LAMINAR FLOW OF A NON-NEWTONIAN SLURRY THROUGH A PIPE

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Abstract. *In the mining industry the most economical way to transport mineral slurry is through the pipeline. These kinds of slurries are non-newtonian fluids. In these study the viscosity of the slurry is described by the Carreau-Yassuda model. A dimensionless unidirectional axisymmetric model was proposed for laminar and fully developed flow through a pipe with circular cross section. A non-linear differential problem arises due to the non-newtonian viscosity model. With dimensionless appears two dimensionless numbers: the Reynolds number and the other parameter defined by* De*. The* De *is defined by* $\lambda GR/\eta_0$*, where* λ^-1 *is the critical shear rate at which viscosity begins to decrease,* R *is the radius of circular cross section,* G *is the ratio of pressure to duct length and* η_0 *is the viscosity at zero shear rate. The numerical solution of the non-linear differential problem was derived by the Runge-Kutta* 2^{nd} *order method. The successive substitution process was used to obtained the analytical solutions. The asymptotic solutions proposed for low and high ranges of De were obtained by Taylor expansion series. The results of the analytical and numerical solutions are shown by profiles of velocity, volumetric flow rate and* De*. The asymptotic theory agrees with numeric results for the De range indicated in this paper.*

Keywords: *mineral slurry pipe flow, Carreau-Yasuda, assymptotic solutions, non-newtonian fluid*

1. INTRODUCTION

In now a days the transport of the high quantities of ore and petroleum is performed by pipelines. These kind of transport is the most viable solution (Chang et al, 1999) for great distances between mine and refinery when it is done without interruptions (Cunningham, 2008). These solution is appropriate to explore ore from remotely site, since the railway and trucking itself as too costly (Weir Minerals Netherlands, 2008, Cunningham, 2008). Additionally, the pipeline option is environmentally friendly.

In 2007, Brazil was the first country to transport bauxite slurry by long-distance pipelines. Bauxite is used to extract aluminum. The achievement of this long-distance transport is attributed to the Mineração Bauxite Paragominas which pumping 13.5 MTA. The pipeline extension from Paragominas Miltonia mine to Alunorte Refinery is about 245 km (Weir Minerals Netherlands, 2008).

To make bauxite slurry is necessary add a quantity of water about 50% of solids. This slurry is taken to the refinery through the pipelines. At refinery the slurry is dewatered. The dewater process increases the bulk costs of the slurry transport (Cunningham, 2008). So the pipeline solution is the most cost effective for long distances. For short distances the caustics is used instead the water. The caustics eliminate the dewatering issue (Weir Minerals Netherlands, 2008).

For transport bauxite slurry is ideal that the larger particles have a size about 250 μm and about 50% of the particles with less than 45 μm (Gandhi et al,2008). However, the viscosity is a relevant factor to prevent settling out during short flow interruptions. The particle sizes influences the viscosity. So to obtain the proper viscosity the slurry needs to have a sufficient quantity of ultra-fine particles (less than 10 μm). On other hand the increase of the viscosity causes an increase on the head requirement.

In this study the slurry bauxite is carry out by a straight pipeline with circular cross section. A dimensionless unidirectional axisymmetric model was proposed for laminar and fully developed conditions. Asymptotic solutions was derived for low and high Deborah regimes. For each regime the profile of the velocity, flow rate and viscosity dimensionless was derived. Since bauxite slurry is a non-newtonian fluid its viscosity can be modeled by the Carreau-Yasuda model. The parameters for the viscosity model were extracted from experimental Nascimento and Sampaio (1999) study. In that experiment the bauxite is from Pará and have a 50% w-w. The size of the smaller particles is about 37 μm.

2. Mathematical Model

In this section the unidirectional and fully developed laminar flow of a slurry through a pipe is described. The slurry is modeled as a generalized Newtonian fluid with variable viscosity. The viscosity is calculated by the Carreau-Yasuda model. The governing equation of the problem was derived with the constitutive equations for newtonian fluid and the viscosity model. This equation is then non-dimensionalized by the characteristic length scale of the pipe and the constant pressure gradient of the flow. From the process of non-dimensionalization appears a dimensionless group similar to the Deborah number, and some important relations between fluid and flow parameters.

2.1 Unidirectional Flow

The constitutive equations for flows are derived by the continuity and conservative momentum equations. These equations are written in cylindrical coordinates for a pipe with a circular cross section. If it is assumed a unidirectional flow, the velocity field is reduced to $u_\theta = 0, u_r = 0$ and $u_x = u_x(r)$, where θ is the angular direction, r is the radial direction and x is the axial direction. This is equivalent to say that for a fully developed flow through a pipe with length dimension greather than the diameter the velocity does not vary both in radial and angular directions. In axisymetric condition all derivatives of the constitutive equation are zero at angular direction. This allows us to write $\mathbf{u} \cdot \nabla \mathbf{u} = \mathbf{0}$ and

$$-\nabla p + \nabla . \boldsymbol{\tau} = \mathbf{0} \,, \tag{1}$$

where p is the pressure field and $\boldsymbol{\tau}$ is the deviatoric part of the stress tensor given by $\boldsymbol{\tau} = 2\eta \boldsymbol{D}$, η is the dynamic viscosity coefficient and $\mathbf{D}$ is the symmetric part of the gradient velocity tensor which is equal to

$$\mathbf{D} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right) .$$

It is important to observed that $\boldsymbol{\tau}$ model do not predicts the elastic effects (Phan-Thien, 2002). The Eq. (1) said that the flow through a pipe is given by the balance between pressure and viscous forces. With above assumptions only the axial direction component of the Eq. (1) exists. In cylindrical coordinate the Eq. (1) is written as:

$$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial r}\left(\eta r \frac{\partial u}{\partial r}\right) , \tag{2}$$

where the viscosity coefficient η is modeled by Carreau-Yasuda model given by:

$$\eta = \eta_0 + (\eta_\infty - \eta_0)\left[1 + (\lambda \dot{\gamma})^a\right]^{\frac{n-1}{a}} , \tag{3}$$

where η_0 and η_∞ are respectively the viscosity at zero and at infinite shear rate, λ^{-1} is the critical shear rate at which viscosity begins to decrease and $\dot{\gamma} = \partial u / \partial r$ is the shear rate. The a and n are material constants of the Carreau-Yasuda model. From Eq.(1) and Eq.(2) is possible to demonstrate that the pressure gradient is actually a constant (Bird et al, 1987). This fact emerges from the inspection that the pressure is only a function of the axial coordinate x. In the other hand, the second term of the Eq.(2) can be written using the shear rate definition:

$$\frac{\partial}{\partial r}\left(\eta \, r \, \dot{\gamma}\right) .$$

This term say that the viscous stress is only a function of the radial coordinate r. By this way, the terms of the Eq. (2) with distinct and independents variables leads to a scenario where the two tems are equals.This is possible only if these two functions are, in fact, constants.

2.2 Nondimensionlization of the mathematical model

The pressure gradient was selected to be a reference quantity of the nondimensionlization process. This selection is due to the pressure gradient being a constant as was described above. In this sense, and using the radius of the pipe, R, as characteristic length, is defined that:

$$U_c = \frac{R^2}{\eta_0} G \quad ; \quad \dot{\gamma_c} = \frac{U_c}{R} = \frac{G}{\eta_0} R \ , \tag{4}$$

where U_c and $\dot{\gamma_c}$ are respectively the characteristic velocity and characteristic shear rate. The quantity G is usually defined as $-\partial p / \partial x$. By integrating the Eq. (2) with respect to r and considering that $\dot{\gamma}$ is finite in the flow domain, it is possible to write that $\eta \dot{\gamma} = -G r / 2$. The η is modeled by the Carreau-Yasuda model. This model describe the shear rate as a function of the viscosity. However, remember that Carreau-Yasuda model is not able to simulate elastic behavior. The characteristic quantities are inserted in the viscosity model. The Deborah number arises of this process as is highlighted by the next equation:

$$\tilde{\dot{\gamma}} + \tilde{\dot{\gamma}} \eta^* \left[1 + \left(\tilde{\dot{\gamma}} De\right)^a\right]^{\frac{n-1}{a}} = -\frac{\tilde{r}}{2} \ , \tag{5}$$

where $\tilde{\dot{\gamma}} = \dot{\gamma} / \dot{\gamma}_c$, $\tilde{r} = r/R$ and

$$De = \frac{\lambda G R}{\eta_0} \quad , \quad \eta^* = \frac{\eta_\infty}{\eta_0} - 1 \, .$$

The parameter η^* is a dimensionless group which is directly related to the viscosity variation profile. For $\eta^* > 0$ $\left(\frac{\eta_\infty}{\eta_0} > 1\right)$, the slurry behaves as a shear thickening fluid ; for $\eta^* < 0$ $\left(\frac{\eta_\infty}{\eta_0} < 1\right)$, the slurry behaves as a shear thinning fluid. Numerical results was obtained with $\eta^* = -0.5$. The dimensionless group De may be understood as a ratio between two time scales: a time constant related to the sensivity of the fluid to shear rate, (λ), and a time scale related to the flow $(\eta_0/(GR)$. In this sense, the group $\lambda GR/\eta_0$ is similar to the Deborah number.

3. APPARENT VISCOSITY

The fully developed flow through a pipe may be studied by comparing the pressure gradient and the consequent flow rate, Q, for any fluid. In other words, by the classical Hagen-Poiseuille equation is possible to isolate the viscosity in such a way that:

$$\eta_{ap} = \frac{\pi\, R^4\, G}{8\, Q}$$

where Q is the flow rate and η_{ap} is an apparent viscosity. For laminar flow and Newtonian fluid the η_{ap} is a constant equal to the dynamic molecular viscosity. For a non-newtonian fluid η_{ap} is a function of the flow rate for a given pressure gradient. The characteristic flow rate Q_c is derived as:

$$Q_c = U_c \pi R^2 = \frac{\pi\, R^2\, G}{\eta_0} \ .$$

The dimensionless apparent viscosity, $\tilde{\eta}_{ap}$, is then:

$$\tilde{\eta}_{ap} = \frac{1}{8\, \tilde{Q}} \ , \tag{6}$$

where $\tilde{\eta}_{ap} = \eta_{ap}/\eta_0$ and $\tilde{Q} = Q/Q_c$.

4. FINAL DIMENSIONLESS MODEL

The n and a are material constants of the Carreau-Yasuda model. For the bauxite slurry the values of these constants were extracted from experimental results of Nascimento and Sampaio (1999). The values $n = 1/2$ and $a = 2$ are a good approximations for this experiment. In order to make the notation easier the tilde notation for dimensionless quantities will be suppressed. After last considerations the final dimensionless model is given by next equations:

$$\dot{\gamma} + \dot{\gamma}\, \eta^* \left[1 + (\dot{\gamma}\, De)^2\right]^{-\frac{1}{4}} = -\frac{r}{2} \ \ , \ \ r \in [0, 1] \ \ , \tag{7}$$

subject to the boundary condition $u(1) = 0$,

$$Q = 2 \int_0^1 u\, r\, dr \ , \tag{8}$$

and by Eq. (6).

5. NUMERICAL MODEL

In order to produce a numerical solution was employed the Newton-Raphson, Runge-Kutta 2^{nd} Order and Trapezoidal methods. This methods allow to calculate the roots, the numerical derivative and integral of an equation, respectivelly (Hoffman, 2001). The radial axis was discretized into N equality spaced control points, r_i. In this study $N = 300$. For each value of r_i the $\dot{\gamma}$ of the Eq. (7) was derived by Newton-Raphson method. The tolerance used in the Newton-Raphson procedure was 10^{-12}. The initial guess for the method was $\dot{\gamma} = r/2$. After this procedure, the initial differential problem is rewritten in the form:

$$f(n) = \begin{cases} \frac{\partial u}{\partial r} = f(r_i) & , \quad i = 1, ..., N \\ \\ u(1) = 0 \end{cases} \tag{9}$$

where $\dot{\gamma}$ was replaced by $\partial u/\partial r$ and $f(r_i)$ is a numerical function (tabulation) resulting of the Newton-Raphson procedure. So the problem is described by an initial value problem, which can be solved in a straight way by a 2^{nd} Order Runge-Kutta method. The flow rate, Q, was calculated by using the Trapezoidal Rule wich is function of the velocity derived by Runge-Kutta method. The study was done for the range $10^{-4} \leq \text{De} \leq 10^7$.

6. ASYMPTOTIC SOLUTIONS

The asymptotic solutions were derived by the successive substitution method (Hinch,1995). This method consists in to generate a sequence using a recursive formula obtained from the governing equation. The recursive formula is the Eq. (7) wich can be rewritten in the form:

$$\dot{\gamma}_n = -\frac{\eta^* \dot{\gamma}_{n-1}}{\left[1 + (\dot{\gamma} De)^2\right]^{\frac{1}{4}}} - \frac{r}{2} \,. \tag{10}$$

For study the limit of $De << 1$ the first term of the sequence is found taking $De = 0$,

$$\dot{\gamma}_0 = \frac{-\frac{r}{2}}{\eta^* + 1} \,. \tag{11}$$

The $\dot{\gamma}_1$ is calculated by the substitution of $\dot{\gamma}_0$ in the right side of the Eq. (10). The dimensionless velocity profile was derived by solving the dimensionless shear rate with velocity zero at wall pipe as boundary condition. The dimensionless flow rate was calculated by integral of velocity profile for a circular cross section area with limits of the integration between zero (center of the pipe) and 1 (maximum dimensionless radius). The dimensionless expressions for shear rate, velocity, volumetric flow rate and apparent viscosity are, respectively:

$$\dot{\gamma} = -\frac{1}{2}\frac{r}{\eta^* + 1} + \frac{1}{32}\frac{\eta * r^3 (\eta^* - 1)}{(\eta^* + 1)^3} De^2 \;, \tag{12}$$

$$u(r) = \frac{1}{128}\frac{\eta^* (\eta^* - 1)\left(r^4 - 1\right)}{(\eta^* + 1)^3} De^2 - \frac{1}{4}\frac{r^2 - 1}{\eta^* + 1} \;, \tag{13}$$

$$Q = -\frac{1}{192}\frac{\eta^* (\eta^* - 1)}{(\eta^* + 1)^3} De^2 + \frac{1}{8(\eta^* + 1)} \;, \tag{14}$$

$$\eta_{ap} = -24\frac{(\eta^* + 1)^3}{\eta^* (\eta^* - 1)} De^{-2} + (\eta^* + 1) \;. \tag{15}$$

For the study of the limit of $De >> 1$ it was necessary to make a chance of variable. In this way we take $\varepsilon = 1/De^2 = De^{-2}$. Note that when $De^{-2} \to \infty$, $\varepsilon \to 0$. The same process of substitution for $De << 1$ was done for $De >> 1$ but here the $\dot{\gamma}_0 = -r/2$. So the dimensionless analytical expressions for High Deborah are:

$$\dot{\gamma} = -\frac{1}{2} r + \frac{1}{2}\eta^* \, r^{1/2} \left(\frac{1}{De^2}\right)^{1/4} \;, \tag{16}$$

$$u(\,r) = \frac{1}{4}\left(1 - r^2\right) + \frac{1}{3}\sqrt{2}\,\eta^* \left(r^{3/2} - 1\right)\left(\frac{1}{De^2}\right)^{1/4} \;, \tag{17}$$

$$Q = \frac{1}{8} - \frac{1}{7}\sqrt{2}\,\eta^* \left(\frac{1}{De^2}\right)^{1/4} \;, \tag{18}$$

$$\eta_{ap} = 1 - \frac{7}{8\sqrt{2}\,\eta^*}\left(\frac{1}{De^2}\right)^{1/4} \;. \tag{19}$$

The results for velocity, flow rate and viscosity dimensionless are presented and commented below.

7. RESULTS

In this section the results of velocity, flow rate and apparent viscosity for numerical results and ansympotic solutions are presented. The Fig. 1 shows the velocity profile for three De values. This graphic shows that the velocity at the center of pipe decreases when $De \to \infty$. The numerical curve for $De = 1$ is between the two analytical curves. However, if the value of the η^* is changed the numerical curve changes its position in graph.

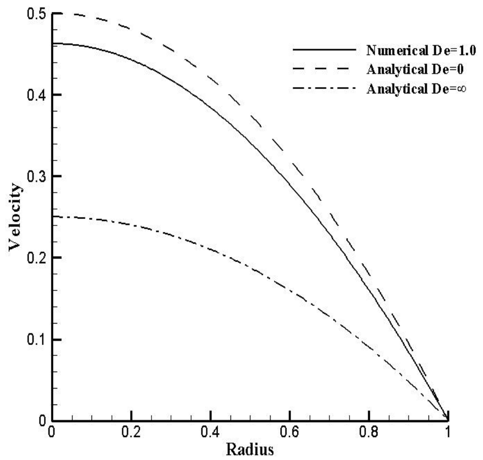

Figure 1. Dimensionless velocity profile at low and high De

The Fig. 2 shows the flow rate in function of De. The graphic shows the fully agreement between numerical results and analytical solutions.

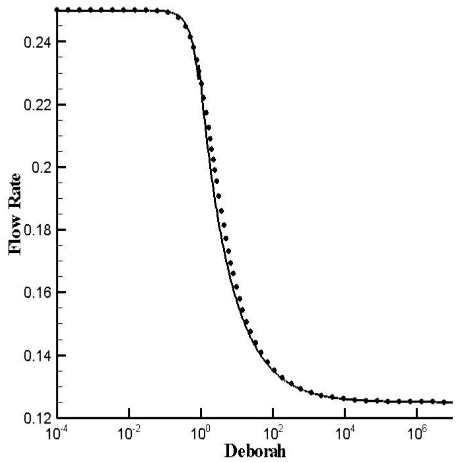

Figure 2. Numerical (points) and analytical (line) dimensionless flow rate at low and high De

The graphic of the Fig. 3 shows the viscosity in function of De. The two plateaus appears again because the apparent viscosity is only function of flow rate. This plateaus describe the shear thinning regime (Low Deborah) and shear thickenning (High Deborah).

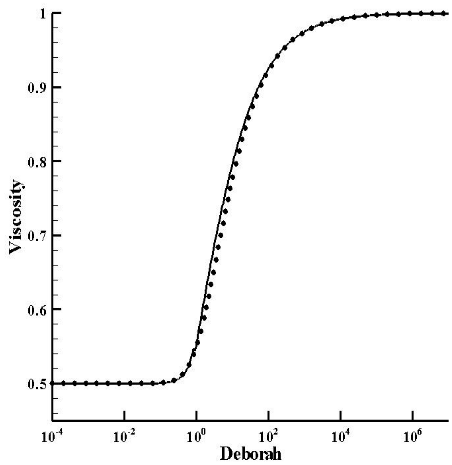

Figure 3. Numerical (points) and analytical (line) dimensionless viscosity at low and high De

The results of the three graphs are consistent with each other. Comparing the Fig. 2 with the Fig. 3 we can see two situations: one in which the region of the low viscosity corresponds to the region of the higher flow rate and the lowest range of Deborah; and the other in which the region of the higher viscosity corresponds to the region of the lowest flow rate and the higher range of Deborah. The Fig. 1 shows that when Deborah tends to higher values the velocity in the center of the pipe decreases and in consequence the flow rate decrease too. Therefore lower viscosity corresponds to the shear thinning regime and higher viscosity corresponds to the shear thickenning regime. This regimes can be explained based on the existence of water in the interstice between the bauxite solid particles. In the shear thinning situation the water acts as a lubricant and the particles can slide one relative to each other. However in the shear thickenning situation the water comes out of the interstices and the particles come into contact with each other. In that situation the apparent viscosity increases and the velocity and flow rate decreases.

8. CONCLUSIONS

Since Brazil was the first country to transport bauxite slurry by long pipelines it is interesting study the behavior of the bauxite slurry flow. The interest of this kind of studies can, for exemple, help to predict the pressure necessary to bauxite slurry flows and under what conditions the flow can show critical situations. This work presented a mathematic model for the laminar flow of the bauxite slurry through a straight pipe with circular cross section. The behaviour of the slurry was modeled by the Carreau-Yasuda model. In this model the viscosity in function of the shear rate. The values for the material constants of the Carreau-Yassuda model were extracted from the bauxite slurry experimental results. The mathematical model was adimensionalized in order to study the velocity profile, flow rate and viscosity. The shear rate profile was derived by Newthon-Raphson. The zero velocity was the boundary condition at the pipe wall. The velocity profile was obtained by 2^{nd} Order Runge-Kutta and the flow rate was calculated by Trapezoidal Rule. The asymptotic solutions for higher and lower Deborah were derived by a recursive formula with Taylor series expansion. The results shows the shear thinning regime for lower De and shear thickenning for the higher De. For shear thinning regime the velocity at the center of the pipe is higher than the velocity for shear thickenning regime. The results shows the viscosity is lower for shear thinning and higher for shear thickenning regime. In this situations the flow rate is lower for the shear thickenning regime.

9. REFERENCES

Bird, R. B., Armstrong, R., C., Hassager, O.,1987, "Dynamics of Polymeric Liquids: Volume 1", Second Edition, John Wiley & Sons

Chang, C., Nguyen, Q. D., Ronningsen, H.P., 1999, "Isothermal start-up of pipeline transporting waxy crude oil", Journal of Non-Newtonian Fluid Mechanics, Volume 87, pp. $127-154$

Cunningham T., 2008, "Long-Distance Transport of Bauxite Slurry by Pipeline", Bechtel Technology Journal, Volume 1, No. 1

Gandhi,R., Weston, M., Talavera,M., Brittes, G.P., Barbosa, E., 2008, "Design and Operation of the World's First Long Distance Bauxite Slurry Pipeline",Light Metals 2008, Volume 1,pp. $95-100$

Hinch, E. J.,, 1995, "Perturbation Methods", Cambridge University Press

Hoffman, J. D., 2001, "Numerical Methods for Engineers and Scientists: Second Edition Revised and Expanded", Marcel Dekker Inc.

Nascimento,C. R., Sampaio,J. A.,1999, "Estudo Reológico da Polpa de bauxite", I Jornada do Programa de Capacitação Interna − CETEM

Phan-Thien, N., 2002, "Understanding Viscoelasticity: Basics of Rheology", Springer − Verlag Berlin Heidelberg

Weir Minerals Netherlands, 2008, "Brazilian Paragominas bauxite pipeline: proven economical!",GEHO Info for sales representatives, Volume 18, No.2

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