SIMULATION OF BINGHAM-PAPANASTASIOU INERTIA FLOWS THROGH A SUDDEN EXPANSION BY A MULTIFIELD GLS FINITE ELEMENT METHOD

Hilda Pari Soto, hps@mec.uff.br
Maria Laura Martins-Costa, laura@mec.uff.br
Laboratory of Theoretical and Applied Mechanics (LMTA) – Mechanical Engineering Graduate Program (PGMEC) – Fluminense Federal University – Rua Passo da Pátria, 156 – 24210-240 – Niterói, RJ, Brazil

Cleiton Fonseca, cfonseca@mecanica.ufrgs.br
Sérgio Frey, frey@mecanica.ufrgs.br
Laboratory of Computational and Applied Fluid Mechanics (LAMAC) – Mechanical Engineering Department – Federal University of Rio Grande do Sul – Rua Sarmento Leite, 425 – 90050-170 – Porto Alegre, RS, Brazil

Abstract. *A stabilized multifield finite element methodology, the Galerkin least-squares (GLS) one, is employed to simulate inertia flows of regularized Bingham fluids (Papanastasiou, 1987) through a one-to-four sudden planar expansion. The GLS strategy modifies the classical Galerkin formulation in terms of extra-stress velocity and pressure, no longer requiring the satisfaction of the compatibility conditions involving the finite element sub-spaces for the pairs pressure-velocity and stress-velocity. Viscoplasticity and inertia effects on the morphology of yielded and unyielded surfaces are accounted for by first ranging the Bingham number up to Bn=100 for creeping flows and, in the sequence, by considering a mild Bn value and ranging Reynolds number from Re=0 to Re=50. The numerical strategy remained stable for very high yield stresses and high inertia flows.*

Keywords: *Bingham- Papanastasiou model, multifield Galerkin least-squares method, inertia effects, expansion flow.*

1. INTRODUCTION

Viscoplastic materials are characterized by a certain rigidity of its internal structure, leading to a minimum critical shear stress, τ_0 – an yield stress, which must be overcome before the material starts to flow. This concept has already been broadly discussed. Barnes (1999) presented a survey on yield-stress fluids behavior, covering the 20th century, emphasizing that although the concept of a true yield stress remains a controversial issue, an apparent yield stress would be valid for Engineering purposes. Among the viscoplastic models, the classical Bingham model, proposed by Bingham in 1922 (Bird et al., 1987), is the simplest and the most employed one in industrial applications. Bingham model presents two rheological parameters only: a yield stress limit – beyond which the material flows like a Newtonian fluid – and a constant viscosity, being characterized by the presence of a yield surface separating yielded and unyielded regions.

The classical Bingham model is discontinuous and provides no information concerning the stress distribution whenever the extra-stress τ is smaller than the yield-stress τ_0, characterizing the material rigid zones. Papanastasiou (1987) introduced a modified constitutive relation – essentially an exponential function controlled by a non-rheological parameter – allowing the shear stress to be described by a single equation that may be used in "both yielded and practically unyielded regions", eliminating "the necessity for tracking yield surfaces in the flow field". Papanastasiou's regularization (1987) for Bingham fluids – referred as Bingham-Papanastasiou equation – has been largely employed (see Zinani and Frey, 2007 and references therein) thanks to its easy computational implementation.

This work employs a Galerkin least-squares multifield finite element methodology, based on the formulation introduced by Behr et al. (1993), to simulate the flow of a Bingham-Papanastasiou fluid through a 1:4 sudden expansion in order to investigate influence of yield-stress and inertia on the morphology of yielded and unyielded material regions. The Galerkin least-squares methodology, introduced by Hughes et al. (1986) modifies the classical Galerkin formulation though the addition of mesh-dependent residuals, resulting from the least-squares of the Euler-Lagrange equations, circumventing the compatibility conditions between pressure and velocity finite element subspaces (the Babuška-Brezzi condition) and between stress and velocity subspaces (present in a formulation in terms of extra-stress velocity and pressure). Also, an appropriate design of the stability parameters of the least-squares terms of the balance equations for the fluid problem (Franca et al., 1992) assures stability even in the approximation of advective-dominated fluid flows.

2. MECHANICAL MODEL

The mechanical model for an isothermal flow requires the solution of mass and linear momentum balance equations only, since the symmetry of Cauchy stress tensor automatically satisfies the balance of angular momentum. Considering a steady-state flow of an incompressible fluid, continuity and motion equations may be stated as:

$$\begin{array}{l} \mathrm{div}\,\mathbf{u}=0 \\ \rho([\nabla\mathbf{u}]\mathbf{u})=\mathrm{div}\,\mathbf{T}+\rho\mathbf{f} \end{array} \tag{1}$$

where **u** represents the fluid velocity, ρ its mass density, **f** the body force per unit volume and **T** the Cauchy stress tensor.

The generalized Newtonian constitutive equation (GNL) relates the internal stresses to kinematic variables, in which the deviatoric stress tensor ($\boldsymbol{\tau}$) may present a non-linear dependence on the rate of strain tensor:

$$\mathbf{T}=-p\mathbf{I}+\boldsymbol{\tau}=-p\mathbf{I}+2\eta(\dot{\gamma})\mathbf{D}(\mathbf{u}) \tag{2}$$

where p is a mean pressure, $p\equiv-1/3\mathrm{tr}(\mathbf{T})$, **I** the unit tensor, $\boldsymbol{\tau}$ the viscous stress tensor and **D(u)** is the rate of strain tensor whose magnitude is given by $\dot{\gamma}=(2\mathrm{tr}\,\mathbf{D}^2)^{(1/2)}$. Besides, $\eta(\dot{\gamma})\equiv\tau/\dot{\gamma}$ is the GNL viscosity function – the shear viscosity of the fluid, which depends on the magnitude of the tensor **D** (Bird el al, 1983).

The constitutive relation considered in this work is a viscoplastic one, characterized by an yield stress, $\tau_{0,}$, a minimum critical shear stress, which must be overcome before the material starts to flow. A particular viscoplastic model is considered, a Bingham one, presenting two rheological parameters only: the yield stress limit τ_0 and a constant viscosity η_p. Considering the shear-stress τ as the Frobenius norm of tensor $\boldsymbol{\tau}$, $\tau=(1/2\,\mathrm{tr}\,\boldsymbol{\tau}^2)^{1/2}$, the following relation characterizes Bingham fluids (Bird et al., 1987):

$$\begin{cases} \tau=\tau_0+\eta_p\dot{\gamma} & \text{if } \tau\geqslant\tau_0 \\ \dot{\gamma}=0, & \text{if } \tau<\tau_0 \end{cases} \tag{3}$$

The Bingham viscosity function may be defined by employing the GNL viscosity concept $\eta(\dot{\gamma})\equiv\tau/\dot{\gamma}$ as

$$\begin{cases} \eta=\eta_p+\dfrac{\tau_0}{\dot{\gamma}}, & \text{if } \quad \tau\geqslant\tau_0 \\ \eta\rightarrow\infty, & \text{if } \quad \tau<\tau_0 \end{cases} \tag{4}$$

The classical Bingham model stated in Eq. (3) provides no information about the stress field whenever $\tau<\tau_0$, and is discontinuous. Papanastasiou (1987) introduced a modified constitutive relation, valid for the whole domain, in which a purely viscous approximation replaces the unyielded regions by “practically unyielded” ones, presenting very high (but finite) viscosity. The modified constitutive relation introduces one more parameter in Bingham model: the non-rheological parameter m. An exponential function assures that, even for very small values of the yield stress τ_0, the shear rate viscosity remains finite. The regularized shear-stress and viscosity functions proposed by Papanastasiou (1987) are given by

$$\tau=\eta_p\dot{\gamma}+\tau_0[1-\exp(-m\dot{\gamma})] \tag{5}$$

$$\eta(\dot{\gamma})=\eta_p+\frac{\tau_0}{\dot{\gamma}}[1-\exp(-m\dot{\gamma})] \tag{6}$$

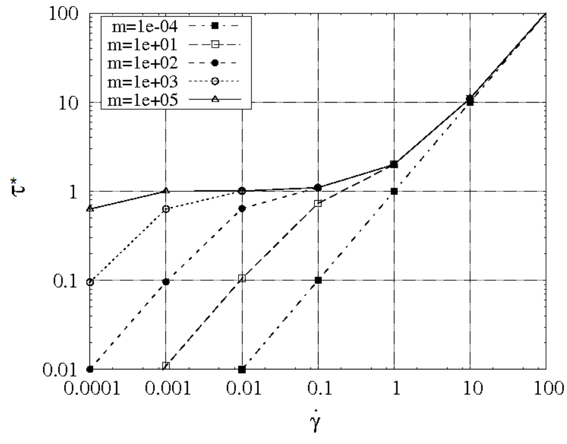

Figure 1. The flow curve for Bingham-Papanastasiou fluids, for m=10^{-4}-10^{5}.

Figure 1 presents flow curves for a Bingham fluid regularized by Papanastasiou equation – defined in Eq. (5) – by plotting the dimensionless shear stress $\tau^*=\tau/\tau_0$ versus the shear rate for several values of the regularizing parameter *m*. As expected, when *m* is very high the Bingham-Papanastasiou model tends to the classical Bingham one, while for very small values of *m*, the Bingham-Papanastasiou model tends to a Newtonian one.

3. FINITE ELEMENT MODELING

In order to introduce the stabilized finite element modeling employed in this work, some preliminary definitions are requested. First the considered domain is a bounded one $\Omega\subset\mathbb{R}^2$ with a polygonal or polyhedral boundary Γ, formed by the union of Γ_g – the portion of Γ where Dirichlet conditions are imposed – and Γ_h – the portion subjected to Neumann boundary conditions. Second, for the discretization, a partition Ω_h of $\overline{\Omega}$ into convex quadrilateral elements is performed in the usual way: no overlapping is allowed between any two elements, the union of all element domains Ω_K reproduces $\overline{\Omega}$ and a combination of triangles and quadrilaterals for the two-dimensional case can be accommodated. Quasi uniformity is not assumed (Ciarlet, 1978).

The finite element approximation for the multifield problem is built with usual finite subspaces for pressure (P^h), velocity ($\mathbf{V}^h$) and stress ($\boldsymbol{\Sigma}^h$) fields,

$$\begin{aligned}
&P^h=\{q\in C^0(\Omega)\cap L_2^0(\Omega)\,|\,q_K\in R_l(K),\ \ K\in\Omega^h\}\\
&\mathbf{V}^h=\{\mathbf{v}\in H_0^1(\Omega)^N\,|\,\mathbf{v}_K\in R_m(K)^N,\ \ K\in\Omega^h\}\\
&\mathbf{V}_g^h=\{\mathbf{v}\in H^1(\Omega)^N\,|\,\mathbf{v}_K\in R_m(K)^N,\ \ K\in\Omega^h,\ \ \mathbf{v}=\mathbf{u}_g \text{ on } \Gamma_g\}\\
&\boldsymbol{\Sigma}^h=\{\mathbf{S}\in C^0(\Omega)^{NxN}\cap L_2(\Omega)^{NxN}\,|\,S_{ij}=S_{ji},\ \ i,j=1,N\,|\,S_K\in R_k(K)^{NxN},\ \ K\in\Omega^h\}
\end{aligned}\tag{7}$$

in which R_l, R_m, and R_k denote, respectively, polynomial spaces of degree *l, m* and *k* (Ciarlet, 1978), $C^0(\Omega)$ represents the space of continuous functions in Ω and, as usual, $L^2(\Omega)$, $L_0{}^2(\Omega)$, and $H^1(\Omega)$, $H_0{}^1(\Omega)$, s stand for Hilbert and Sobolev functional spaces, respectively (Rektorys,1975).

The multifield boundary-value problem, in terms of extra-stress, pressure and velocity is obtained by combining continuity and motion equations, Eq.(1), with Bingham-Papanastasiou model, Eq. (6), and incorporating suitable velocity and extra-stress boundary conditions as follows:

$$\begin{aligned}
&\rho(\nabla\mathbf{u})\mathbf{u}+\nabla p-\operatorname{div}\boldsymbol{\tau}=\rho\mathbf{f} && \text{in } \Omega\\
&\boldsymbol{\tau}-2(\eta_p+\frac{\tau_0}{\dot{\gamma}}[1-\exp(-m\dot{\gamma})])\,\mathbf{D}(\mathbf{u})=0 && \text{in } \Omega\\
&\operatorname{div}\mathbf{u}=0 && \text{in } \Omega\\
&\mathbf{u}=\mathbf{u}_g && \text{on } \Gamma_g\\
&[\boldsymbol{\tau}-p\mathbf{I}]\,\mathbf{n}=\mathbf{t}_h && \text{on } \Gamma_h
\end{aligned}\tag{8}$$

where $\mathbf{t}_h$ is an imposed stress vector and $\mathbf{u}_g$ is an imposed velocity.

A GLS multifield approximation for the boundary-problem defined in Eq. (8) is stated as: find the triple $(\boldsymbol{\tau}^h,p^h,\mathbf{u}^h)\in\boldsymbol{\Sigma}^h\times P^h\times\mathbf{V}_g^h$ such that:

$$\begin{aligned}
&\int_\Omega(2(\eta_p+\frac{\tau_y}{\dot{\gamma}}[1-\exp(-m\dot{\gamma})]))^{-1}\boldsymbol{\tau}^h\cdot\mathbf{S}^h\,d\Omega-\int_\Omega\mathbf{D}(\mathbf{u})^h\cdot\mathbf{S}^h\,d\Omega+\varepsilon\int_\Omega p^hq^h\,d\Omega\\
&+\int_\Omega\rho(\nabla\mathbf{u}^h)\mathbf{u}^h\cdot\mathbf{v}^h\,d\Omega+\int_\Omega\boldsymbol{\tau}^h\cdot\mathbf{D}(\mathbf{v}^h)\,d\Omega-\int_\Omega p^h\operatorname{div}\mathbf{v}^h\,d\Omega+\int_\Omega\operatorname{div}\mathbf{u}^hq^h\,d\Omega\\
&+\delta\int_\Omega\operatorname{div}\mathbf{u}^h\operatorname{div}\mathbf{v}^h\,d\Omega+\sum_{K\in\Omega^h}\int_{\Omega_K}(\rho(\nabla\mathbf{u}^h)\mathbf{u}^h+\nabla p^h-\operatorname{div}\boldsymbol{\tau}^h).\alpha(\mathrm{Re}_K)(\rho(\nabla\mathbf{v}^h)\mathbf{u}^h+\nabla q^h-\operatorname{div}\mathbf{S}^h)\,d\Omega\\
&+2\eta(\dot{\gamma})\beta\int_\Omega((2(\eta_p+\frac{\tau_y}{\dot{\gamma}}[1-\exp(-m\dot{\gamma})]))^{-1}\boldsymbol{\tau}^h-\mathbf{D}(\mathbf{u})^h).((2(\eta_p+\frac{\tau_y}{\dot{\gamma}}[1-\exp(-m\dot{\gamma})]))^{-1}\mathbf{S}^h-\mathbf{D}(\mathbf{v})^h)\,d\Omega\\
&=\int_\Omega\mathbf{f}\cdot\mathbf{v}^h\,d\Omega+\int_{\Gamma_h}\mathbf{t}_h\cdot\mathbf{v}^h\,d\Gamma+\sum_{K\in\Omega^h}\int_{\Omega_K}\mathbf{f}\cdot(\alpha(\mathrm{Re}_K)(\rho(\nabla\mathbf{v}^h)\mathbf{u}^h+\nabla q^h-\operatorname{div}\mathbf{S}^h))\,d\Omega
\end{aligned}\tag{9}$$

where $\boldsymbol{\tau}^h$, $\mathbf{u}^h$ and p^h denote the admissible extra-stress, velocity and pressure fields and $\mathbf{S}^h$, $\mathbf{v}^h$ and q^h the virtual extra-stress, velocity and pressure fields, respectively. Also, $\varepsilon<<1$, the stability parameter of the material equation β is set as 0.5, as suggested by Behr et al. (1993), and the stability parameters of continuity, δ, and motion, $\alpha(\mathrm{Re}_K)$, equations are defined as in Franca and Frey (1992):

$$
\begin{aligned}
&\delta=\chi|\mathbf{u}|_p h_K \xi(\mathrm{Re}_K)\\
&\alpha(\mathrm{Re}_K)=\frac{h_K}{2|\mathbf{u}|_p}\xi(\mathrm{Re}_K) \quad \text{with} \quad \xi(\mathrm{Re}_K)=\begin{cases}\mathrm{Re}_K, & 0<\mathrm{Re}_K<1\\ 1, & \mathrm{Re}_K>1\end{cases}\\
&\mathrm{Re}_K=\frac{m_k|\mathbf{u}|_p h_K}{4\eta(\dot{\gamma})} \quad \text{with} \quad m_k=\min\{1/3,\ 2C_k\} \quad \text{and} \quad C_k\sum_{K\in\Omega^h} h_K^2\|\mathrm{div}\,\mathbf{D}(\mathbf{u})^h\|_{0,K}^2\geq\|\mathbf{D}(\mathbf{u})^h\|_0^2 \quad \forall\,\mathbf{u}^h\in\mathbf{V}^h
\end{aligned}
\tag{10}
$$

in which Re_K is the grid Reynolds number, χ is a scalar positive constant, h_K is the K-element size, and $|\mathbf{u}|_p$ is the p-norm of $\mathbb{R}^n$. Also, the parameter m_k, included in the Reynolds mesh number is derived from error analysis of the GLS formulation (Franca and Frey, 1992), allowing to characterize advective-dominated regions of the flow by $\mathrm{Re}_K>1$ and the diffusive-dominated ones by $\mathrm{Re}_K<1$.

It is interesting to note that making the stability parameters α, β and δ equal to zero on the GLS formulation defined by Eq. (9), the classical Galerkin multifield approximation for Eq. (8), is recovered.

The discrete residual equation is obtained by substituting trial and test functions for extra-stress, velocity and pressure fields in the GLS formulation defined by Eq. (9), being given by:

$$
\begin{aligned}
\mathbf{R}(\mathbf{U})=&[(1+\beta)\mathbf{E}(\eta(\dot{\gamma}))+(1-\beta)\mathbf{H}+\mathbf{E}_\alpha(\eta(\dot{\gamma}),\mathbf{u})]\boldsymbol{\tau}\\
&+[\mathbf{N}(\mathbf{u})+\mathbf{N}_\alpha(\eta(\dot{\gamma}),\mathbf{u})+\beta\mathbf{K}-(1+\beta)\mathbf{H}^T-\mathbf{G}^T+\mathbf{M}]\mathbf{u}\\
&+[\mathbf{G}+\mathbf{G}_\alpha(\eta(\dot{\gamma}),\mathbf{u})+\boldsymbol{\epsilon}]\mathbf{p}-\mathbf{F}-\mathbf{F}_\alpha(\eta(\dot{\gamma}),\mathbf{u})=0
\end{aligned}
\tag{11}
$$

The above residual equation, $\mathbf{R}(\mathbf{U})=0$, is solved by a quasi-Newton incremental method which employs a frozen gradient strategy and a continuation procedure on the geometrical and material non-linearity terms, explained in details by Zinani and Frey (2008). The solution procedure may be summarized as:

1) Calculate analytically the Jacobian matrix for a generic iteration k, given by: $\mathbf{J}(\mathbf{U}_k)=\left.\frac{\partial\mathbf{R}(\mathbf{U})}{\partial\mathbf{U}}\right|_{\mathbf{U}_k}$;

2) Estimate number of iterations m to update $\mathbf{J}$ and the initial guess for the three-field problem, namely: $\mathbf{U}_0^h=[\tau_{12}^h,\tau_{11}^h,\tau_{22}^h,u_1^h,u_2^h,p]_0^T$;

3) Make k=0, j=0 and set the tolerance as $\epsilon=10^{-7}$;

4) If $k-int(k/m)\times k=0$, then j=k.

5) Solve the linear system: $\mathbf{J}(\mathbf{U}_k^h)\mathbf{A}_{k+1}^h=-\mathbf{R}(\mathbf{U}_k^h)$; to calculate the incremental vector: $\mathbf{A}_{k+1}^h=\mathbf{U}_{k+1}^h-\mathbf{U}_k^h$;

6) Calculate $\mathbf{U}_{k+1}^h$ and verify the convergence criterion: If $\left|\mathbf{R}(\mathbf{U}_k^h)\right|_\infty<\epsilon$ then store the solution and exit the algorithm, else do k=k+1 and go to step (4).

It is worth noting that the continuation procedure employed to approximate the residual equation (11) is used in both material and geometrical non-linearity terms, in such a way very high values of Bingham and Reynolds numbers may be considered.

4. NUMERICAL RESULTS

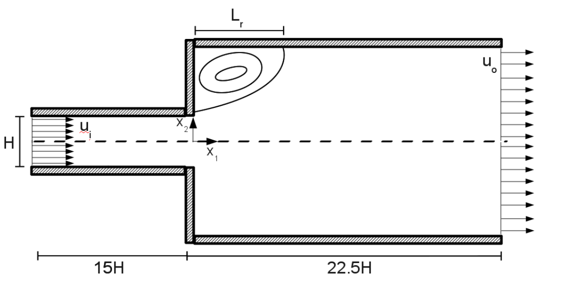

Fig. 2. Flow through a 1:4 sudden expansion: problem statement.

In this section, the multifield GLS formulation previously defined is used to approximate Bingham-Papanastasiou fluids flowing through a sudden planar expansion. Taking advantage of the channel symmetry, only half the expansion channel with aspect ratio fixed as 1:4, as depicted in Fig. 2, is simulated. In order to avoid entrance and exit effects, the length of both channels is considered sufficiently large – namely the smaller channel with a length of 15 H, being H=1m its height and the larger channel with 22.5 H length. In all presented computations, a mesh containing 19,800

Lagrangian bi-linear finite elements and 121,686 degrees-of-freedom has been employed, with the mesh being refined at the expansion neighborhood.

On the channel walls, no-slip and impermeability boundary conditions are imposed while at the centerline, symmetry conditions are assumed. Besides, flat velocity profiles are considered both at the channel inlet (with u_i=1m/s) and at the outlet (u_o obtained by considering the flow mass conservation); the fluid density is set as ρ=1.0kg/m^3, the plastic viscosity as η_p=1.0Pa.s and Papanastasiou regularization parameter as m=1000s (Mitsoulis and Huilgol, 2004).

Bingham and Reynolds numbers are defined by

$$\mathrm{Bn} = \frac{\tau_0 H}{\eta_p u_i} \qquad \mathrm{Re} = \frac{\rho u_i H}{\eta_p} \tag{12}$$

The effect of the the yield stress limit on morphology of yielded surfaces for creeping flows is depicted in Fig. 3. The unyielded zones (black regions, characterized by a dimensionless shear stress $\tau^*=\tau/\tau_0<1$) may be either unmoving (or dead) zones at the corner of the expansion plane or moving unyielded ones – the plug flows at the channel centerline, while the yielded regions (white ones) are characterized by $\tau^*=\tau/\tau_0>1$. As Bingham number increases, both the unmoving and moving unyielded zones monotonically increase, too. The monotonic growth of unyielded regions with the increasing of Bingham number seems to be broken when Bingham reaches the value 20, after which both the plug-flow and dead zone in the larger channel are almost insensitive to the Bingham increase.

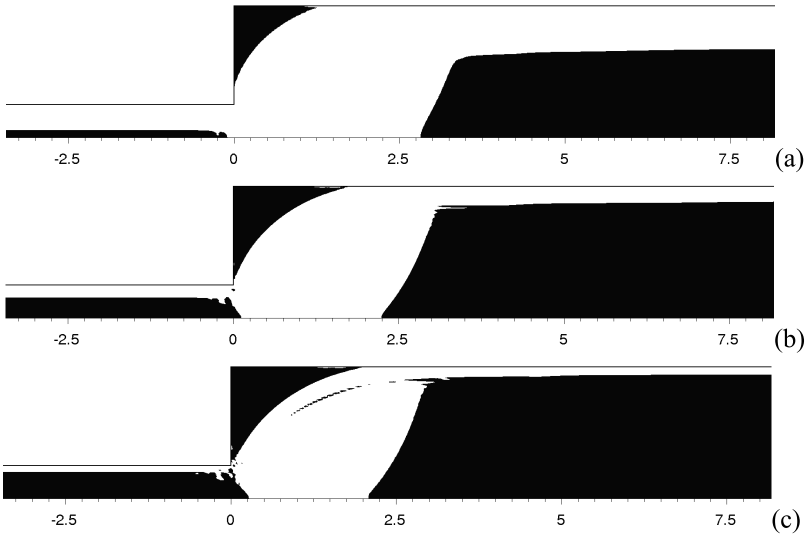

Figure 3. τ*-isobands, for Re=0: (a) Bn=2; (b) Bn=20; (c) Bn=100.

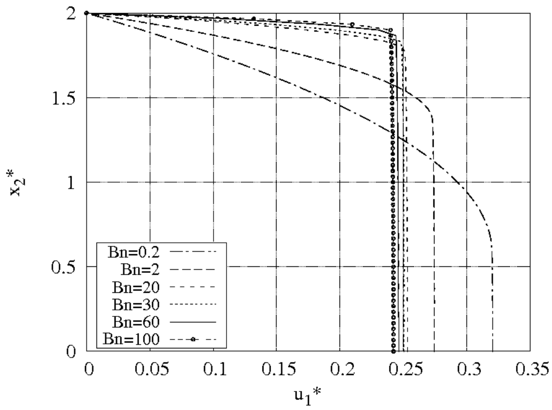

Figure 4. Transverse profiles of dimensionless axial velocity, for Re=0 and Bn=0.2-100 at x_1^*=+10.

Figure 4 presents transverse axial velocity profiles (in which the velocity along the flow direction, u_1, is scaled by the inlet velocity u_i, so that $u_1^*=u_1/u_i$) in a fully-developed viscoplastic region in the larger channel – namely at $x_1^*=x_1/H$=+10. The profiles consider creeping flow, $x_2^*=x_2/H$ and Bingham number ranging from 0.2 to 100. It may be noted that the more viscoplastic is the flow, the flatter are the profiles – from the quasi-parabolic profile, for Bn=0.2 up to the almost completely flat profile for Bn=100.

Figure 5 investigates the influence of viscoplasticity on the pressure drop along the channel, by considering inertialess flow and varying Bingham number from Bn=0 to Bn=100. The dimensionless pressure drop is given by $p^*=2(p-p_o)/\rho u_i^2$, with p_o standing for a reference pressure at the channel outlet, while the dimensionless length is $x_1^*=x_1/H$. In all cases, the greater the Bingham number is, the more the pressure drops – with all flows presenting a steeper slope in the smaller channel. Since Papanastasiou (1987) regularization substituted the material unyielded regions prescribed by the Bingham model for yielded regions submitted to very high (but finite) viscosity, viscoplastic flows are much more viscous than the quasi-Newtonian case (for Bn=0.2) as Bingham number increases.

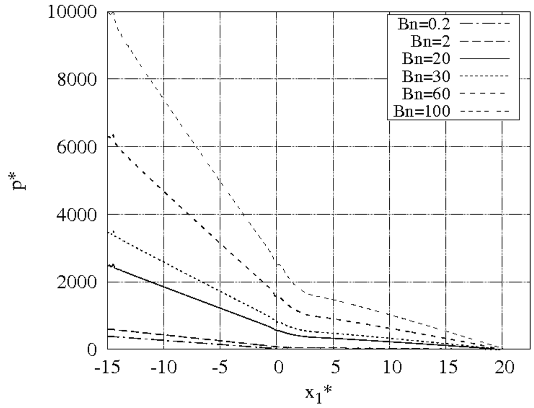

Figure 5. Dimensionless longitudinal profile for pressure drop along the channel considering for Re=0 and Bn=0.2-100.

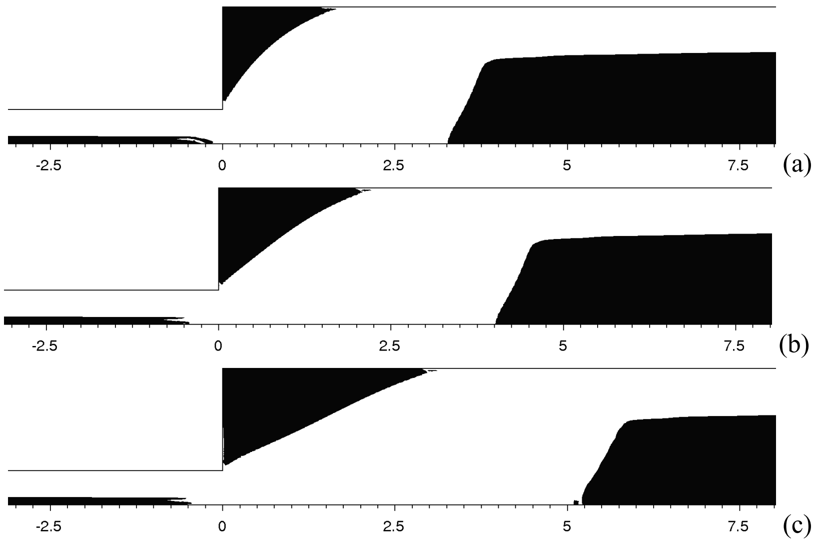

Fig. 6. τ^*-isobands for Bn=2: (a) Re=15; (b) Re=30; (c) Re=50.

Figure 6 analyzes the influence of inertia effects on the morphology of yielded and unyielded material regions, considering Bn=2 and ranging Reynolds number up to 50. Two major effects may be noticed: the former is the displacement from the expansion plane of plug-flows in the larger channel, with the a distance of downstream plug-flows from the expansion plane of $x_1^*\simeq 2.8$, for Re=0 (depicted in Fig. 3a) and of $x_1^*\simeq 5$, for Re=50. The latter is the stretching of dead zones at the expansion corner, along the flow streamlines, so that as Re increases the end of dead zones moves away from the expansion plane – for Re=0 its location is at $x_1^*\simeq 1.25$ while for Re=50 it is at $x_1^*\simeq 3$.

The inertia influence on the vortex length at the expansion corner is shown in Fig. 7, which presents the dimensionless reattachment length $Lr^*=Lr/H$ – as defined in Fig. 2 plotted for a fixed value of Reynolds equal to 50 while Bingham number varies from 0 to 5. It may be noted that the reattachment length decreases with the increasing of the Bingham number, due to the increasing of yield stress effect. Considering a comparison for the Newtonian case in Fig. 7 (the point Bn=0) with the results of Dagtekin and Ünsal (2010) it is observed that both articles present a good agreement: the latter work yields a value Lr=8.785 and the current work yields Lr=8.6.

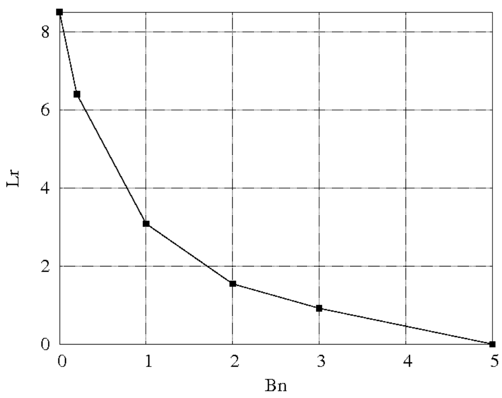

Figure 7. Reattachment Lengths versus Bingham numbers, for Re=50.

5. FINAL REMARKS

In this article, inertial flows of Bingham-Papanastasiou fluids through a 1:4 sudden expansion were approximated by a multifield GLS method in terms of extra-stress, velocity and pressure; which circumvents the need to satisfy pressure-velocity and stress-velocity compatibility conditions, remaining stable for high Bingham and Reynolds fluid flows. First the yield stress influence on the morphology of material yield surfaces was analyzed for creeping flows, by ranging Bingham number from 0.2 to 100. Bingham number increase provoked a monotonic increase of moving and unmoving unyielded material regions and of the pressure drop on the flow. Then inertia influence was considered by varying Reynolds number: its increase drives plug-flow zones away from the expansion plane, stretches dead zones at expansion corner along the main flow streamlines and severely restrains the reattachment length in the larger channel.

6. ACKNOWLEDGEMENTS

The author C. Fonseca thanks CAPES for his graduate scholarship, and authors S. Frey acknowledges CNPq and author M. L. Martins-Costa acknowledges CNPq and FAPERJ, for financial support and grants.

7. REFERENCES

Behr, M.A., Franca, L.P., Tezduyar, T.E., 1993, “Stabilized Finite Element Methods for the Velocity-Pressure-Stress Formulation of Incompressible Flows”, Computer Methods Appl. Mech. Engng., vol.104, pp.31-48.

Bird, R.B., Armstrong, R.C., Hassager, O., 1987, “Dynamics of Polymeric Liquids”, vol. 1: Fluid Mechanics”, 2nd ed., John Wiley & Sons, U.S.A.

Ciarlet, P.G., 1978, “The Finite Element Method for Elliptic Problems”. North Holland, Amsterdam.

Dagtekin, I, Ünnsal, M. 2010, “Numerical analysis of axisymmetric and planar sudden expansion flows for laminar regime”, Int. J. Numer. Meth. Fluids, DOI: 10.1002/fld.2239.

Franca, L.P., Frey, S., Hughes, T.J.R., 1992, “Stabilized finite element methods: I. Application to the advective-diffusive model”, Comput. Methods Appl. Mech. Engrg., vol. 95, pp. 253-276.

Franca, L.P., Frey, S., 1992, “ Stabilized finite element methods: II. The incompressible Navier-Stokes equations”, Comput. Methods Appl. Mech. Engrg., vol. 99, pp. 209-233.

Hughes, T.J.R., França, L., Balestra, M., 1986, “A New Finite Element Formulation for Computational Fluid Dynamics: V. Circumventing the Babuska-Brezzi Condition: A Stable Petrov-Galerkin Formulation of the Stokes Problem Accommodating Equal-Order Interpolations”, Computer Methods Appl. Mech. Engrg., vol. 59, pp. 85-99.

Mitsoulis, E., Huilgol, R.R, 2004, “Entry flows of Bingham plastics in expansions”, J. Non-Newtonian Fluid Mech., vol. 122, pp. 45-54.

Papanastasiou, T.C., 1987, “Flows of Materials with Yield”, J. Rheology, vol. 31, pp.385-404.

Rektorys, K., 1975, “Variational Methods in Mathematics, Science and Engineering”, Reidel, U.S.A.

Zinani, F., Frey, S., 2008, “Galerkin Least-Squares Multi-Field Approximations for Flows of Inelastic Non-Newtonian Fluids”, J. Fluids Engineering, vol. 130, pp. 081507/1-14.

Zinani, F., Frey, S., 2007, “Galerkin Least-Squares Solutions for Purely Viscous Flows of Shear-Thinning Fluids and Regularized Yield Stress Fluids ”, J. of the Braz. Soc. of Mech. Sci. & Eng., vol. 29/4, pp. 432-443.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.