One-dimensional simulation of superficial warming in canine knee joint

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Abstract. *The present study aimed at a simulation of transient heat transfer in the canine knee, resulting from the application of superficial thermotherapy. The dog knee joint was considered to be comprised of three tissues layers: the epidermis, the subcutaneous tissue and the fat tissue. To solve the problem, a software based on the finite element discretization method was used (FEHT- academic version). Validation of the software was based on a heat transfer problem in cartesian coordinates, considering only one tissue layer. The results were compared with the analytical solution of the Pennes model, and showed an average percentage difference of 0.0012% between simulation and analytical solution. The average temperature results obtained in reference to the simulation of heat transfer in response to surface warming in the joint, considering the three tissue layers and cylindrical geometry, were higher in comparison to experimental data available in literature, due to the fact that the model did not take into account the effect of heat losses. The results presented in this study show that computer simulation can be an important tool for assessing the effectiveness of thermal treatments in biological tissues, since it allows the calculation, analysis and visualization of the temperature changes that occur over time in tissues.*

Key words: *bio heat transfer, thermotherapy, computational simulation, Pennes.*

1. INTRODUCTION

Heat in its various forms has been employed for therapeutic purposes since ancient times. One of its first uses (~3000 B.C.) aimed at the destruction of tumors and benign lesions (Shitzer and Eberhart, 1985). Therapeutic modalities such as superficial applications of heat use the body's natural thermoregulatory mechanisms to relieve pain or provide healing. However, tissue temperatures must be in the range of 40 to 45 °C in order to ensure attainment of therapeutic benefits (Draper *et al.* 1999; Kitchen, 2003; Robertson, *et al.* 2005). According to Kitchen (2003), lower temperatures are not sufficient for promoting the desired therapeutic effects, whereas higher values can cause thermal damage to the tissue, with consequent destruction of the cytoskeleton, cell membrane and micro vessels. Such lesions can interrupt the cell reproduction cycle and local blood flux, leading to tissue necrosis (Knight *et al.* 2001). Therefore, monitoring of these temperatures is an important strategy for safe and effective treatments (Karaa *et al.* 2005; Araújo, 2006).

In vivo tissue temperature determinations have been employed in order to better understand and improve therapeutic strategies (Shrivastava and Vaughan, 2009). Nevertheless, there are several difficulties associated with this type of experimental procedures, given its invasive nature and the lack in precision of several parameters in association to time constraints and the complexity of control mechanisms. Furthermore, such measurements are limited to a few locations in the tissue and do not provide a detailed description of the temperature distribution (Trobec *et al.* 2008). Given the aforementioned problems associated to *in vivo* temperature determinations, numerical simulations can be viewed as a promising alternative for the assessment of temperature distribution in several tissues layers exposed to different thermotherapy interventions. Therefore, this study aimed at the simulation of transient heat transfer in the canine knee, resulting from the application of superficial thermotherapy. The dog knee joint was considered to be comprised of three tissues layers: the epidermis, the subcutaneous tissue and the fat tissue.

2. MATERIALS AND METHODS

The canine knee joint was represented by a cylinder, considering transient heat transfer in the radial direction and including the effects of blood perfusion and metabolic heat, according to the model proposed by Pennes (1948). The corresponding governing equation can be written as:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla(k\nabla T) + (\rho\omega)_b c_{pb}(T_a - T) - \dot{q}_{met} \tag{1}$$

where k, ρ and c_p correspond to the values of thermal conductivity [W/m·ºC], density [kg/m^3] and specific heat [J/kg·ºC], respectively. The subcript b is in reference to blood properties, with w_b representing the blood perfusion [m^3.s^{-1}.m^{-3}]. $\dot{q}_{met}$ is the metabolic heat [W/m^3],T is the tissue temperature [ºC] and T_a is the arterial temperature (37ºC). All simulation studies were based on the finite element discretization method, employing a commercially available software (FEHT - academic version – Incropera and Dewitt, 2003).

2.1. Software Validation

Validation of the software was based on a one-dimensional heat transfer problem in cartesian coordinates, considering only one tissue layer of 0.03 m. The employed values for tissue and blood properties are displayed in Tab. 1. The results were compared to the analytical solution of the steady-state one-dimensional Pennes model that can be represented by the following equation:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_{met} + (\rho c_p w)_b (T_a - T)}{k} = 0 \tag{2}$$

Or in non-dimensional form:

$$\frac{d^2\theta}{dx^2} - \overline{m}^2\theta = 0 \tag{3}$$

where

$$\overline{m}^2 = \frac{(\rho c_p w)_b}{k} \tag{3.1}$$

$$\theta = T - T_a - \dot{q}_{met} / (\rho c_p w)_b \tag{3.2}$$

The corresponding analytical solution, considering that the temperature values at the boundaries are known, i.e., T (x=0) = Ti = 37 ºC and T (x=0.03) = Ts, is:

$$\theta_{(x)} = \theta_{(b)} / senh\overline{m}L[\theta_{(l)} / \theta_{(b)} senh\overline{m}x + senh\overline{m}(L - x) \tag{4}$$

where L represents the width of the tissue layer and:

$$\theta_{(b)} = T_i - T_a - \dot{q}_{met} / (\rho c_p w)_b \tag{4.1}$$

$$\theta_{(l)} = T_s - T_a - \dot{q}_{met} / (\rho c_p w)_b \tag{4.2}$$

Table 1. Thermal properties for the tissue and blood – software validation.

Material	Width (m)	k (W/m.K)	Cp (J/kgK)	ρ (kg/m^3)	w (s^{-1})	$\dot{q}_{met}$ (W/m^3)	T (ºC)
Tissue (muscle)	0.03	0.5	-	-	0.0005	700	-
Blood	-	-	3600	1000	-	-	37

Two different surface temperatures were evaluated: (i) *situation 1:* T_s = 35.8 ºC and (ii) *situation 2:* T_s = 28.7 ºC. Simulation parameters are displayed in Tab. 2.

Table 2. Simulation parameters – software validation.

	$\overline{m}$	θ_b	θ_l
Heating (Ts = 35.8 ºC)	60 m^{-1}	- 0.389 K	-1.589 K
Cooling (Te = 28.7 ºC)			-8.689 K

2.2. Simulation Study

Simulations aimed at the evaluation of the tissue temperature profile under heating conditions during 40 min. The external heat source was a thermal blanket (50 cm x 29 cm), providing an average heat amount of 31.2 W and presenting an average temperature of 57.6 ^oC, according to experimental measurements reported by Araújo (2009). All simulations were based on the assumption that the blanket was in perfect contact with the epidermis and two types of boundary conditions were evaluated: (i) *simulation a*: the surface temperature was considered to be equivalent to the average blanket temperature 57.6 ^oC, and (ii) *simulation b*: no heat losses from the thermal blanket (q_x"=215.7 W/m²). The employed values for tissue and blood properties were based on the study by Araújo (2009) and are displayed in Tab. 3.

Table 3. Thermal properties for the tissue and blood – simulation.

Material	Width (m)	c_p (Jkg^{-1o}C^{-1})	ρ (kgm^{-3})	k (Wm^{-1o}C^{-1})	$\dot{q}_{met}$ (Wm^{-3})	w (m^{3}s^{-1}m^{-3} tec)	Initial temperature (ºC)
Epidermis	8.0x10^{-5}	3593	1200	2.28x10^{-1}	0	0	34.9
Subcutaneous tissue	9.0x10^{-3}	3365	1200	4.64x10^{-1}	200	1.3x10^{-3}	35.5
Fat tissue	1.3x10^{-3}	2678	937	2.03x10^{-1}	3.9	2.9x10^{-4}	36.1

3. RESULTS AND DISCUSSION

3.1. Software Validation

The temperature variations within the muscle tissue, obtained from the software validation tests, are displayed in Fig. 1. In both cases (*situation 1:* T_s = 35.8 ºC and *situation 2:* T_s = 28.7 ºC), it can be observed that the tissue temperature decreases from the interior towards the surface, with such decrease being larger for situation 2, as a consequence of the larger values of temperature gradients in comparison to situation 1. A comparison between simulation and analytical results is displayed in Fig. 2 and Fig. 3. The average percentage differences between simulation and analytical solution were 0.0011% and 0.0012% for situation 1 and situation 2, respectively. Such results confirm that the employed software is appropriate for solution of one-dimensional bio-heat transfer problems.

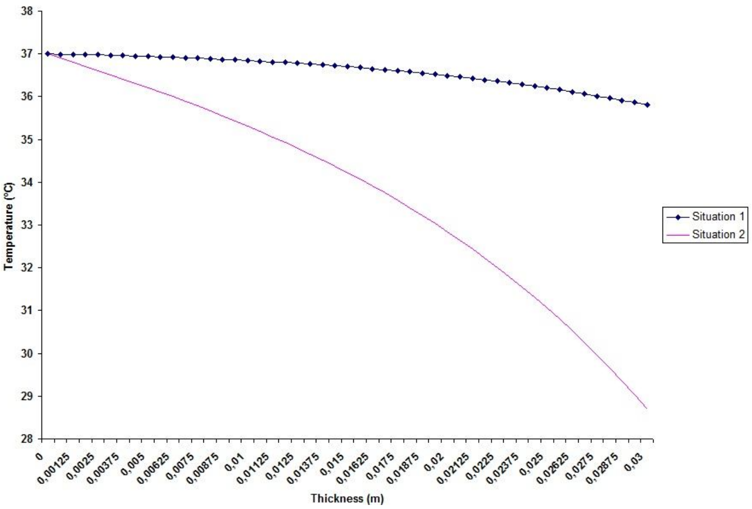

Figure 1. Temperature variation along the muscle layer – validation study.

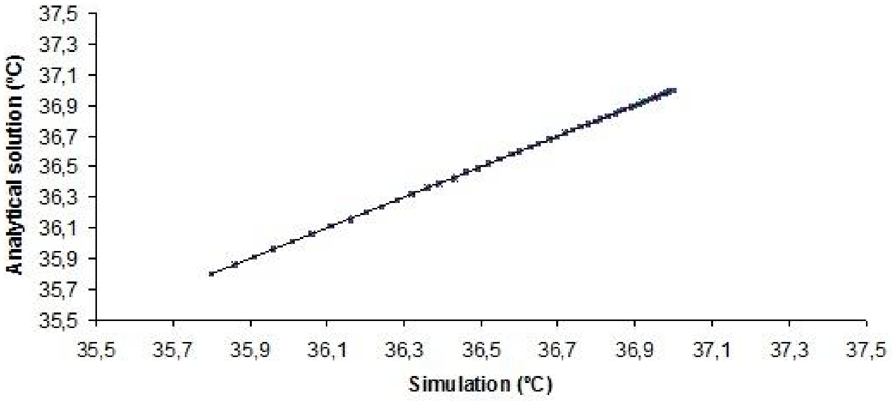

Figure 2. Comparison between simulation and analytical results – validation situation 1.

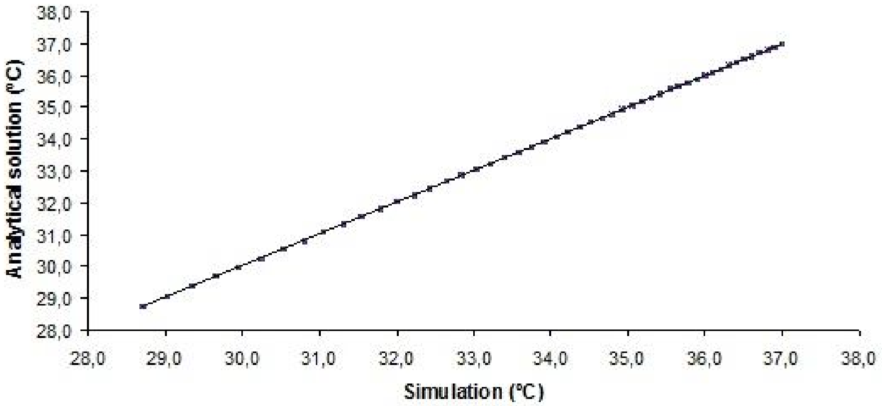

Figure 3. Comparison between simulation and analytical results – validation situation 2.

3.2. Simulation Study

Simulations aimed at the evaluation of the transient behavior of the tissue temperature. Average results from *simulation a*, based on the assumption that the external epidermis temperature was considered to be equivalent to the average blanket temperature 57.6 ^{o}C, are shown in Figures 4 to 6. Higher temperatures are observed for the epidermis in comparison to the other tissues. This is attributed to the fact that the epidermis is the layer that is in direct contact with the heat source and also to the fact that it is the only layer in which there is no effect of blood perfusion. A qualitative evaluation of the temperature variation, in Figures 4 to 6, points towards an exponential behavior, as also reported by Araújo (2006). While the temperature increases rapidly for both the epidermis and subcutaneous tissue, there is a small delay (lag phase) in the case of the fat tissue. Such behavior was also reported by Chaui-Berlinck *et al.* (2005) and Araújo (2009). This is attributed to a more pronounced effect of blood perfusion in that specific tissue layer at the beginning of heating. A comparison between experimental and simulation results shows a significant difference between them, with the calculated values being much larger than the experimental ones. Furthermore, calculated temperature values are above the recommended range of 40 to 45ºC (Draper *et al.* 1999; Kitchen, 2003; Robertson, *et al.* 2005).

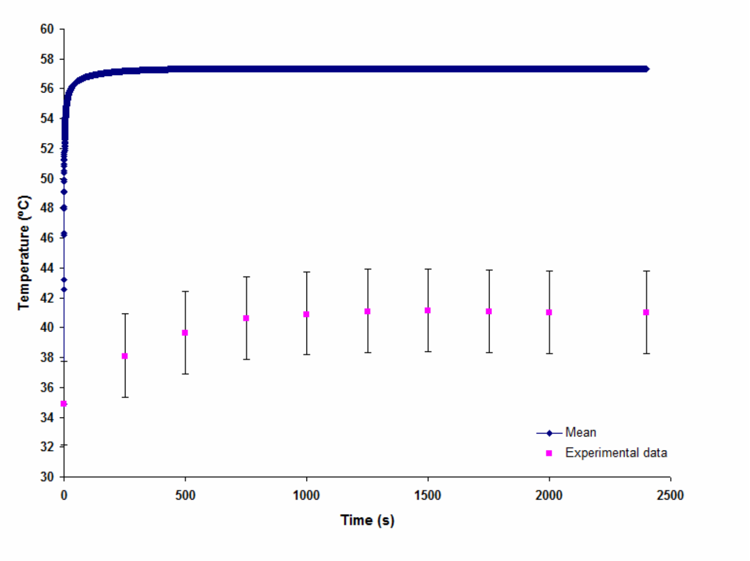

Figure 4. Variation of the average temperature of the epidermis with time – *simulation a.*

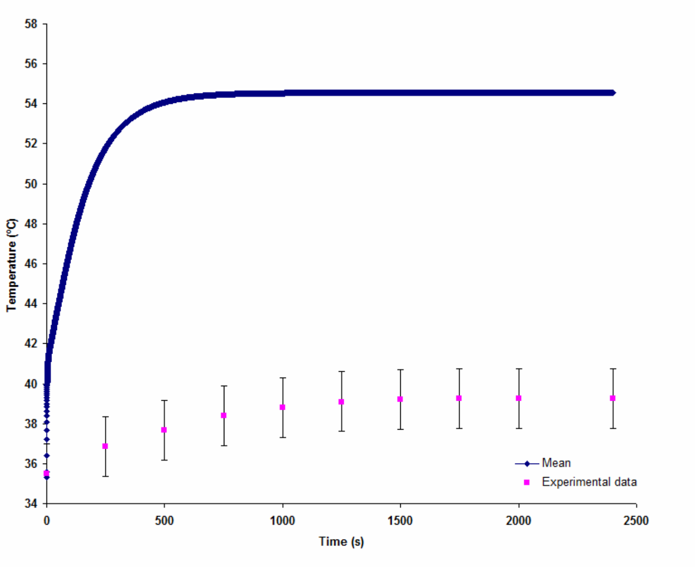

Figure 5. Variation of the average temperature of the subcutaneous tissue with time – *simulation a*.

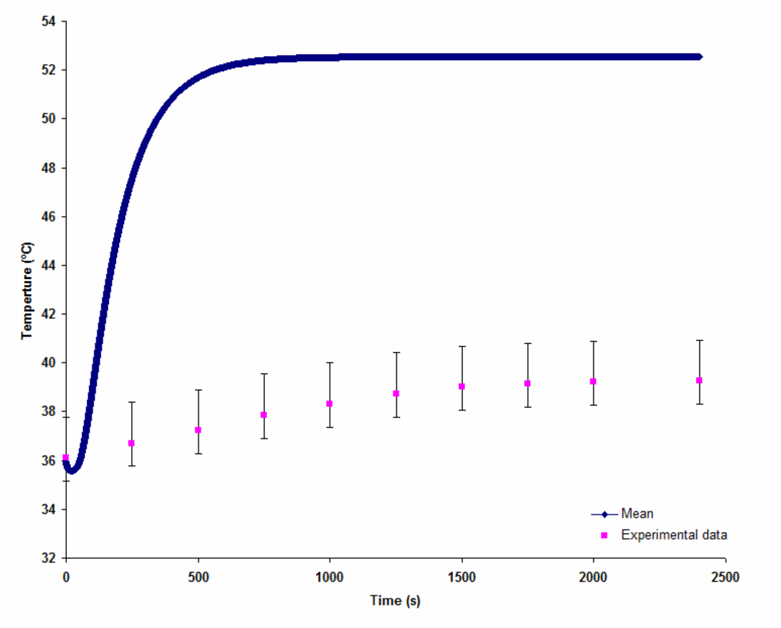

Figure 6. Variation of the average temperature of the fat tissue with time – *simulation a*.

Average results from simulation b, based on the assumption that the epidermis will receive all the heat provided by the thermal blanket (q_x”=215.7 W/m²) are shown in Fig. 7 to Fig. 9. In this case, the temperature increases slower as time progresses in comparison to simulation a. Also, the period of time associated with the “lag phase” is longer (see Fig. 6 and Fig. 9). Results from simulation b show better agreement with experimental data in comparison to simulation a. Such results indicate that specification of a boundary condition in terms of heat flux provides a more accurate description of the heat transfer phenomena. The differences between simulation and experimental results are attributed to the facts that (i) the heat transfer losses between the thermal blanket and the external environment were not taken into account, and (ii) the contact between the blanket and the epidermis is not perfect, and the air between the two solids acts as a thermal insulator. Another simulation was performed (simulation c), considering a 30 % reduction in the amount of heat received by the epidermis (q_x”=151 W/m²). Results for the variation of the average temperature of the epidermis with time are shown in Fig. 10 in comparison to experimental data. Such results confirm that, for the model to provide an accurate of the temperature variation associated with surface heating of the knee joint, the external boundary condition should be modified in order to include the effect of heat losses.

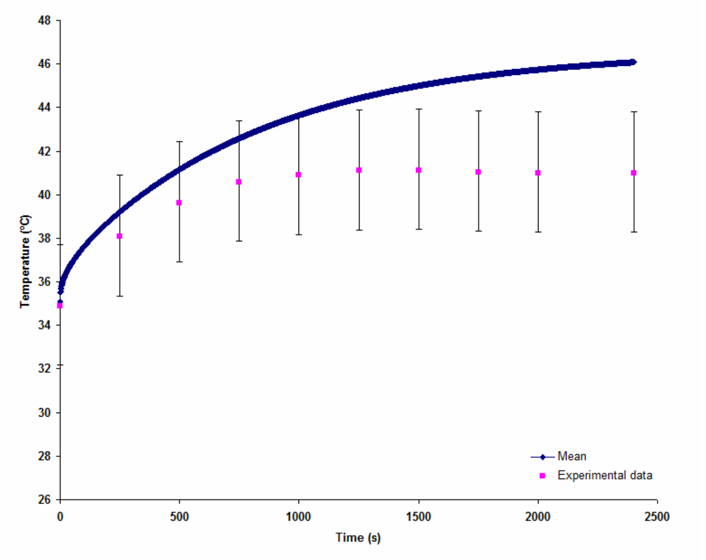

Figure 7. Variation of the average temperature of the epidermis with time – *simulation b*.

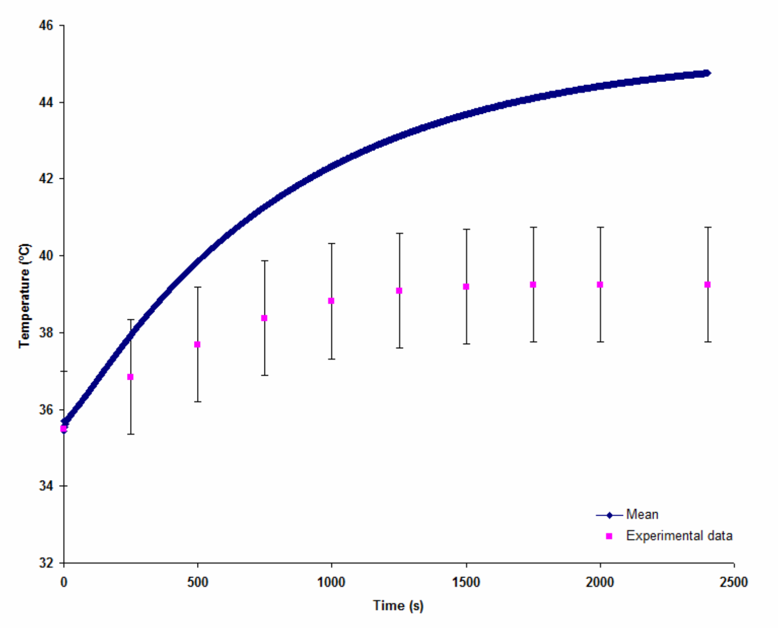

Figure 8. Variation of the average temperature of the subcutaneous tissue with time – *simulation b*.

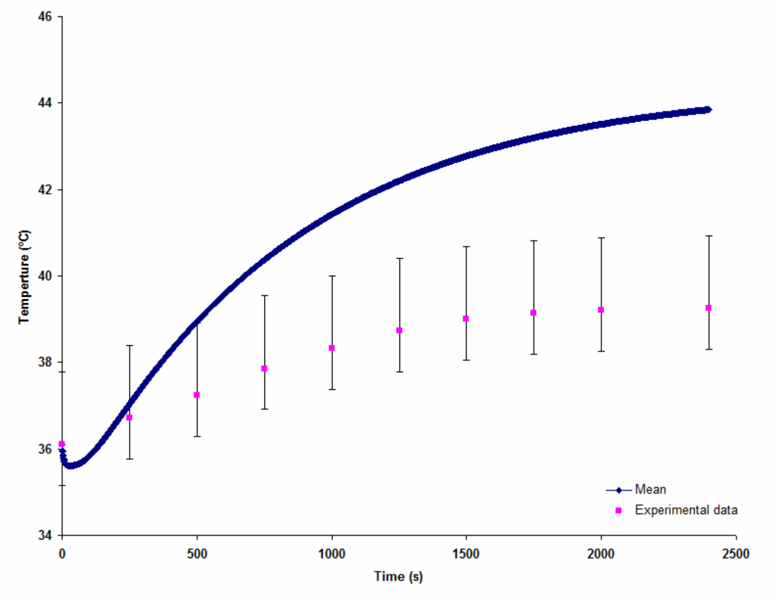

Figure 9. Variation of the average temperature of the fat tissue with time – *simulation b*.

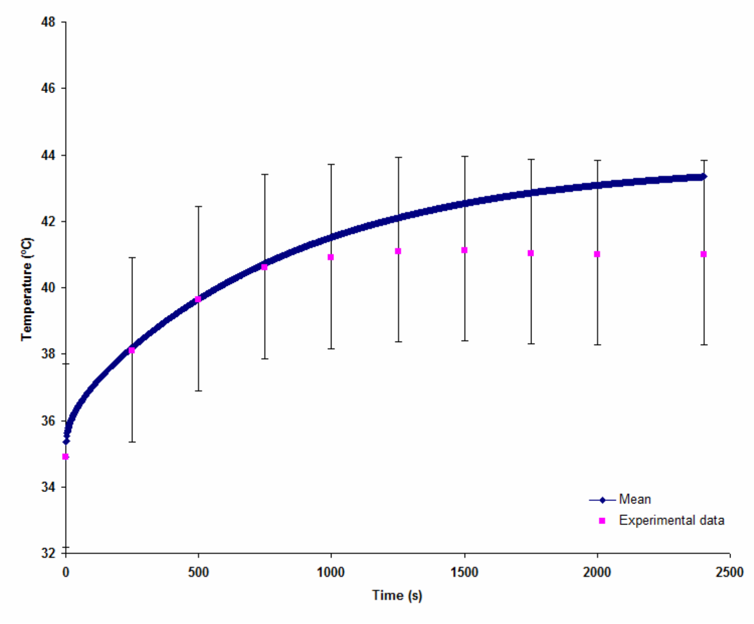

Figure 10. Variation of the average temperature of the epidermis with time – *simulation b.*

4. CONCLUSIONS

The present study aimed at a simulation of transient heat transfer in the canine knee as a result from the application of superficial thermotherapy. Validation tests indicated that the software provides accurate calculations of the bio-heat transfer (Pennes) equation. Simulations took into account two types of boundary conditions, based on known values of temperature and heat flux at the epidermis. A comparison of the simulations and experimental data showed that the boundary condition based on heat flux provided a better description of the heat transfer phenomena. Further studies will target model improvements by including the effect of heat losses.

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6. RESPONSIBILITY NOTICE