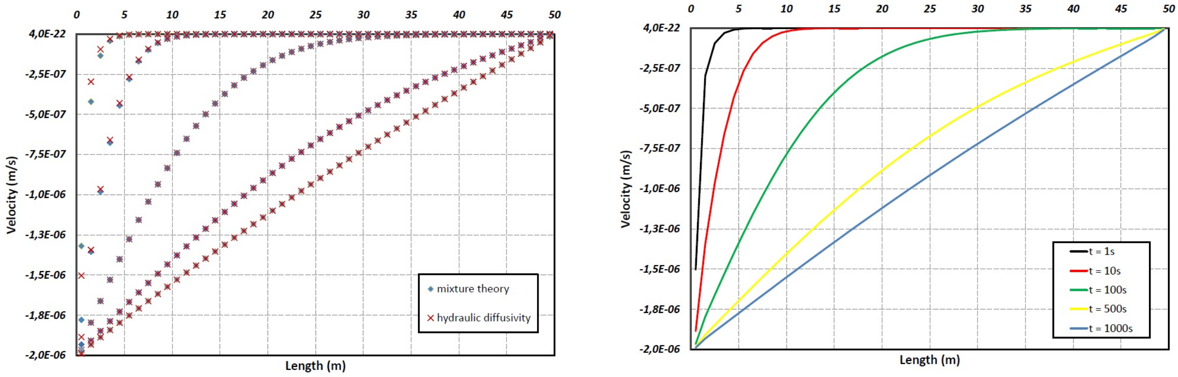
In this case, velocity is imposed at $x = 0$ and wall at $x = L$, the pressure at $x = 0$ falls indefinitely. It is interesting to note that the initial instants not all of the fluid inside domain moves, because the pressure is constant, until $t = 500s$, in some places in the domain, Fig. 3.



(a) Pressure Comparison between the two approaches. (b) Time analysis of pressure behavior.

Figure 3. Pressure Field for Velocity - Sealed Porous Media.

As might be expect the velocity profile when we change the boundary condition is totally different. The manner in which the fluid flows in the reservoir is totally dependent on the boundary condition imposed on the boundary (well),Fig. 4.



(a) Velocity Comparison between the two approaches. (b) Time analysis of velocity behavior.

Figure 4. Velocity Field for Velocity - Sealed Porous Media.

6.3 Pressure and Supply Pressure

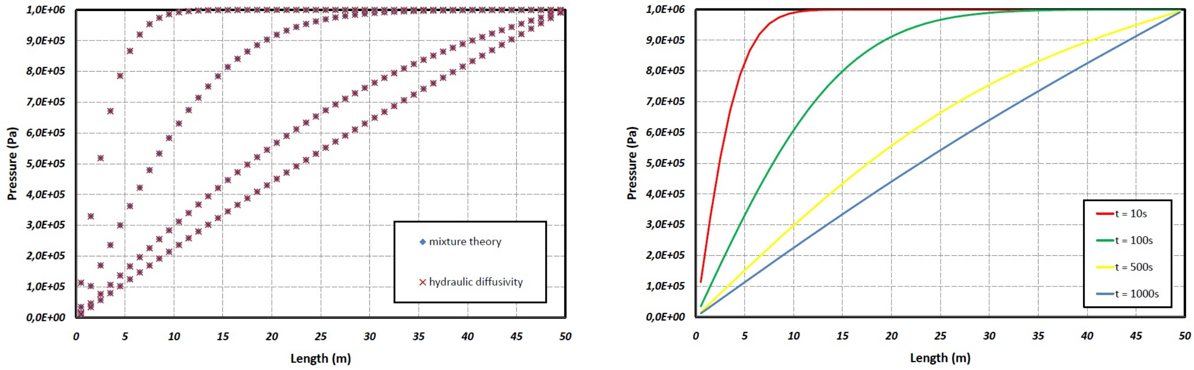
This case has as boundary conditions, pressure at $x = 0$ and $x = L$, Fig. 5. The results found for regime permanent reservoir through both theories is expected. This solution can be easily found analytically.

One can make two important analysis of this results. The first, when the transient reservoir regime is disregarding the pressure information arrives instantly at $x = L$, ie, the pressure along the domain has a linear behavior immediately. The second is that all the fluid in the porous medium moves with the same velocity (constant) after the well opening in the case permanently reservoir regime.

Considering the fluid and formation compressibility, the pressure boundary condition information at $x = 0$ delay to reach the other boundary ($x = L$).

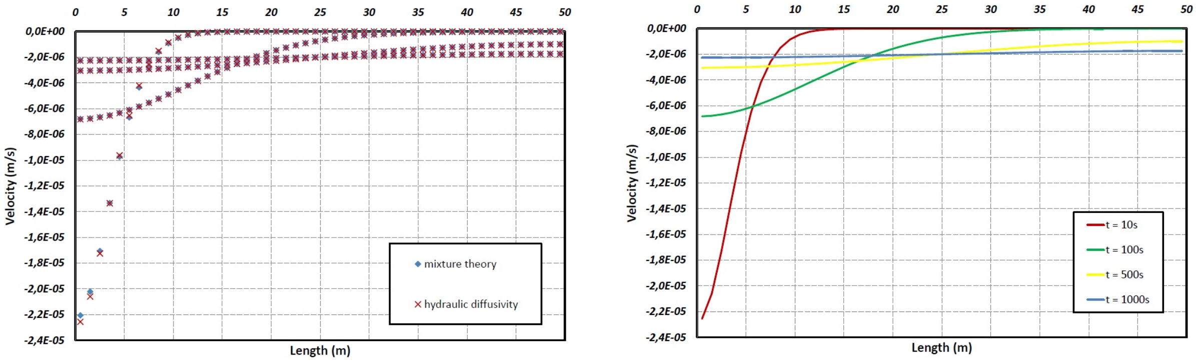
At $t = 100s$ there is still fluid immobile in the domain. It can be see that just in the initial instants the velocity at the beginning of the domain increases while in the final, (at $x = L$) the velocity is zero,"Fig. 6". After a certain period the velocity begins to decrease at the beginning and the end the velocity begins to increase. This behavior occurs until the velocity reaches the velocity found in the steady state. Typical behavior of pressure pre-written in the well ($x = 0$).

Another important fact is that even though the coefficient of hydraulic diffusivity is very small, for a more realistic estimate of the behavior of the fluid within the porous medium, the transient term must be taken into account.



(a) Pressure Comparison between the two approaches. (b) Time analysis of pressure behavior.

Figure 5. Pressure Field for Pressure - Supply Pressure Porous Media.



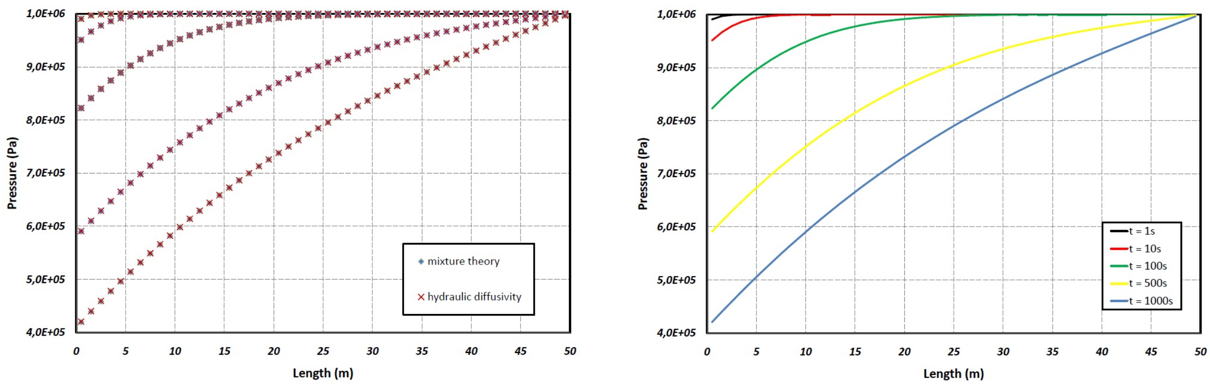
(a) Velocity Comparison between the two approaches. (b) Time analysis of velocity behavior.

Figure 6. Velocity Field for Pressure - Supply Pressure Porous Media.

6.4 Velocity and Supply Pressure

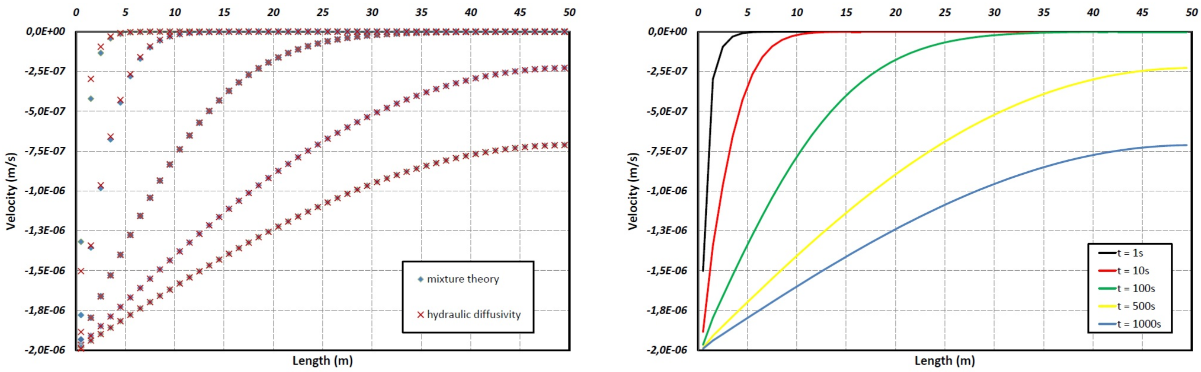
The results found for steady state for the cases "Pressure and Supply Pressure" and "Velocity and Supply Pressure", changing the boundary condition of pressure at $x = 0$ for velocity at $x = 0$, should be equal. The velocity chosen for this case was exactly the response velocity found for the previous case in steady state.

Although the final result is the same, the behavior of pressure and velocity in porous media during the transient process is totally different when changing the boundary condition. The results for this case can be seen in Fig. 7 and Fig. 8.



(a) Pressure Comparison between the two approaches. (b) Time analysis of pressure behavior.

Figure 7. Pressure Field for Velocity - Supply Pressure Porous Media.



(a) Velocity Comparison between the two approaches.

(b) Time analysis of velocity behavior.

Figure 8. Velocity Field for Velocity - Supply Pressure Porous Media.

7. CONCLUSIONS

This paper provided a comparison between two theories for reservoir simulation. The diffusivity hydraulic equation and mixture theory are used to solve the compressible single phase flow in porous media. This comparison is made by pressure transient data analysis. A practical application for these types of pressure data is in well tests or in decline curve analysis. The mixture theory formulation has been implemented in an advanced, commercial, general-purpose CFD code, FLUENT. The compared results using the two approaches represents the same behavior.

The transient pressure measurement depends on such reservoir and well properties as permeability, large-scale reservoir heterogeneities, and well damage (skin factor). It also determine, effective fracture lengths and conductivities and distances to boundaries. Information obtained from transient pressure measurement was being used routinely by drilling, completion, production, and reservoir engineers.

This article also describes in detail the importance of accounting transient reservoir regime in reservoir simulation. The present study compares two different formulations and also aimed to show the importance of boundary conditions on the behavior of pressure and velocity fields when the transient reservoir regime is taken into account.

In some cases, although the final result is the same, the behavior of pressure and velocity in porous media during the transient process is totally different when changing the boundary condition at $x = 0$ (well).

When the transient reservoir regime is disregarding the pressure information arrives instantly at $x = L$, ie, the pressure along the domain has a linear behavior immediately. If only the steady state reservoir regime is taken into account, all the fluid in the porous media moves with the same velocity (constant) after the well opening.

8. ACKNOWLEDGEMENTS

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