

RESIDUAL MASS FOR THE IMMISCIBLE LIQUID-LIQUID DISPLACEMENT OF TWO VISCO-PLASTIC MATERIAL IN A PLANE CHANNEL

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Abstract. *The motion of two immiscible visco-plastic materials in a capillary plane channel is analyzed for the case in which the flow conditions and the interactions between the liquids and the solid surface maintain the displaced SSuid attached to the wall. An elliptic mesh generation technique, coupled with the Galerkin Finite Element Method is used to compute the velocity Þeld and the conÞguration of the interface between the two SSuids. Examples of important applications where this kind of problem appears are the oil recovery processes from porous media and the cementation of oil wells, where the liquid Þlm of the displaced material that remains attached to the rocks plays a signiÞcant role on the e?ciency of the operation. Typically, because of the high viscosity levels and slow displacement velocities involved, these kind of processes occur with negligible inertial effects. Besides that, especially in oil recovery, as a consequence of the small length scale, the capillary forces have a fundamental importance on the physics of the phenomenon. We can find in the literature interesting investigations involving non-Newtonian liquid-liquid displacement. However, in the most of them, the interfacial tension was neglected. The objective of the present work is to present results for the thickness of the displaced liquid Þlm attached to the channel walls as a function of the non-Newtonian properties of the two fluids considered for a large range of capillary number and viscosity ratio.*

Keywords: *liquid-liquid displacement, visco-plastic materials, Galerkin Finite Element Method*

1. INTRODUCTION

The displacement of a fluid by another is an important problem in fluid mechanics that has received significant attention from the literature. Among the numerous industrial applications, we can mention the process of mud removal during the primary cementing of an oil well and the oil recovery in porous media. This last process is sketched in Fig. 1. In this case, the mechanical properties of the oil that fills the rocks of the reservoir are fixed. On the other hand one can test different injected materials so as to take advantage of the rheological character of the displacing fluid to optimize this process.

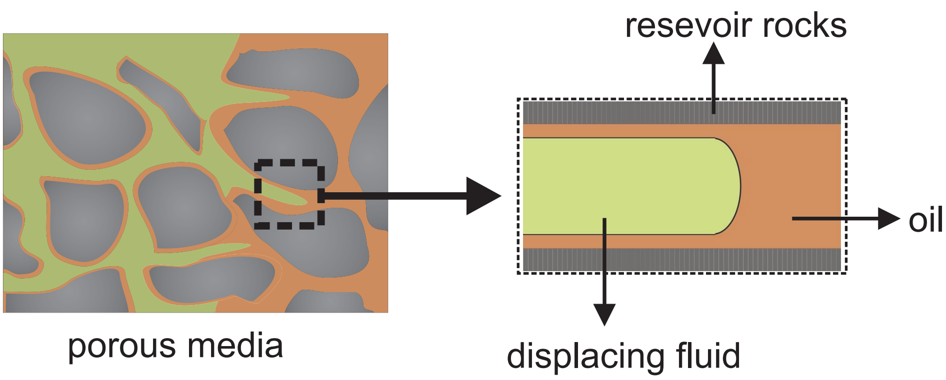

Figure 1. On the left: injection of Fluid 1 into a porous media filled with Fluid 2. On the right: simplified 2-D problem.

The idealized problem conceived to model the industrial process described can be simplified into a 2-D approach in two manners: the fluid-fluid displacement in a capillary tube or in a capillary plane channel, often called Hele-Shaw cell. The former represents a useful simplification for the case where the cross area of the porous passage has an aspect ratio around the unity while the latter is more adequate to mimic the case of high values of aspect ratio. A comparison between the axi-symmetric and Cartesian approaches for the case of Newtonian-Newtonian displacement can be found in [28] .

The gas-liquid (Newtonian) displacement has received considerable attention in the literature and was investigated by several authors such as [13] (experimentally), [5] (theoretically) , [32] (experimentally), [7] (experimentally), [25] (theoretically), [6] (experimentally), [21] (numerically), [15] (numerically).

The gas displacement of non-Newtonian fluids has also received some attention. [17], [30], [34] investigated the gas-displacement of power-law fluids. [1], [11], [**?**], [30], [34] studied the problem of gas displacing viscoplastic materials. The gas displacement of viscoelastic material was studied by [19], [20], [12], among others.

The liquid-liquid displacement of two immiscible Newtonian fluids has been investigated since the pioneering work of [16]. Since than, important contributions were made by [35], [22], [18], [27], [29] and [28], among others. In the liquid-liquid displacement problem the conservation equations are solved in the displacing fluid domain also and the force balance at the interface needs an additional term that takes into account the extra normal stress at the displacing fluid side, besides the pressure.

[2] and [26] investigated the liquid-liquid displacement when the two fluids are non-Newtonian. They recovered the immiscible problem as an approximation of the miscible one for the limit of high values of the Peclet number and, therefore, capillary effects were not analyzed.

A recent study of the Newtonian displacement by a non-Newtonian liquid when capillary stresses are important was conducted by [34] for the case of power-law displacing fluid.

Viscoplastic materials are widely used in industrial processes. Since the seminal work of [4] till nowadays (e.g. [9]), modeling yield-stress materials is an active area of research. It is not an easy task to implement a numerical scheme to solve complex flows of yield-stress materials in the sense originally conceived by [4], i.e. with a vanishing rate of deformation when the applied stress is below the material yield stress. In this case one can determine an yield surface in the frontier between unyielded and yielded zones. The options to implement an approximation of this kind of behavior are: a) A bi-viscosity model; b) a viscoplastic model with a regularization parameter; c) a viscoplastic physically-regularized model.

Bi-viscosity models are characterized by a two-rule expression for the viscosity function: a very high value when the stress is below the yield stress and the classic expression proposed by [4]: $\eta = \mu_P + \frac{\tau_0}{\dot{\gamma}}$.

The models with a regularization parameter are inspired by the original work developed by [24]. In this case, an ad-hoc (regularization) parameter is introduced and the proposed model has a smooth transition between the region where the stress is below the material yield stress to the region where the stress is above the material yield stress. The choice of the regularization parameter is a matter of discussion in the literature. Important analyses on the subject are made by [14] and [26]. Ideally the problem should be independent of the chosen value. Generally it is accepted that the position and shape of the "yield surface" is an interesting parameter to evaluate the regularization-parameter independence. It is worth noticing that [36], by comparing analytical results in viscometric flows between Bingham and Papanastasiou's models, recommend a dimensionless regularization parameter of a thousan d. Generally, as the regularization parameter is increased, the model leads to an yield-surface model.

A first viscoplastic physically-regularized model was presented by [10]. In this case, the model is conceptually more aligned to the idea, discussed by [3], that the yield stress does not bound a region where there is no deformation rate, but simply is a level of stress below which the fluid has a very high level of viscosity. When the stress achieves the material yield stress, the micro-structure starts to be destroyed and there is a dramatic decrease of viscosity. In this case, there also appears a parameter that regularizes the model, but now with a physical interpretation. This parameter is a rheological material function to be determined by rheometric characterization and its dimensionless form is called *jump number*.

In the present work, we use the second approach to analyze the displacement of two Papanastasiou materials in a plane channel. We focus our investigation on the influence of the yield number on the displacement efficiency.

2. PHYSICAL FORMULATION

The scheme of the problem analyzed is depicted in Fig. 2. The reference frame is attached to the tip of the drop and, therefore, the wall moves with its velocity, U. The channel has a $2H_0$ gap. We assume that there is a part of the domain, Region IV, where the flow is fully-developed with a fixed distance from the centerline till the interface, H_b. With this assumption, the problem in this reference frame is steady. The flow is assumed to be isothermal, inertialess, and incompressible.

2.1 Governing equations

The governing equations are presented here in a dimensionless form using appropriate characteristic velocity and length scales, the wall velocity U and the half distance of the capillary gap, H_0, respectively, together with a characteristic stress, $\eta_{C2}U/H_0$, where η_{C2} is a characteristic viscosity of the viscoplastic displaced fluid.

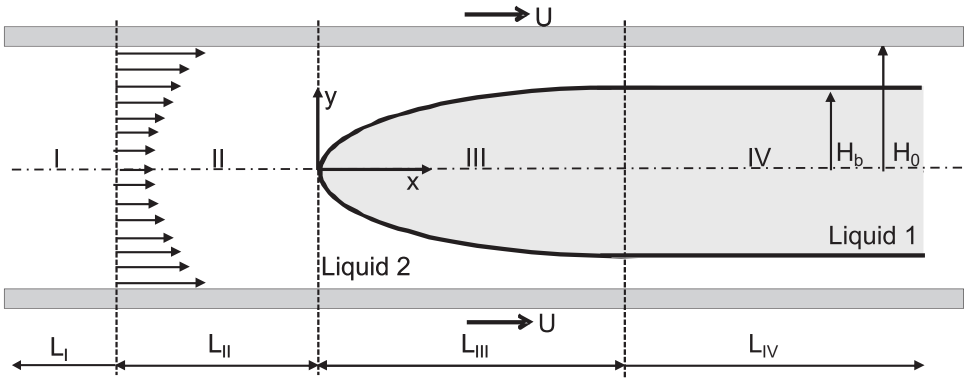

Figure 2. Scheme of the problem.

The conservation of mass equation is given by

$$\nabla \cdot \mathbf{u}'_k = 0. \tag{1}$$

Considering the Papanastasious equation to model the visco-plastic materials, the momentum conservation equations for Liquid 1 and Liquid 2 are given by

$$-\nabla p'_1 + \frac{1}{N_\eta}\nabla' \cdot \{1 - \tau'_{01} + \frac{\tau'_{01}}{\dot{\gamma}'_1}[(1 - exp(-\alpha'\dot{\gamma}'_1)]\}\mathbf{D'}_1 = 0 \tag{2}$$

$$-\nabla p'_2 + \nabla' \cdot \{1 - \tau'_{02} + \frac{\tau'_{02}}{\dot{\gamma}'_2}[(1 - exp(-\alpha'\dot{\gamma}'_2)]\}\mathbf{D'}_2 = 0. \tag{3}$$

Where, N_η, τ'_{01} and τ'_{02} are the viscosity ratio, yield number for liquid 1 and yield number for liquid 2 defined by the equations below.

$$N_\eta = \frac{\eta_2}{\eta_1} = \frac{\mu_{p2} + \tau_{02}(H_0/U)}{\mu_{p1} + \tau_{01}(H_0/U)} \tag{4}$$

$$\tau'_{01} = \frac{\tau_{01}}{\tau_{01} + \mu_{p1}(U/H_0)} \tag{5}$$

$$\tau'_{02} = \frac{\tau_{02}}{\tau_{02} + \mu_{p2}(U/H_0)} \tag{6}$$

2.2 Boundary conditions

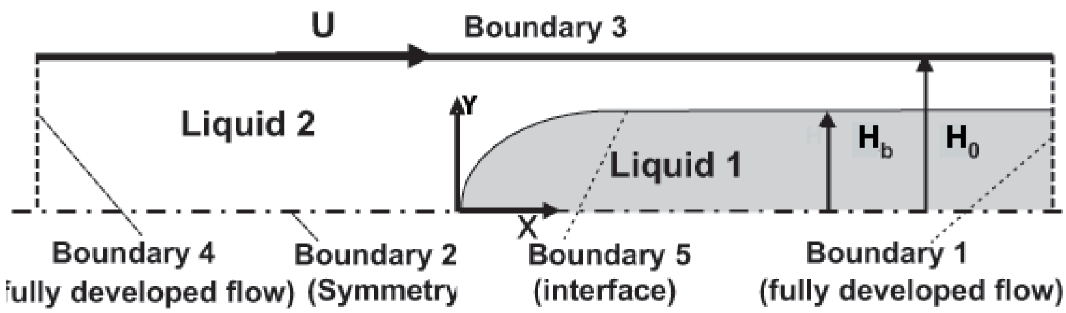

Figure 3. Boundary conditions.

The boundary conditions are exactly the same as described in [28]. Here we make a brief description. We labelled the boundaries from 1 to 5 (see Fig. **??**). At Boundary 4 and Boundary 1, we impose fully-developed flow and uniform pressure. Along the symmetry axis, Boundary 2, both the shear stress and the radial velocity vanish. No-slip and impermeability conditions are imposed along the tube wall, Boundary 3. Along the interface, Boundary 5, **where $\mathbf{t}$ is the tangent vector**,

$$\mathbf{u_1} = \mathbf{u_2} = (\mathbf{u_k} \cdot \mathbf{t})\,\mathbf{t}, \tag{7}$$

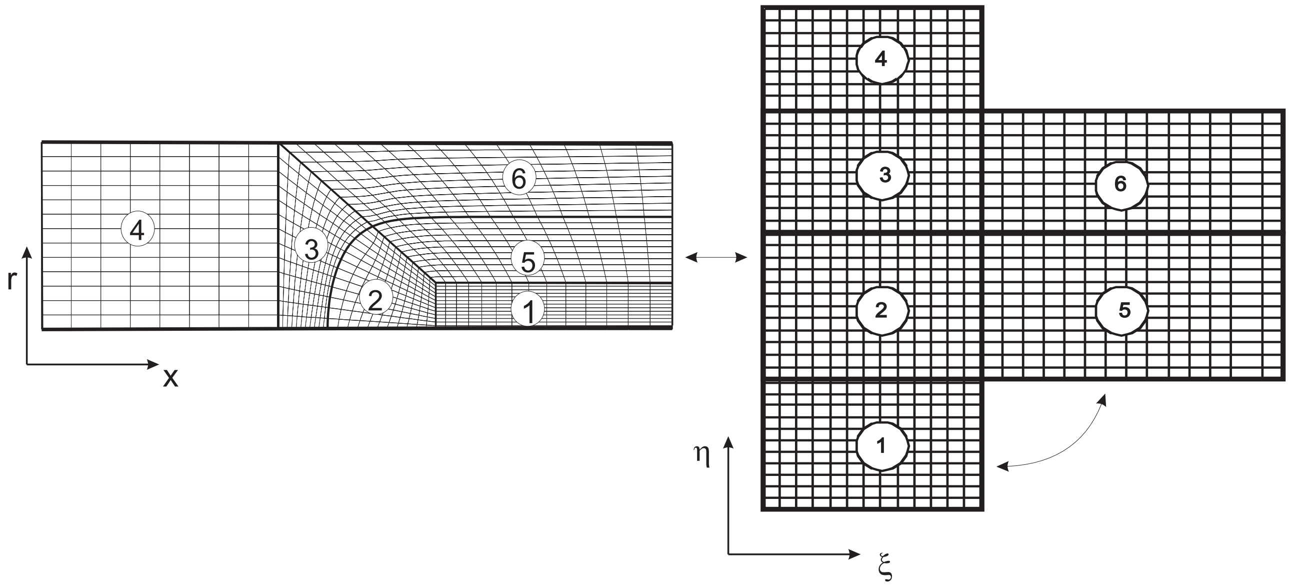

Figure 4. Mapping between reference and physical domains.

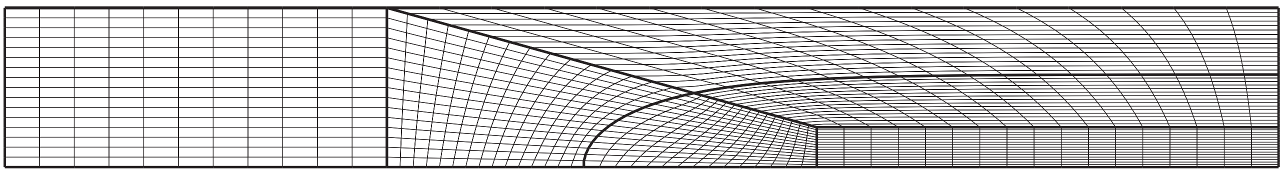
Figure 5. Typical mesh.

which states that there is no flux crossing the interface. Besides that, at this same Boundary 5, the traction balances the capillary pressure. The dimensionless form of the traction balance is given by the equation below.

$$\mathbf{n}(p'_1 - p_2) + \mathbf{n}\cdot\{[1-\tau'_{02} + \frac{\tau'_{02}}{\dot{\gamma}'_2}(1-e^{-\alpha'\dot{\gamma}_2})]\mathbf{D}'_2 - [1-\tau'_{01}+\frac{\tau'_{01}}{\dot{\gamma}'_1}(1-e^{-\alpha'\dot{\gamma}_1})]\frac{\mathbf{D}'_1}{N_\eta}\} = \frac{1}{R'_m}\frac{1}{Ca} \qquad (8)$$

where Ca is the the capillary number, defined below, p'_1 and p'_2 are the dimensionless pressures on Phase 1 and Phase 2, respectively, and $1/R'_m$ is the dimensionless local mean curvature of the interface.

2.3 Numerical implementation

The details concerning the elliptic grid generation, the solution of the equation system by Galerkin Finite Element Method, including the basis functions used to represent position, velocity and pressure are described in [29] with some adaptation to cartesian coordinates as presented by [28]. Since the domain of the problem is part of the solution, the problem is formulated in a reference domain and mapped onto the physical domain as shown in Fig. 4.

For the solution of the non-linear system of algebraic equations by Newton's Method, a frontal solver was used. The initial guess to solve the Newtonian displacement of a viscoplastic material was the Newtonian-Newtonian displacement with the same viscosity ratio. Figure 5 shows a typical mesh used in our simulations.

2.4 RESULTS

We present here the residual mass fraction as a function of capillary number and viscosity ratio for a wide range of yield number on Liquid 1 and 2.

Figure 6 shows m times Ca for a fixed viscosity ratio and yield number 1($N_\eta = 4$ and $\tau'_{01} = 0.4$). The yield number for liquid 2 is variated from zero, the Newtonian case, to 0.8, a high visco-plastic material ($0 < \tau'_{02} < 0.8$). As the yield number 2 is increased the fraction of mass deposited on the wall falls. The same tendency is observed on Figure 7 where the viscosity ratio is increased, $N_\eta = 8$. However, the difference from Newtonian curve is much less pronounced.

Figure 6 shows the residual mass, now, fixing the yield number of Liquid 2, changing the the yield number 1. Again, the fraction of mass falls, but the difference from Newtonian case is not so pronounced as it is shown in Figure 6.

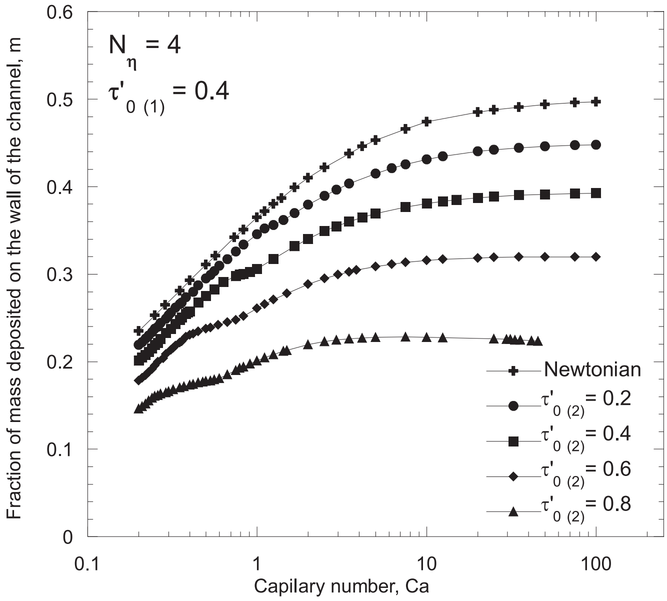

Figure 6. Diagram of shear modulus versus frequency at 303 K

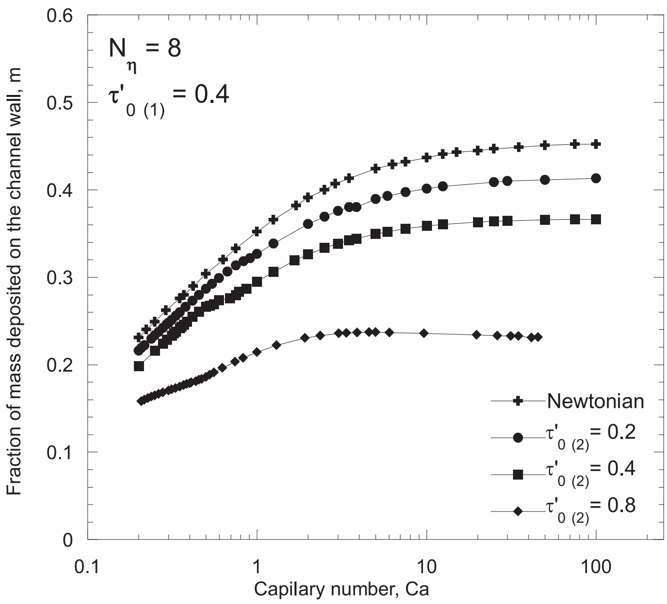

Figure 7. Diagram of shear modulus versus frequency at 303 K

3. FINAL REMARKS

The numerical analysis by finite element method of displacement of two visco-plastic material in a capillary plane channel was presented. The results were focused on the effect of yield number 1 and 2 on the fraction of mass attached to the wall of the plates. The simulations indicate that the residual mass always reach an asymptotical value in high capillary numbers. It is clear that the influence of viscosity ratio and yield numbers are in the same direction: the residual mass decreases as these parameters are increased. Though, the difference from the Newtonian results are more pronounced when N_η is smaller. More over, the effect of yield number 2 is much more pronounced that yield number 1.

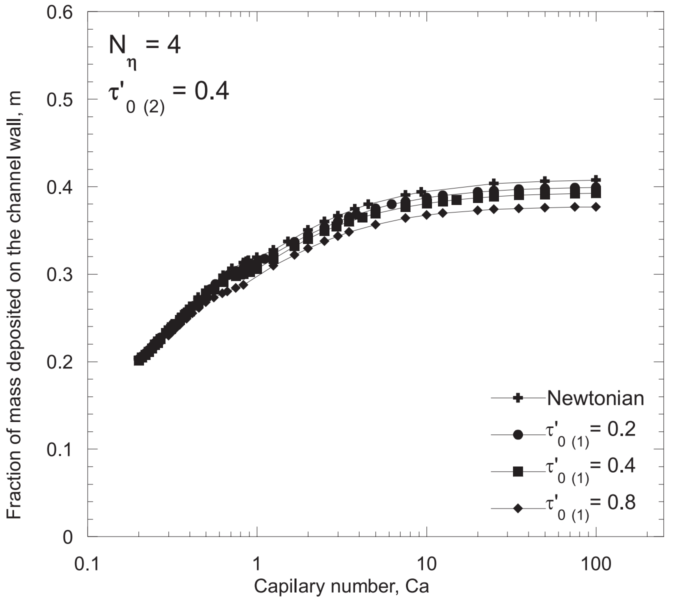

Figure 8. Diagram of shear modulus versus frequency at 303 K

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