ANALYTICAL DETERMINATION OF THE TRANSIENT TEMPERATURE DISTRIBUTION IN GUN BARRELS SUBJECTED TO EXTERNAL COOLING

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Abstract. *When firing artillery rounds, heat flows into the gun inner surfaces in large amounts resulting in wear and erosion of the gun bore. This reduces the accuracy of the projectile and ultimately diminishes the life of the gun. Moreover, depending on the number of rounds fired per minute, the chamber surface temperature could reach the cook-off temperature of the propellant. If this happens, the next round will self-ignite, jeopardizing the safety of the gun crew. In order to circumvent this problem, large-caliber weapons subjected to long-burst firing, such as guns and howitzers, must be cooled. In this contribution, a theoretical analysis of the heat transfer in gun barrels is performed and the resulting model is analytically solved thus furnishing the transient temperature distribution for the situation of external wall cooling. Therefore, benchmark results are produced with a very low computational effort. Furthermore, some situations related to barrel external natural cooling are simulated and the results are critically compared to the ones obtained through the finite elements method.*

Keywords*: heat transfer, analytical methods, gun barrel, wall cooling.*

1. INTRODUCTION

It has long been known that the performance of a gun is affected by the heating of its barrel. During the firing, gun bore surface receives large amounts of heat resulting from both combustion of the propellant and friction between the projectile and the inner wall. Although the barrel has some time to cool down before another round is fired, only a small amount of heat is transferred to the environment by means of convection and radiation, which results in increasing temperatures in the barrel as rounds are shot in sequence. The limiting temperature for a barrel is known as the “cook-off” temperature above which the propellant of the next round will self-ignite thus causing an explosion that could severely injure or even kill the gun crew. In addition to this safety issue, gun bore erosion is closely related to high temperatures and heat fluxes. Barrel wear reduces the accuracy of the projectile which is also an undesirable effect.

Wu et al. (2008) analyzed the heat transfer in a 155 mm midwal cooled compound gun barrel. The finite elements method was employed to validate the theoretical analysis showing that natural air cooling is ineffective to transferring heat out of the barrel and that forced midwall cooling has great extraction capabilities. The problem of manufacturing such midwall cooled compound gun barrel was not addressed. A heat balance estimated typical cook-off temperatures to be around 200°C.

Lawton (2001) derived a simple equation of the Arrhenius type relating the wear per round to the initial temperature, maximum surface temperature and the erosivity of the propellant. The equation was verified by experiments and by data on the wear rates of numerous guns and propellant combinations.

A brief literature review also suggests that inverse heat conduction strategies have been devised to determine quantities of practical interest in the heating of gun barrels. For example, Chen et al. (2007) employed a recursive input estimation algorithm based on Kalman filters and real-time least square methods to determine the heat flux and the temperature at the inner wall in rifles and in machine guns subjected to continuous firing situations. The gun barrel wss modeled as a hollow two-layered (chrome-steel) cylinder subjected to a one-dimensional transient heat transfer process. Their findings indicate that the proposed technique is capable of reconstructing triangle and square wave types of heat fluxes at the inner wall with a good accuracy and, moreover, the authors claim that such numerical tests can be useful to develop strategies aimed at prolonging barrel life and reducing maintenance costs. In a more recent contribution Chen and Liu (2008) carried out basically the same study but taking into account a two-dimensional transient heat transfer study in gun barrels while disregarding the contribution of the chrome layer. Finite element schemes were used to solve the direct problem and triangular, sine wave and Weilbull distributions were taken as case studies that simulate the heat flux at the inner tube wall. Once again, excellent reconstruction patters were obtained validating the numerical strategy described in their research. Finally, Lee and his collaborators (2009) have recently described an inverse algorithm based on the Conjugate Gradient Method that also attempts to estimate the unknown time-varying heat flux at the inner wall in a 5.56 mm machine gun. Here, the direct problem was solved by an efficient hybrid analytical-numerical procedure based on Laplace transforms and finite difference schemes. Consistent with previously published works, the inner surface of the gun barrel was coated with a thin layer of chrome and the thermal contact resistance between the chrome and steel layers was taken into account in the heat transfer formulation. An interesting aspect of this particular contribution is that they were also concerned with the determination of the thermal stresses that arise from the

continuous firing of the gun. They performed a critical evaluation of the proposed methodology and apparently highly accurate estimations of the time dependent inner wall heat flux and radial stress distributions were obtained.
A common trait to most of the above-mentioned papers is that the direct problem is solved by using numerical strategies. However, purely analytical solution schemes are always desirable especially within the context of this particular problem in which steep temperature gradients are expected to arise due to the rapid and intricate phenomena related to the combustion gases and the sliding of the projectile in the machine chamber. Consequently, the purpose of this contribution is to perform a theoretical analysis of the transient temperature distribution on a gun barrel subjected to external cooling solely by natural convection and radiation to the ambient. This is the typical situation encountered in the fields of war. As the solution to the mathematical problem is analytical, it outputs highly accurate benchmark results. This allows the assessment of the performance of commonly employed artillery guns furnishing a valuable insight for the gun designer. Moreover, it is a simple matter to extend the simple mathematical model here advanced in order to simulate other convective heat transfer situations of interest like forced air and forced liquid cooling.

2. ANALYSIS

In this section, we present and discuss a theoretical analysis of the transient temperature field of a gun barrel upon a first round which is being cooled at its outer surface by a mixed linearized convection-radiation process. Even though more involved problem formulations are possible, in this contribution we closely follow the formulation presented in Wu et al. (2008) in which the gun barrel is modeled as a hollow cylinder of inner and outer radius given by r_{in} and r_{out} under a transient one dimensional, temperature-independent property situation. Accordingly, the radial transient temperature distribution T(r,t) is supposed to be governed by the following heat transfer equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]=\frac{1}{\alpha}\frac{\partial T}{\partial t}\,,\ r_{in}<r<r_{out}\,,\ t>0 \tag{1}$$

where α is the thermal diffusivity of the material of the gun barrel. Next, an initial condition related to a uniform temperature distribution is assumed, and therefore we have:

$$T(r,0) = T_0\,,\ r_{in} \le r \le r_{out} \tag{2}$$

In order to account for the complex set of phenomena related to the combustion in the inner surface of the barrel, an exponentially time decaying heat flux is proposed (Wu et al., 2008; Lawton, 2001) and thus, the boundary condition at r_{in} is given by:

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_{in}} = q_0\, e^{-bt}\,,\ t>0 \tag{3}$$

In the above relation, k is the thermal conductivity of the barrel, q_0 is a heat flux at in the earliest stage of the first round and b is the exponent related to rate of the decay of the heat transfer from combustion process to the inner wall. At the outer wall, r_{out} , a convection and radiation process with the ambient air is supposed to take place. If h is the combined convection-radiation heat transfer coefficient and T_∞ is the air temperature, the boundary condition at r_{out} takes the form:

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_{out}} = h\left[T(r_{out},t)-T_\infty\right],\ t>0 \tag{4}$$

Having completed the problem formulation, we now seek an analytical approach to the determination of the transient temperature distribution of the gun barrel during and after the first round. We make use of the so-called “split-up” procedure described in Özisik (1980) and construct the solution as:

$$T(r,t) = T_\infty + T_{aux}(r)\,e^{-bt} + T_h(r,t) \tag{5}$$

where the auxiliary problem for the determination of T_{aux} is given by:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT_{aux}(r)}{dr}\right]+\frac{b}{\alpha}T_{aux}(r)=0\,,\ r_{in}<r<r_{out} \tag{6}$$

$$-k\left.\frac{dT_{aux}(r)}{dr}\right|_{r=r_{in}}=q_0 \tag{7}$$

$$-k\left.\frac{dT_{aux}(r)}{dr}\right|_{r=r_{out}}=h\ T_{aux}(r_{out}) \tag{8}$$

The homogeneous problem for $T_h(r,t)$ is now determined by inserting the proposed solution represented by Eq. (5) in the original problem formulation, Eqs. (1) – (4). With the aid of the relations (7) and (8), it is a relatively straightforward matter to show that the set of relations governing $T_h(r,t)$ is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_h}{\partial r}\right]=\frac{1}{\alpha}\frac{\partial T_h}{\partial t}\,,\ r_{in}<r<r_{out}\,,\ t>0 \tag{9}$$

$$T_h(r,0) = T_0 - T_\infty - T_{aux}(r),\quad r_{in}\le r\le r_{out} \tag{10}$$

$$\left.\frac{\partial T_h}{\partial r}\right|_{r=r_{in}}=0\,,\ t>0 \tag{11}$$

$$-k\left.\frac{\partial T_h}{\partial r}\right|_{r=r_{out}}=h\,T_h(r_{out},t)\,,\ t>0 \tag{12}$$

Through inspection, we can determine the solution of the steady-state auxiliary problem as a linear combination of first and second type Bessel functions of order zero in the form of:

$$T_{aux}(r)=C_1^{\ *}\,J_0(\sqrt{(b/\alpha)}\,r)+C_2^{\ *}\,Y_0(\sqrt{(b/\alpha)}\,r) \tag{13}$$

Upon substitution of Eq. (13) on boundary conditions (7) and (8), the two constants are determined as being:

$$C_1^{\ *}=-q_0\,\frac{S_{0m}}{k\sqrt{(b/\alpha)}}\left[S_{0m}\ J_0'(\sqrt{(b/\alpha)}\,r_{in})-V_{0m}\ Y_0'(\sqrt{(b/\alpha)}\,r_{in})\right]^{-1} \tag{14}$$

$$C_2^{\ *}=q_0\,\frac{V_{0m}}{k\sqrt{(b/\alpha)}}\left[S_{0m}\ J_0'(\sqrt{(b/\alpha)}\,r_{in})-V_{0m}\ Y_0'(\sqrt{(b/\alpha)}\,r_{in})\right]^{-1} \tag{15}$$

where the variables S_{0m} and V_{0m} are evaluated as:

$$S_{0m}=(h/k)\,Y_0(\sqrt{(b/\alpha)}\,r_{out})+\sqrt{(b/\alpha)}\ Y_0'(\sqrt{(b/\alpha)}\,r_{out}) \tag{16}$$

$$V_{0m}=(h/k)\,J_0(\sqrt{(b/\alpha)}\,r_{out})+\sqrt{(b/\alpha)}\ J_0'(\sqrt{(b/\alpha)}\,r_{out}) \tag{17}$$

Once the solution for the auxiliary problem is available, the homogeneous problem is readily solved through the classical method of separation of variables (Özisik, 1980). Therefore, the homogeneous transient temperature distribution is determined through an eigenfunction expansion of the form:

$$T_h(r,t)=\sum_{i=1}^{\infty}C_i\ \psi_i(r)\,e^{-\alpha\mu_i^{\,2}t} \tag{18}$$

where,

$$C_i = \frac{1}{N_i}\left[(T_0 - T_\infty)\frac{r_{out}}{\mu_i^2}\left(\frac{h}{k}\right)\psi_i(r_{out}) + \frac{r_{in}\ \psi_i(r_{in})(q_0/k)}{(b/\alpha) - \mu_i^2}\right] \tag{19}$$

The norms N_i and eigenfunctions ψ_i of the associated Sturm-Liouville problem are (Ozisik, 1980)

$$\frac{1}{N_i} = \frac{\pi^2}{2}\frac{\mu_i^2\ J_0'^2(\mu_i\ r_{in})}{\left[(h/k)^2 - \mu_i^2\right]J_0'^2(\mu_i\ r_{in}) - V_0^2} \tag{20}$$

$$\psi_i(r) = S_0\ J_0(\mu_i\ r) + V_0\ Y_0(\mu_i\ r) \tag{21}$$

where S_0 and V_0 are defined as:

$$S_0 = (h/k)\,Y_0(\mu_i\ r_{out}) + \mu_i\ Y_0'(\mu_i\ r_{out}) \tag{22}$$

$$V_0 = (h/k)\,J_0(\mu_i\ r_{out}) + \mu_i\ J_0'(\mu_i\ r_{out}) \tag{23}$$

and the eigenvalues μ_i are determined as the positive roots of the following transcendental equation:

$$S_0\ J_0'(\mu_i\ r_{in}) - V_0\ Y_0'(\mu_i\ r_{in}) = 0 \tag{24}$$

3. RESULTS AND DISCUSSION

The methodology described in the previous section was employed to solve the problem described by Eqs. (1) - (4). In order to obtain results that could be compared to the literature, two practical situations were simulated. The first one was the traditional 155 mm artillery gun with an external radius of 107.5 mm subjected to natural air cooling and radiation to the environment which results in a combined heat transfer coefficient (h) equal to 30 W/m²/K (Wu et al., 2008). The second one was a 155 mm artillery gun with an extended external radius of 127.5 mm that simulates the presence of an outer jacket subjected again to natural air cooling and radiation. The latter geometry is closer to the more sophisticated one analyzed by Wu et al. (2008) that adopted a midwall cooled compound gun barrel composed of an outer jacket and a barrel (liner) with an array of axial semicircular grooves on its exterior. In both situations studied here, the 155 mm gun barrel was assumed to be a cylinder made of steel and to bear the typical geometrical, mechanical and thermal properties as listed in Tab. 1. The ambient temperature was taken as being 27°C. For the heat flux input into the gun bore, Eq. (3), the values adopted for q_0 and b were, respectively, 1.927 x 10^8 W/m² and 210,97 (1/s) (Wu et al., 2008; Lawton, 2001).

Table 1. Geometrical, mechanical and thermal properties of the gun barrel

Inner Radius (mm)	77.5
Density (kg/m³)	7833
Specific Heat (J/kg.K)	460
Thermal Conductivity (W/m.K)	40
Thermal Diffusivity (m^2/s)	1.1 x 10^{-5}

To illustrate the accuracy of the proposed approach in predicting the temperature distribution across the gun barrel, a thorough study was performed on the convergence behavior of the solution for different truncation orders N in the infinite summation present in Eq. (18). The roots of Eq. (24) were computed through the Newton-Raphson method thus furnishing as many eigenvalues as needed for a desired numerical precision. Figure (1) and Figure (2) show such convergence behavior in the calculation of the transient temperature during the heating phase for the inner surface of the gun barrel for the first artillery round. It is observed that the convergence ratio of the series expansion as the number of

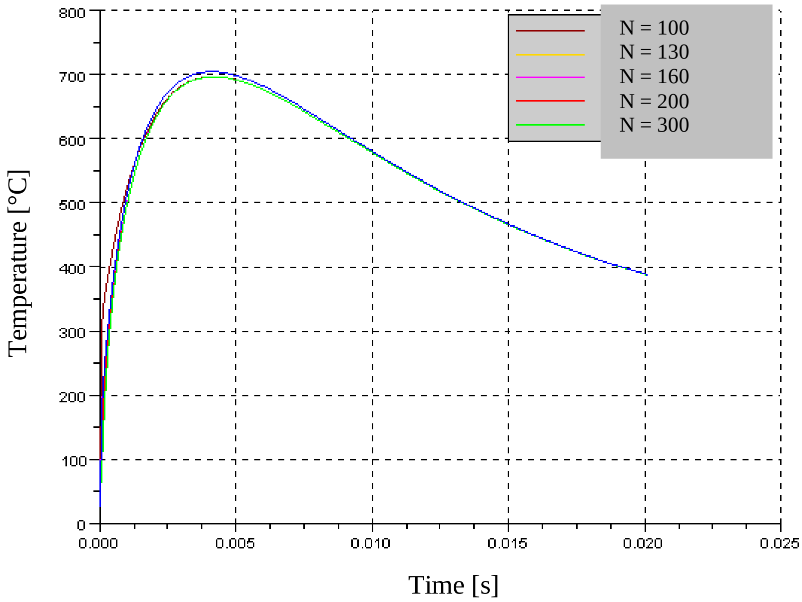

Figure 1. Transient temperature distribution at the inner surface of the barrel (r_{out} = 107.5 mm) for natural cooling (h = 30 W/m²K) and for several truncation orders.

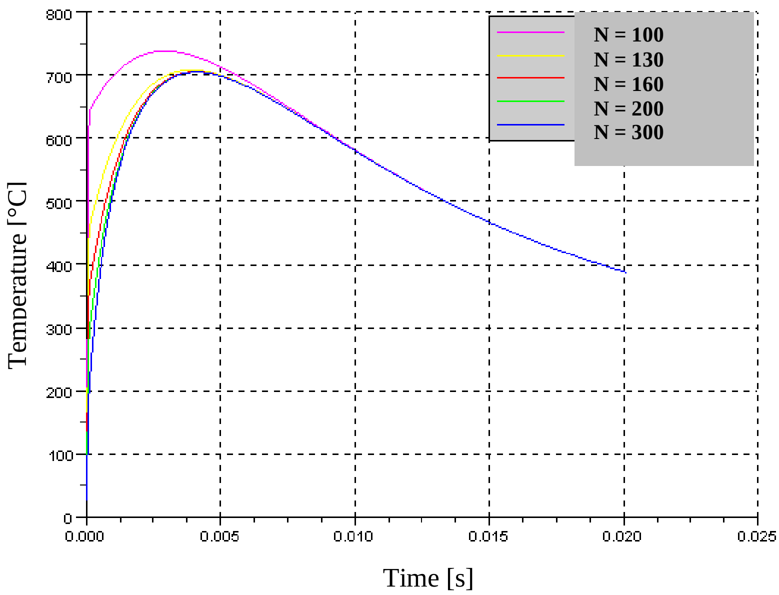

Figure 2. Transient temperature distribution at the inner surface of the barrel (r_{out} = 127.5 mm) for natural cooling (h = 30 W/m²K) and for several truncation orders.

eigenquantities retained (N) increases is greater for the case of the thinner wall. In both cases, it was found that an expansion up to 300 terms ensures 3 digit accuracy for times below 8 ms. This relatively high expansion order can actually be anticipated if one considers that a very steep temperature gradient is present in the early stages of the first round. Therefore, it is no surprise that in order to accurately capture the sharp rise in temperature during the first 5 ms which are depicted in Figs. (1) and (2), an unusually elevated truncation order is needed. Furthermore, an examination of these two figures reveals that the maximum temperature attained was 696.39°C for the first case (thinner wall) and 700.67 °C for the second case (thicker wall), both around t = 4 ms. The finite element solution by Wu et al. (2008) indicated that a maximum temperature of 672.8 °C is reached at t = 4.4 ms. Lawton (2001) describes how the inner surface temperature varies with time as a gun fires based on experimental data. This study reveals that the maximum bore temperature computed for the first situation is 701.8°C at t = 4 ms which is in close agreement with the peak value of Fig. (1). Since the modified 155 mm gun studied by Wu et al. (2008) has a complex geometry that includes cooling channels embedded in an outer jacket exposed to the ambient, the mathematical model that led to a peak temperature of 672.8 °C at t = 4.4 ms is somewhat different from the one presented in this contribution. In that model, the cooling jacket is represented by two extra convective boundary conditions and therefore a smaller peak temperature is expected due to the added heat removal. Nonetheless, the results obtained by our methodology strongly suggest that the present formulation is capable of furnishing realistic values for the transient temperature distribution in gun barrels either cooled by natural means or by embedded water jackets.

In order to further compare the analytical solution proposed here with traditional numerical methods, the FlexPDE® finite element software was employed to solve case 1 above (r_{out} = 107.5 mm). A finite elements grid was generated with 3002 nodes and 1350 triangular elements with mesh refinement at the inner surface of the tube. Figure (3) shows the 3D temperature distribution across the wall thickness for t = 20 ms. The software solution indicates a steadily decreasing temperature from 350°C at the inner surface to 27°C at the outer surface. As noted by Wu et al. (2008) the relative error between the results obtained by theoretical analysis method and the finite element method is 15%. In fact, for t = 20 ms, the temperature computed by the analytical method is close to 390°C.

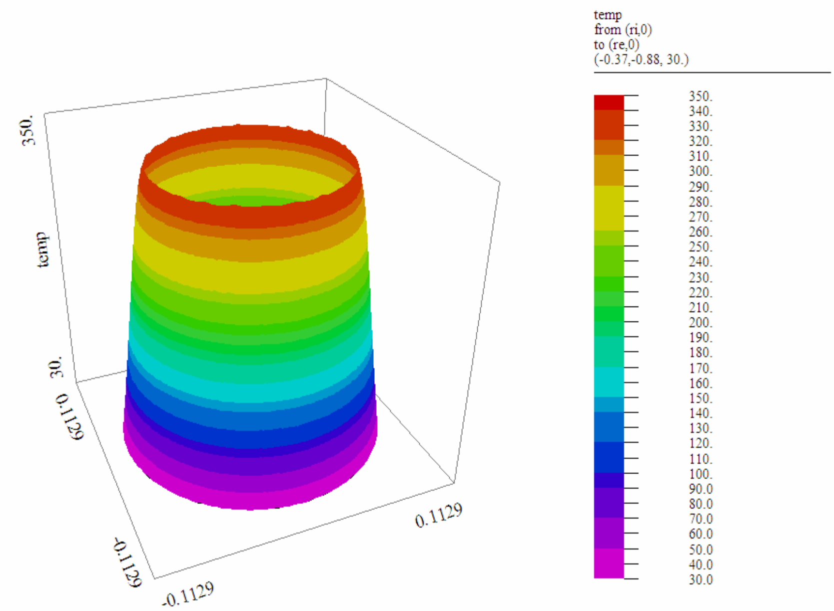

Figure 3. Surface temperature distribution for t = 20 ms and natural cooling (h = 30 W/m²K) – FlexPDE® results

Figure (4) shows the transient temperature distribution computed by FlexPDE® at different radial positions, namely, inner radius (a), midwall (b) and outer radius (c). It can be observed from this figure that the outer surface temperature remains unchanged during the heating phase (t ≤ 20 ms) since heat cannot be transferred across the 30 mm thickness in such a short period of time. This same trend was observed by Wu et al. (2008).

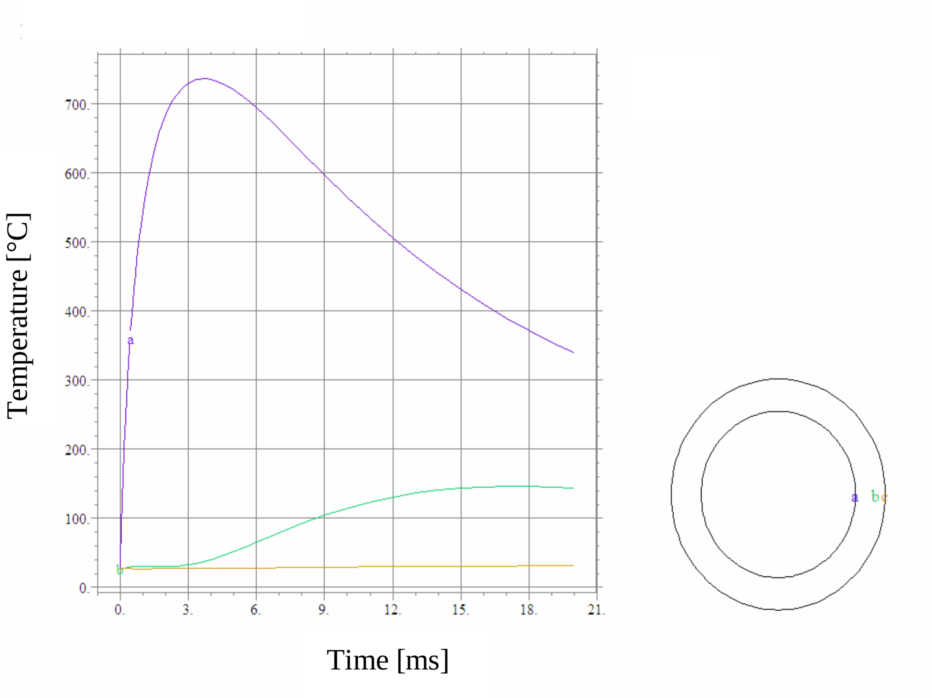

Figure 4. Transient temperature distribution at different radial positions for natural cooling (h = 30 W/m²K) – FlexPDE® results

Figure (5) shows the temperature distribution across the gun wall at t = 4 ms as predicted by FlexPDE®. When the peak temperature is reached, the temperature gradient induced by thermal input from the propellant detonation is very sharp and the bulk of the thermal energy is concentrated in a layer roughly 13 mm thick. This reinforces the reasoning that heat cannot be transferred far from the bore surface during the heating phase. As suggested by Wu et al. (2008), the thermal energy is first stored in a thin layer and then transferred to the outer surface during the cooling phase due to heat conduction. Therefore, the cooling mechanism of a gun firing passes through the stages of thermal input, thermal storage and thermal output.

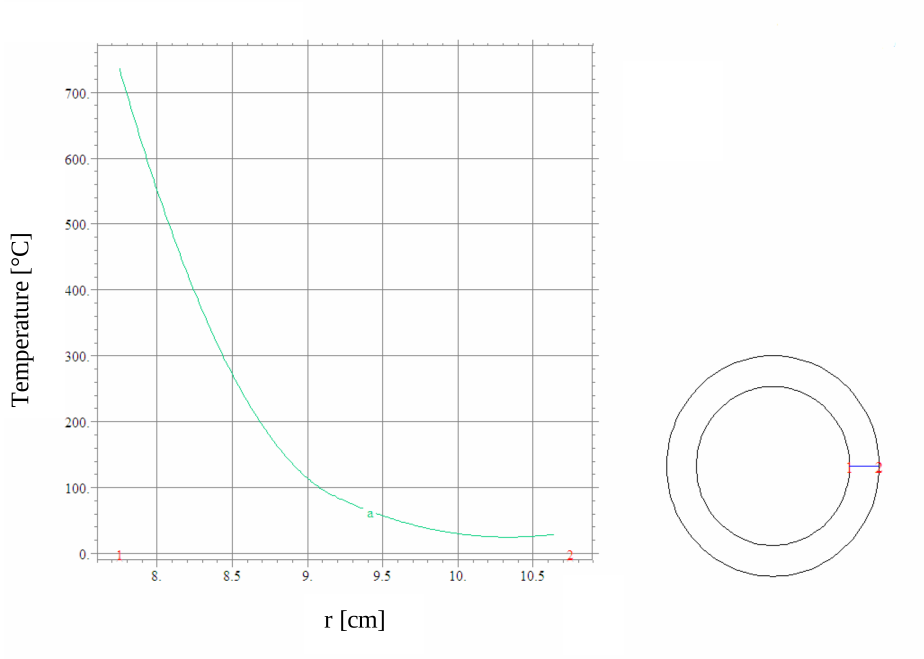

Figure 5. Temperature distribution across the gun wall at t = 4 ms for natural cooling h = 30 W/m²K) – FlexPDE® results

Now we turn our attention to the more involved situation in which a thermal analysis in gun barrels is performed when a certain number of rounds per minute are fired. It is a simple matter to show that this problem can be treated by the analysis outlined in the previous section but taking into account that the initial condition for round "j" is computed from the transient temperature distribution given by Eq. (5) at the end of round "j-1" (Gonzaga, 2009). Therefore, temperature distribution for the second artillery round fired after a given time interval is here obtained by substituting the solution T(r,t) at time t equal to the time of the second detonation for the initial condition described by Eq. (2) . The solution thus attained is also used as the initial condition for the third round and so on. This simple procedure furnishes a straightforward means of obtaining the transient temperature distribution after a desired number of rounds being shot. The procedure here advanced for treatment of multiple shots is purely analytical in contrast to the numerical ones found in the archival literature. Figure (6) depicts the analytical solution for the transient temperature distribution of the inner surface of the barrel under natural cooling and for a firing rate of 10 rounds per minute. Clearly, a heating phase and a cooling phase can be observed at every round fired. The temperature increment induced by every shot leads to an accumulated temperature that approaches the safe temperature limit (200°C) under natural cooling in about 60 s. Therefore, in a mission that fires a shot at every 6 seconds the eleventh artillery round would jeopardize the safety of the gun crew due to the probability of self-detonation.

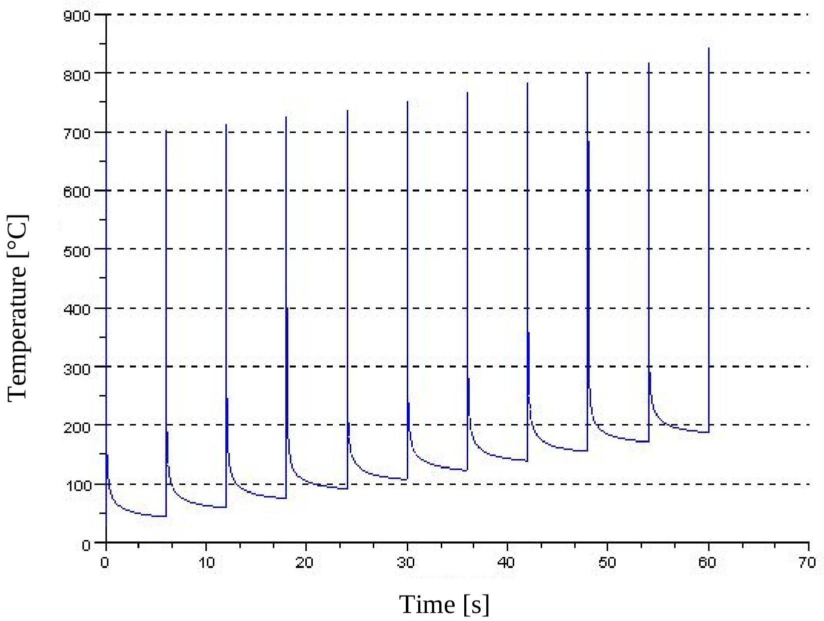

Figure 6. Transient temperature distribution at the inner surface of the barrel (r_{out} = 107.5 mm) for natural cooling (h = 30 W/m²K) and for a firing rate of 10 rounds per minute.

As a conclusion, we have successfully demonstrated the feasibility of applying classical techniques such as the "Split-Up" procedure together with an eigenfuction expansion to provide for a fully analytical solution of the one-dimensional transient temperature distribution of a gun barrel in a single or multiple shot situations. Our simulations revealed that the proposed strategy is capable of precisely capturing the steep temperature gradient that arises from such problems at an extremely low computational cost as a 300 expansion series yields temperature field with three-digit accuracy for the case of a single shot. As a final note, our current research efforts are aimed at employing the present strategy to determine the heat transfer flux at the inner wall by means of inverse problems techniques and real field data provided by the Brazilian Army.

4. REFERENCES

Chen, T.-S., Liu, C.-C., Jang, H.-Y., Tuan, P.-C., 2007, "Inverse Estimation of Heat Flux and Temperature in Multi-Layer Gun Barrel", International Journal of Heat and Mass Transfer, Vol. 50, pp. 2060-2068.

Chen, T.-S., Liu, C.-C., 2008,"Inverse Estimation of Time-Varied Heat Flux and Temperature on 2-D Gun Barrel using Input Estimation Method with Finite-element Scheme", Defense Science Journal, Vol. 58, pp. 57-76.

Gonzaga, R. N., 2009, "Heat Transfer Analysis in Mortar and/or Howitzer Tubes Subjected to Liquid Cooling", Internal Report (in Portuguese), IME, 41 p.

Lawton, B., 2001, "Thermal-Chemical Erosion in Gun Barrels", Wear, Vol. 251, pp.827–838.

Lee, H.-L., Yang, Y.-C., Chang, W.-J., Tser, T.-S., 2009, "Estimation of the Heat Flux and Thermal Stresses in Multilayer Gun Barrel with Thermal Contact Resistance", Applied Mathematics and Computing, Vol. 209, pp. 211-221.

Özisik, M.N., 1980, "Heat Conduction", John Wiley & Sons, Inc., 687 p.

Wu, B., Chen, G, Xia, W., 2008, "Heat transfer in a 155 mm Compound Gun Barrel with Full Length Integral Midwall Cooling Channels", Applied Thermal Engineering, Vol. 28, pp. 881-888.

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