ON THE EXERGY DISAGGREGATION FOR THERMOECONOMIC ANALYSIS OF A GAS TURBINE COGENERATION SYSTEM

José Joaquim Conceição Soares Santos, jjcssantos@yahoo.com.br
Federal University of Espírito Santo – UFES/DEM/CTIII/LFTC
Av Fernando Ferrari 514, CEP 29.075-910, Goiabeiras, Vitória-ES
Brazil

Julio Augusto Mendes da Silva, jams@usp.br
University of São Paulo, São Paulo-SP, Brazil

José Carlos Escobar Palacio, jocescobar@gmail.com
Federal University of Itajubá, Itajubá-MG, Brazil

Hugo Lima Moreira, hugolimabr@yahoo.com.br
Federal Rural University of Amazônia, Belém-PA, Brazil

Abstract. *Most analysts agree that exergy, instead of enthalpy only, is the most adequate thermodynamic quantity to associate with cost. Therefore, all thermoeconomic methodologies use the exergy to define productive structure. Sometimes, under a thermoeconomic analysis point of view, it is necessary to consider a mass or an energy flow rate consisting of several exergy components, for example thermal, mechanical and chemical components. Most analysts agree that by considering separate exergy forms the accuracy of the results is improved in thermoeconomics. However, these analysts recognize that the disaggregation of physical exergy might not be always reasonably because of the increase in the computational efforts. Thus, the main questions are: Is this improvement often marginal? Is exergy disaggregation necessary for extracting the main conclusions? This paper aims at answering these questions for the case of external fuel (natural gas) exergy allocation to the two final products (heat and power) of a gas turbine cogeneration system, by applying four different productive structures based on different kind and level of physical exergy disaggregation. The paper also showns that the model that uses enthalpy together with sintropy presents the closest result in relation to the total disaggregated model, showing the racionality of this methodology that intrisically allocate the residues to the components of the cycle using the entropy variation in each component as a weighting factor.*

Keywords*: Accuracy Improvements, Disaggregation of Exergy, Exergetic Cost, Exergy Components.*

1. INTRODUCTION

The thermoeconomics practitioners agree that exergy is the most adequate thermodynamic property quantity to associate with cost. Therefore, all thermoeconomic methodologies use the exergy to define the productive structure.

Sometimes, under a thermoeconomic analysis point of view, it is necessary to consider a mass or an energy flow rate consisting of several components, for example thermal, mechanical and chemical exergy (Torres *et al.*, 1996).

However, by considering separate exergy forms the accuracy of the results is improved, the increase in the computational efforts is significant, besides the difficulties that might be involved in the separate calculation of mechanical and thermal components. Therefore, the decision to be made in this step should be based on the purpose of the study and on the system being evaluated (Lazzaretto and Tsatsaronis, 2006).

At this point, two questions arise: Is this improvement really often marginal? Is the exergy disaggregation necessary for extracting the main conclusions in thermoeconomic analysis?

In order to answer these questions, this paper compares four different productive structures based on three different levels of physical exergy disaggregation. These productive structures are applied for natural gas exergy allocation to the final products (heat and power) in a gas turbine cogeneration system. The productive structure at disaggregation level I is based on total exergy. In the productive structure at disaggregation level IIa, de physical exergy is split into two components: thermal and mechanical exergy. At disaggregation level IIb the physical exergy is also disaggregated into two components: the enthalpic term ($m.\Delta h$) and the here called syntropic term ($m.T_0.\Delta s$). In the productive structure at disaggregation level III the physical exergy is disaggregated into three components, i. e., besides the enthalpic term, the syntropic term is split into other two terms, here called thermal-syntropic and mechanical-syntropic terms.

2. PHYSICAL MODEL

Figure 1 represents the physical structure of the gas turbine cogeneration system. The cogeneration system is defined as having four units or subsystems: the air compressor (AC), the combustion chamber (CC), the gas turbine (GT) and the recovery boiler (RB). The streams are air, gases, mechanical power and natural gas. The thermodynamic modelling of the physical structure in Fig. 1 considers complete combustion with excess of air.

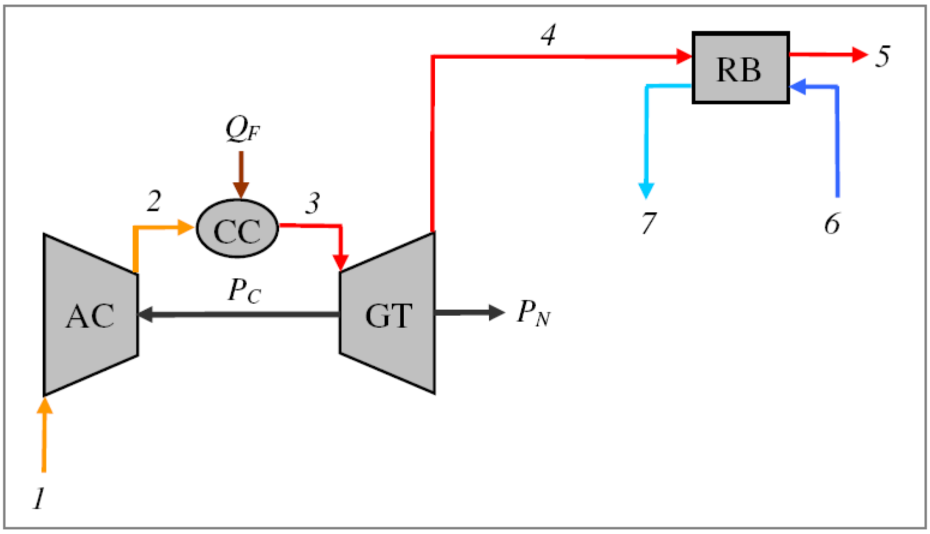

Figure 1. Physical Structure of the Gas Turbine Cogeneration System

The parameters of the main streams of the physical structure of the cogeneration system are presented in Tab. 1.

Table 1. Main Parameters of the Main Physical Flows of the Gas Turbine Cogeneration System

PHYSICAL FLOW		m [kg/s]	p [kPa]	T [°C]
i	Description			
1	Air	14.72	101.32	25.00
2	Air	14.72	510.40	230.20
3	Gases	14.94	484.80	850.00
4	Gases	14.94	102.07	537.30
5	Gases	14.94	101.32	151.10
6	Water	2.49	2,040.00	60.00
7	Steam	2.49	2,000.00	212.42

The mechanical net power (P_N) is 2,433.47 kW and the compressor power (P_C) is 3,113.03 kW. The fuel is natural gas (Q_F), whose consumption in exergetic base is 11,630.96 kW. The recovery boiler produces 2,246.32 kW of heat (exergy). The air and the gases are considered as mixtures of ideal gases. Their molar compositions are in Tab. 2.

Table 2. Molar Composition of Air and Gases Streams present in the Physical Structure of the Cogeneration System

ELEMENT			PERCENTAGE [%]	
n	Description	Symbol	Air	Gases
1	Oxygen	O_2	20.56	14.72
2	Carbon Dioxide	CO_2	0.03	2.67
3	Water Vapor	H_2O	1.88	7.12
4	Nitrogen	N_2	76.61	74.59
5	Argon	Ar	0.92	0.90

The specific heat of the air and the gases varies with their temperature, as it is described by the polynomial equation and the respective coefficients in the Tab. 3.

Table 3. Coefficients for the Specific Heat Polynomial Equation of some Ideal Gases (Lozano and Valero, 1986)

ELEMENTS			$Cp = A + B \cdot T + C \cdot T^2 + D \cdot T^3$ [kcal/kmol.K]			
n	Description	Symbol	A	$B \cdot 10^2$	$C \cdot 10^5$	$D \cdot 10^9$
1	Oxygen	O_2	6.085	0.3631	-0.1709	0.3133
2	Carbon Dioxide	CO_2	5.316	1.4285	-0.8362	1.784
3	Water Vapor	H_2O	7.7	0.04594	0.2521	-0.8587
4	Nitrogen	N_2	6.903	-0.03753	0.193	-0.6861
5**	Argon**	Ar**	4.964**	0.00	0.00	0.00

OBS: ** (Verda *et al.*, 2004)

In order to carry out the thermoeconomic modeling aimed by this paper, the exergy and its component need to be calculated. For the physical flows representing water and steam (i = 6 and 7), the exergy (E_i) is calculated using Eq. (1).

$$E_i = m_i \cdot \left[(h_i - h_0) - T_0 \cdot (s_i - s_0)\right] \tag{1}$$

The total exergy of the streams representing air and gases (i = 1, 2, 3, 4 and 5) is calculated using Eq. (2), which has four components.

$$E_i = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot \left[\sum x_{n,i} \cdot R \cdot T_0 \cdot \ln\left(\frac{x_{n,i}}{x_{n,o}}\right) + \sum x_{n,i} \int_{T_0}^{T_i} Cp_n \cdot dT - T_0 \cdot \sum x_{n,i} \int_{T_0}^{T_i} \frac{Cp_n}{T} \cdot dT + R \cdot T_0 \cdot \ln\left(\frac{p_i}{p_0}\right)\right] \tag{2}$$

The first term is the chemical exergy component. Thus, the remaining three terms of Eq. (2) define de physical exergy, as shown in Eq. (3).

$$E_i^{PH} = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot \left[\sum x_{n,i} \int_{T_0}^{T_i} Cp_n \cdot dT - T_0 \cdot \sum x_{n,i} \int_{T_0}^{T_i} \frac{Cp_n}{T} \cdot dT + R \cdot T_0 \cdot \ln\left(\frac{p_i}{p_0}\right)\right] \tag{3}$$

In order to calculate the exergy of the physical flows, the reference temperature and pressure is fixed at 25^{o}C and 101.32 kPa, respectively.

3. THERMOECONOMIC MODELING

The thermoeconomic model is a set of equations which describes the cost formation process of the system. But, the physical model is not enough to identify the cost formation process. To carry out a thermoeconomic analysis, it is convenient to make up a thermoeconomic model, which define the productive propose of the subsystems (products and fuels), as well as the distribution of the external resources and internal product throughout the system. It could be represented by means of the productive or functional diagram. The only limitation with must be imposed it that it should be possible to evaluate all the flows of the productive structure in relation to the state of the plant as defined by the physical structure (Lozano and Valero, 1993). The way in which we define the productive structure is a key point in thermoeconomic analysis. In other words, the deeper the conceptual disaggregation of the system in components and flows, the better the results (Lozano and Valero, 1993).

3.1. Disaggregation Level I

Figure 2 shows the productive structure of the plant at disaggregation level I, which represents the cost formation process of the system. The external exergy resource consumed by the system is natural gas (Q_F). The functional products are: mechanical net power (P_N) and heat exergy (Q_U). The rectangles are the real units (or subsystems) that represent the actual equipment of the system. The rhombus and the circles are fictitious units called junction (J_E) and bifurcations (B_E and B_P), respectively. Each productive units of Fig. 2 has inlet and outlet arrows, that represent its fuel (or resource) and products, respectively. All of the flows of the productive structure represent total exergy, which can be mechanical power, useful heat exergy or the exergy of air and gases. Each productive flow is defined based on physical flows. The productive flows representing air and gases ($E_{j:k}$) are always exergy variations between two physical flows (E_j and E_k), as shown in Eq. (4).

$$E_{j:k} = E_j - E_k \tag{4}$$

The total exergy of the physical flows (E_j and E_k) are calculated using Eq. (1) for water and steam streams, or Eq. (2) for air and gases streams. The mathematical model for the natural gas exergy allocation is obtained by formulating cost equation balance in each subsystem (or productive unit) of the productive structure, as shown in Eq. (5), where c is the exergetic unit cost of the productive flows (unknown variable) and Y represents the generic productive flow, which can be mechanical power, useful heat exergy or the exergy of air and gases.

$$\sum (c \cdot Y) = 0 \tag{5}$$

In order to formulate the cost equation balance in each productive unit or subsystem, the inlet flows (fuels) assume negative value and the outlet flows (products) assume positive value. Since the number of flows is always greater than the number of productive units, some auxiliary equations attribute the same exergetic unit cost to all of the productive flows leaving the same bifurcation.

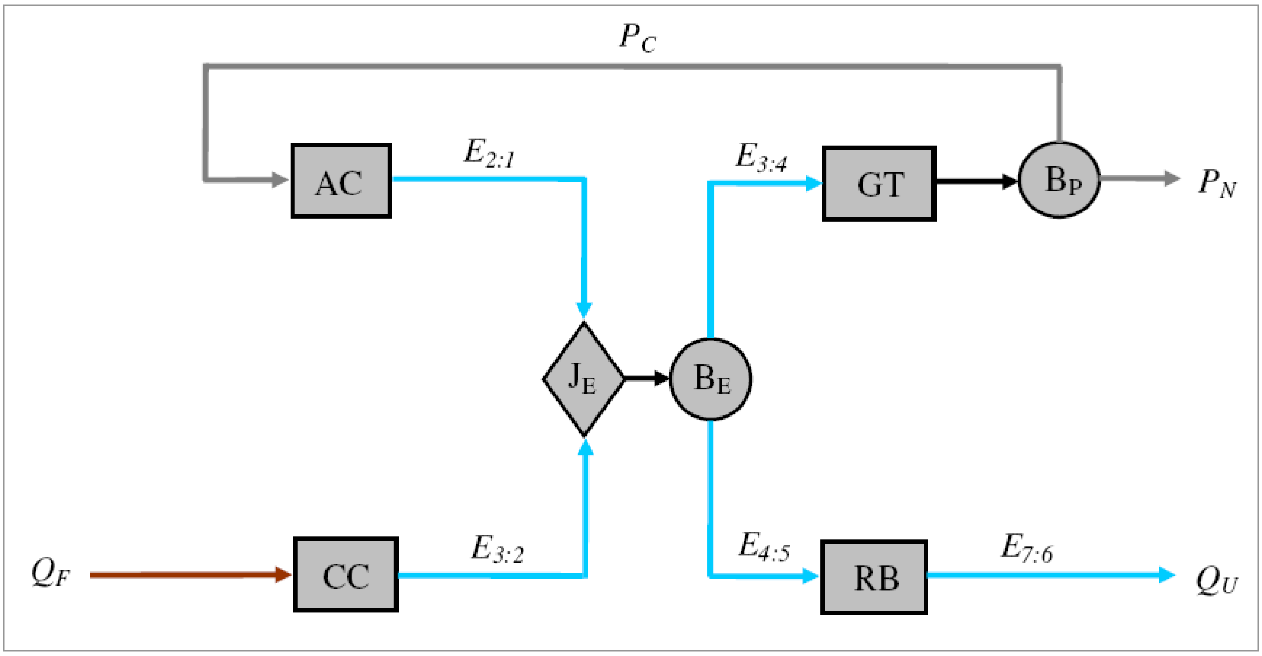

Figure 2. Productive Structure of the Gas Turbine Cogeneration System at Disaggregation Level I

The solution of the set of cost equations allows the attainment of the exergetic unit cost of each internal flow and final product.

3.2. Disaggregation Level IIa

Let us consider a more disaggregated model based on decomposing the flows of the productive structure, shown in Fig. 2, in their thermal and mechanical components. The result is shown in Fig. 3. This productive structure is equivalent to that of the CGAM plant introduced by Frangopoulos (1994) and used by Torres *et al*. (1996).

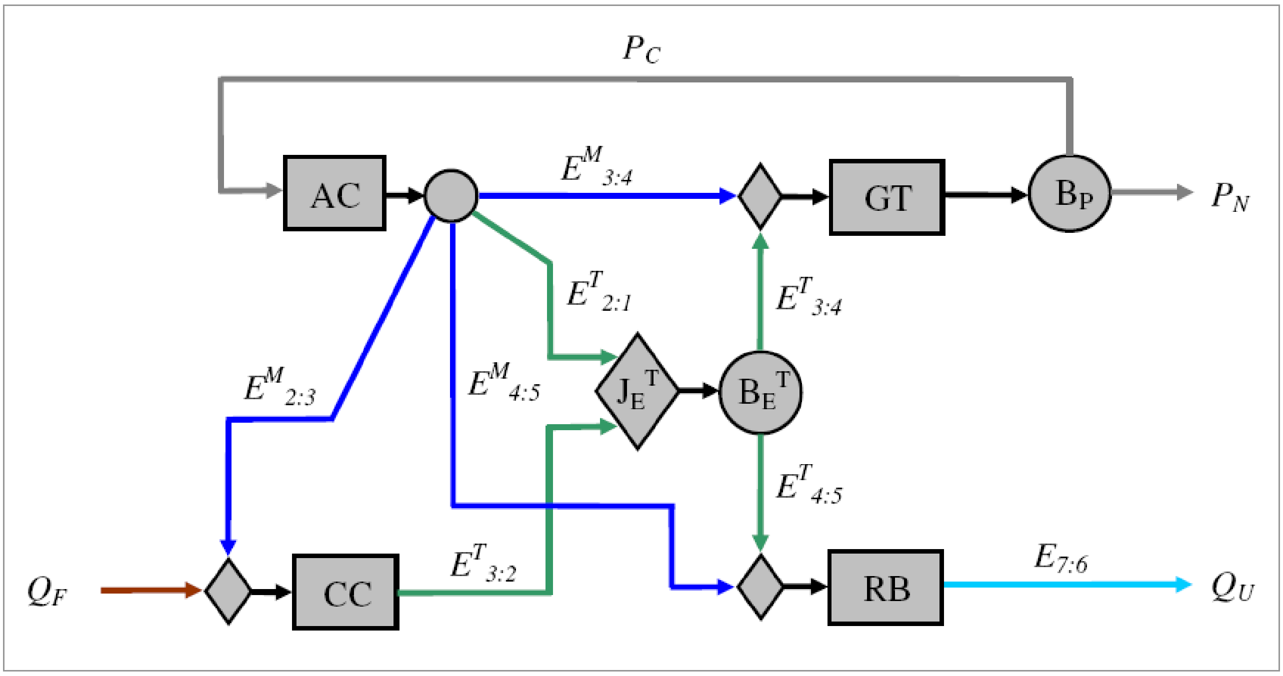

Figure 3. Productive Structure of the Gas Turbine Cogeneration System at Disaggregation Level IIa

The meaning of the internal flows representing the thermal and mechanical components of exergy is explained in Eqs. (6)-(9).

$$E_{j:k}^{T} = E_{j}^{T} - E_{k}^{T} \tag{6}$$

$$E_{j:k}^{M} = E_{j}^{M} - E_{k}^{M} \tag{7}$$

$$E_{i}^{T} = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot \left[\sum x_{n,i} \int_{T_0}^{T_i} Cp_n \cdot dT - T_0 \cdot \sum x_{n,i} \int_{T_0}^{T_i} \frac{Cp_n}{T} \cdot dT \right] \tag{8}$$

$$E_{i}^{M} = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot R \cdot T_0 \cdot \ln\left(\frac{p_i}{p_0}\right) \tag{9}$$

The procedure to obtain the mathematical model for the natural gas exergy allocation to the final product is the same explained in subsection 3.1.

3.3. Disaggregation Level IIb

Another kind of exergy desegregation can be performed, also considering two components. The first component is the enthalpy while the second one is here called syntropy, as shown in Eq. (10) and Eq. (11), respectively.

$$H_i = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot \sum x_{n,i} \int_{T_0}^{T_i} Cp_n \cdot dT \tag{10}$$

$$S_i = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot \left[T_0 \cdot \sum x_{n,i} \int_{T_0}^{T_i} \frac{Cp_n}{T} \cdot dT - R \cdot T_0 \cdot \ln\left(\frac{p_i}{p_0}\right) \right] \tag{11}$$

The result is the productive structure shown in Fig. 4. This kind of physical exergy disaggregation was introduced by Santos *et al.* (2008) and Santos *et al.* (2009). This thermoeconomic approach is called H&S Model.

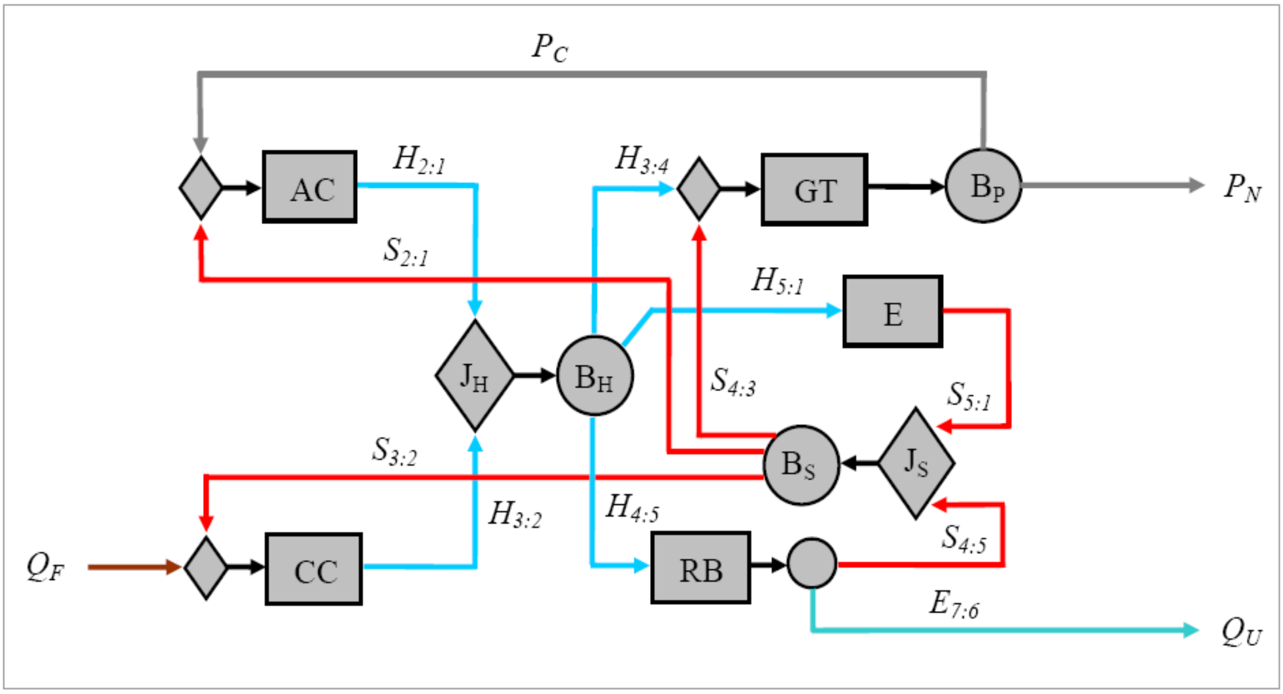

Figure 4. Productive Structure of the Gas Turbine Cogeneration System at Disaggregation Level IIb

This kind of productive structure is similar to the one that uses the negentropy, because syntropy and negentropy are the same magnitude ($m.T_0.\Delta s$) with essentially the same meaning. However, the negentropy is used as a fictitious flow (together with exergy) and the syntropy is a physical exergy component, which must be used together with enthalpy. The meaning of the internal flows of the productive structure at disaggregation level IIb (Fig. 4), representing the enthalpy and the syntropy flows, are explained in Eq. (12) and Eq. (13), respectively.

$$H_{j:k} = H_j - H_k \tag{12}$$

$$S_{j:k} = S_j - S_k \tag{13}$$

In order to obtain the mathematical model, cost equations are formulating in each subsystem, as shown in Eq. (5) and explained in the subsection 3.1. The auxiliary equations are needed to attribute the same exergetic unit cost to all of the productive flows leaving the same bifurcation or subsystem.

3.4. Disaggregation Level III

Equation (11) shows two different components for the syntropy. Thus, the syntropy flows of the productive structure, shown in Fig. 4, can be disaggregated in their thermal and mechanical components, as shown in Eq. (14) and Eq. (15), respectively. Equation (15) shows that the mechanical syntropy is the same term shown in Eq. (9) for the mechanical exergy. The meaning of the internal flows representing the thermal and mechanical syntropy is explained in Eq. (16) and Eq. (17), respectively.

$$S_i^T = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot T_0 \cdot \sum x_{n,i} \int_{T_0}^{T_i} \frac{Cp_n}{T} \cdot dT \tag{14}$$

$$S_i^M = E_i^M = \frac{m_i}{\sum x_{n,i} \cdot M_n} \cdot R \cdot T_0 \cdot \ln\left(\frac{p_i}{p_0}\right) \tag{15}$$

$$S_{j:k}^T = S_j^T - S_k^T \tag{16}$$

$$S_{j:k}^M = S_j^M - S_k^M \tag{17}$$

The result is a more disaggregated productive structure shown in Fig. 5. This is the first work using the physical exergy disaggregated in their three components

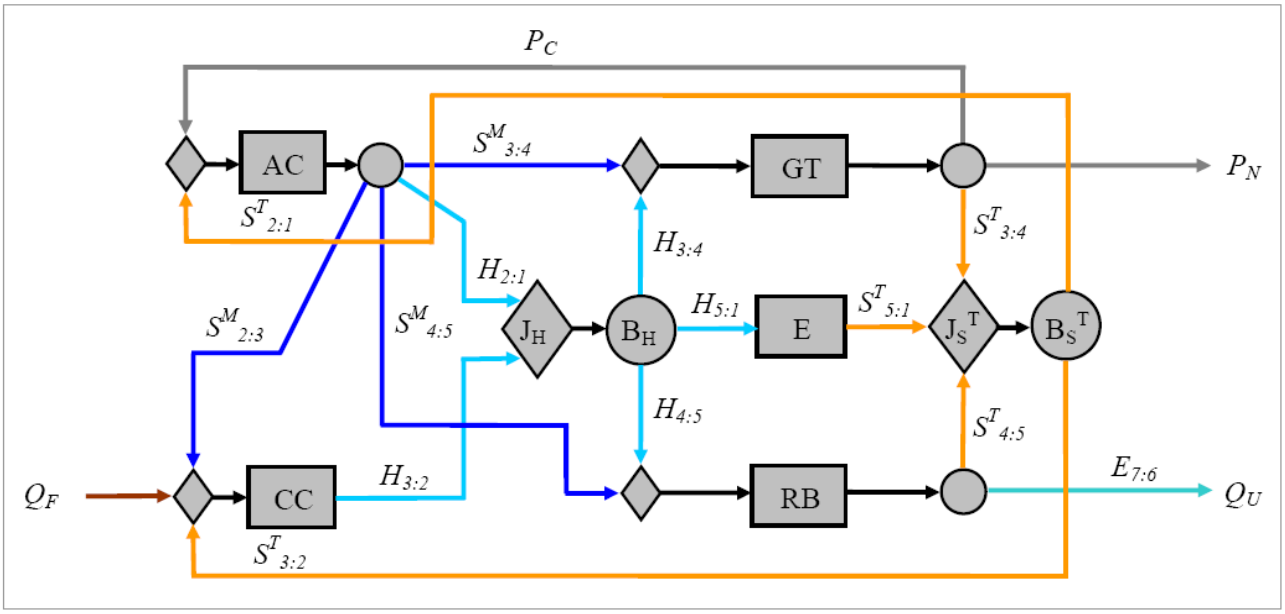

Figure 5. Productive Structure of the Gas Turbine Cogeneration System at Disaggregation Level III

The procedure to obtain the mathematical model is the same explained in subsection 3.1, i. e., the cost equations are formulating in each subsystem, and the auxiliary equations attribute the same exergetic unit cost to all of the productive flows leaving the same bifurcation or the same subsystem.

4. RESULTS AND DISCUSSIONS

Table 4 shows the productive flows, its exergy values and its respective exergetic unit costs, considering each of the four thermoeconomic models based on different disaggregation levels (I, IIa, IIb and III). Figure 6 compares the exergetic unit cost of the final products (heat and power), obtained by the application of each disaggregation level (DL I, DL IIa, DL IIb and DL III). The higher the unit cost of power, the lower the unit cost of heat, and vice-versa.

Table 4. Exergetic Unit Cost of the Productive Flows obtained by using Different Disaggregation Levels

PRODUCTIVE FLOW	VALUE (KW)	EXERGETIC UNIT COST (KW/KW) Disaggregation Level, DL			
		I	IIa	IIb	III
Q_F	11,630.96	1.00	1.00	1.00	1.00
$E_{2:1}$	2,799.13	2.36	-----	-----	-----
$E_{3:2}$	6,794.44	1.71	-----	-----	-----
$E_{3:4}$ / $E_{4:5}$	5,853.64 / 3,219.83	2.01	-----	-----	-----
$E^T_{2:1}$ / $E^M_{2:3}$ / $E^M_{3:4}$ / $E^M_{4:5}$	747.81 / 11.74 / 2,029.98 / 9.61	-----	2.48	-----	-----
$E^T_{3:2}$	6,616.50	-----	1.76	-----	-----
$E^T_{3:4}$ / $E^T_{4:5}$	3,823.66 / 3,210.22	-----	1.92	-----	-----
$H_{2:1}$	3,113.03	-----	-----	2.63	2.61
$H_{3:2}$	10,806.96	-----	-----	2.10	2.07
$H_{3:4}$ / $H_{4:5}$ / $H_{5:1}$	5,546.50 / 6,389.87 / 1,983.62	-----	-----	2.22	2.19
$S_{2:1}$ / $S_{3:2}$ / $S_{4:3}$	313.90 / 4,202.19 / 307.13	-----	-----	2.63	-----
$S_{4:5}$	3,170.04	-----	-----	2.62	-----
$S_{5:1}$	1,653.19	-----	-----	2.66	-----
$S^T_{2:1}$ / $S^T_{3:2}$	2,365.22 / 4,190.46	-----	-----	-----	2.54
$S^T_{3:4}$	1,722.84	-----	-----	-----	2.40
$S^T_{4:5}$	3,179.65	-----	-----	-----	2.58
$S^T_{5:1}$	1,653.19	-----	-----	-----	2.62
$S^M_{2:3}$ / $S^M_{3:4}$ / $S^M_{4:5}$	11.74 / 2,029.98 / 9.61	-----	-----	-----	2.61
P_N / P_C	2,433.47 / 3,113.03	2.12	2.23	2.36	2.40
$Q_U = E_{7:6}$	2,246.32	2.88	2.76	2.62	2.58

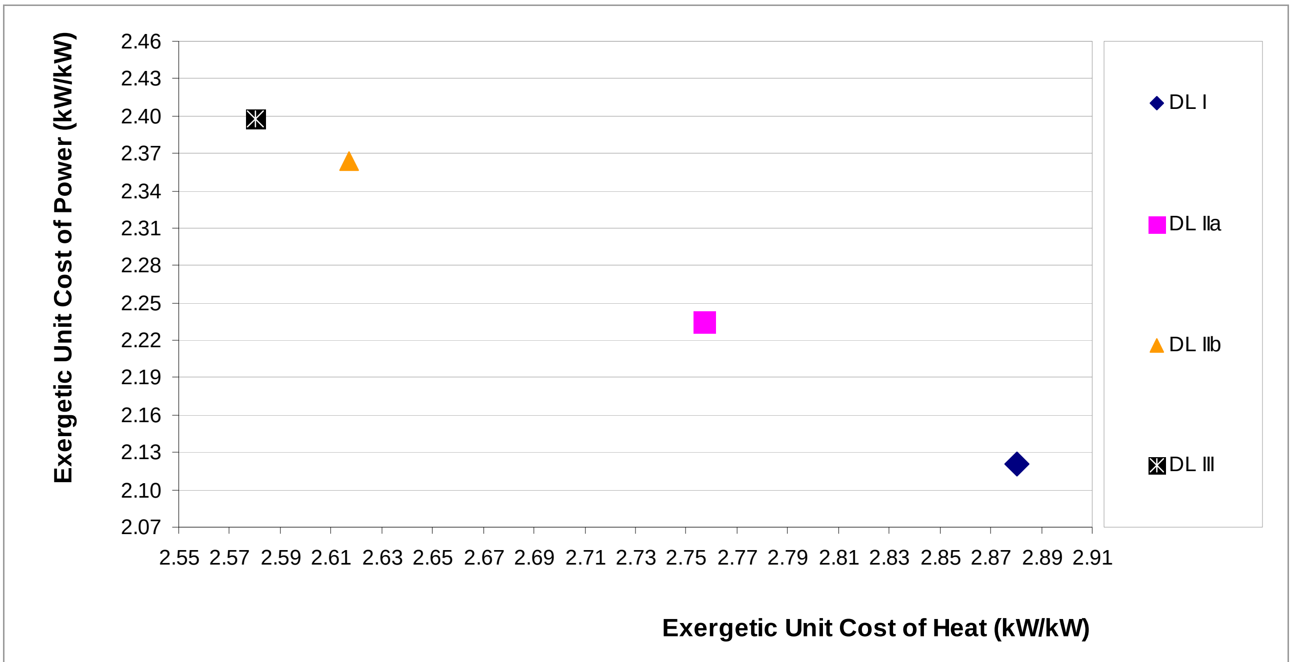

Figure 6. Exergetic Unit Cost of Heat and Power obtained by using Different Disaggregation Levels

From the minimum desegregation level (DL I) to the maximum disaggregation level (DL III), the exergetic unit cost of heat decreases 10.42% and, consequently, the exergetic unit cost of power increases 13.06%. At this point, a very important question arises: is this difference due to the exergy disaggregation level only? The answer is not! Part of this difference is due to the criterion used to allocate the residues (exhaust gases). Using DL I or DL IIa, the residue cost is implicitly distributed to the final products (power and heat) proportionally to the exergy (total or partial) removed from the working fluid by the gas turbine and recovery boiler (respectively). When DL IIb or DL III are applied, the productive structures (Fig. 4 and Fig.5) define explicitly that the enthalpic content of the exhaust gases or residues ($H_{5:1}$) is delivered to an imaginary productive unit, here called environment (E), and afterwards it is redistributed to the real productive units proportionally to the increase of entropy in the working fluid (partial or total) caused by these units. Thus, to evaluate the real effect of physical exergy disaggregation only, this comparison should be made between DL I and DL IIa and between DL IIb and DL III, separately.

From DL I to DL IIa, the exergetic unit cost of heat decreases 4.27% and, consequently, the exergetic unit cost of power increases 5.36%. From DL IIb to DL III the exergetic unit cost of heat decreases 1.40% and, consequently, the exergetic unit cost of power increases 1.43%.

5. CONCLUSIONS

The results obtained for the gas turbine cogeneration system analyzed show that really small accuracy improvements are obtained when the physical exergy is disaggregated. Thus, in agreement with other authors, this splitting might not be always meaningful because of the increase in the computational efforts and the difficulties that might be involved in the separate calculation of mechanical and thermal components (disaggregation level IIa and III), particularly when the working fluid change phases, i. e, when the working fluid is water and steam (or also, refrigerants) and can not be treated as a mixture of ideal gases making incorrect the use of equations (2) and (3).

But, is very important to recognize that by considering separate exergy parcels the accuracy of the results in thermoeconomics is improved, once that the disaggregation make explicit the real exergy parcels that are consumed and produced by each component. Therefore, the disaggregation of physical exergy into enthalpy and syntropy (disaggregation level IIb – H&S Model) has the advantage of being easily applicable to any working fluid, because it does not require the separate calculation of mechanical and thermal components and provide the closest results in relation to the total disaggregated model. Bearing this in mind, the disaggregation level IIb (H&S Model) is a very good way to improve the accuracy of the results in thermoeconomics.

6. ACKNOWLEDGEMENTS

The two first authors of this paper would like to tank ANP and CNPq, respectively, for the financial supports.

7. REFERENCES

Frangopoulos, C.A., 1994, “Application of the Thermoeconomic Functional Approach to the CGAM Problem”, Energy, 19(3), pp. 323-342.
Lazzaretto, A. and Tsatsaronis, G., 2006, “SPECO: Systematic and General Methodology for Calculating Efficiencies and Costs in Thermal Systems”, Energy 31 (2006) 1257-1289.
Lozano, M.A. and Valero, A., 1986, “Determinación de la Exergia para Sustancias de Interes Industrial”, Departamento de Termodinámica y Fisicoquímica, ETSII, Universidad de Zaragoza.
Lozano, M.A. and Valero, A., 1993, “Thermoeconomic Analysis of a Gas Turbine Cogeneration System”. ASME Book no. H00874, WAM 1993, AES, vol. 30, p. 312-20.
Santos, J.J.C.S., Nascimento, M.A.R., Lora, E.E.S. and Martínez Reyes, A.M, 2008, “On The Productive Structure for the Residues Cost Allocation in a Gas Turbine Cogeneration Plant”. In Proceedings of ECOS 2008. Volume 2, Pages 641-648. Cracow, Poland.
Santos, J.J., Nascimento, M.A., Lora, E.E. and Martínez Reyes, A.M, 2009, “On The Negentropy Application in Thermoeconomics: a fictitious or an exergy component flow?”, Int. J. of Thermodynamics, 12(4), pp. 163-176.
Torres, C., Serra, L., Valero, A. and Lozano, M.A., 1996, “The Productive Structure and Thermoeconomic Theories of System Optimization”, ME’96: International Mechanical Engineering Congress & Exposition (ASME WAN’ 96).
Verda, V., Serra, L. and Valero, A., 2004, “The Effects of the Control System on the Thermoeconomic Diagnosis of a Power Plant”, Energy, Vol. 29, pp. 331- 359.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.