

DETERMINATION OF THE DIFFUSION COEFFICIENT OF SALTS IN NON-NEWTONIAN LIQUIDS BY THE TAYLOR DISPERSION METHOD

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Abstract. *This research looked for a method to determine the binary diffusion coefficient D of salts in liquids (especially in drilling fluids) not only accurately, but in a reasonable time. We chose to use the Taylor Dispersion Method. This technique has been used for measuring binary diffusion coefficients in gaseous, liquid and supercritical fluids, due to its simplicity and accuracy. In the method, the diffusion coefficient is determined by the analysis of the dispersion of a pulse of soluble material in a solvent flowing laminarily through a tube. This work describes the theoretical basis and the experimental requirements for the application of the Taylor Dispersion Method, emphasizing the description of our experiment. A mathematical formulation for both Newtonian and non-Newtonian fluids is presented. The relevant sources of errors are discussed. The experimental procedure and associated analysis are validated by applying the method in well known systems, such as NaCl in water.*

Keywords: *Taylor Dispersion Method, Diffusion of salts in liquids, Diffusion Coefficient*

1. INTRODUCTION

One of the main challenges of the petroleum industry is the drilling in salt domes. The presence of salt zones indicates the possibility of the existence of petroleum, since the insulating properties of salt allow the formation of geologic traps that imprison the migrant oil. However, if it is not well designed, many problems may occur due to the diffusion of the salt in the drilling fluid, such as fracture of the region above the saline zone or even the collapse of the well. These problems may increase the costs of the drilling phase (Silva *et al.*, 2006). The study of the impact of the drilling fluid used in salt regions is very important and, to make this analysis possible, it is necessary to know the diffusion coefficient D of the salt in this fluid.

1.1 Theoretical Revision

Diffusion is an irreversible process of mass transport, where a difference of concentration is reduced by a spontaneous flow. Although diffusion is a three-dimensional process, sometimes it can be studied as a one-dimensional case, where the diffusion coefficient D can be defined by the Fick's law:

$$J = -D \frac{\partial c}{\partial x} \quad (1)$$

In this equation J is the particle flux, C is the concentration of the solute and x is the distance into the substrate. The negative sign indicates that the diffusing mass flows in the direction of decreasing concentration. If we combine Fick's law with the conservation of mass, we find Fick's second law, which predicts how diffusion causes the concentration field to change with time t :

$$\frac{1}{D} \frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial x^2} \quad (2)$$

If we consider the problem of one-dimension diffusion of an amount M of solute, situated at the origin at an initial time (Fig. 1), the solution to Fick's Law under these conditions is:

$$c(x, t) = \frac{M}{2\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right) \quad (3)$$

We can observe that this equation gives us a normal distribution of the solute, as seen in Fig. 2.

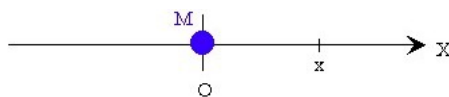


Figure 1. Limited-source one dimension diffusion (Santana, 2010)

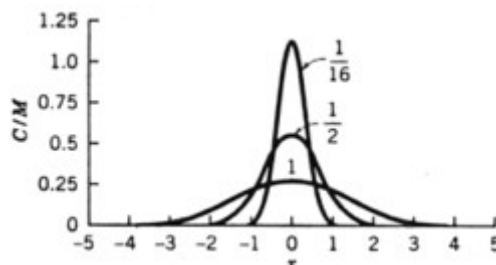


Figure 2. Concentration curves for a limited-source. Numbers on curves are values of Dt (Chiang et al., 1996)

The simplest way to visualize the diffusion process is shown in Fig. 3. At $t = 0$ a concentration difference is established at the interface $z = 0$. But to accomplish an experiment like this, it is difficult and takes a long time: since diffusion is a very slow process, it takes many days to measure a significant difference of concentration and, during this time, temperature and pressure must be constant and uniform (Granger, 1994).

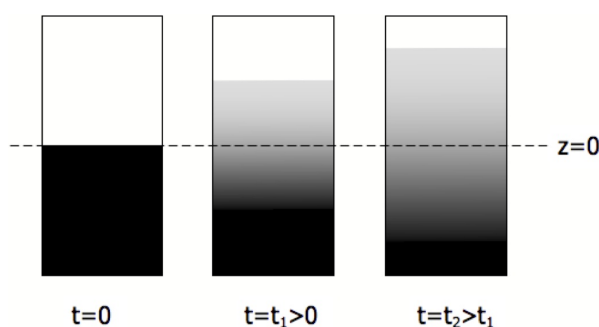


Figure 3. Scheme of a diffusion process (Granger, 1994)

Many different methods have been developed for the experimental determination of diffusion coefficients, but in most of them diffusion happens in steady state (like the process described above). So all of these stationary methods have the same problem: the slowness of the diffusion process. For this reason there are only a few accurate methods to measure diffusion in liquids (Bailey and Gogarty, 1963).

1.2 The Taylor Dispersion Method

The Taylor Dispersion Method, also called Chromatographic Broadening Technique, is a kind of transient response method: measurements of transport properties can be made from small gradients of driving forces, such as from a small change in concentration, and the times required are relatively short when compared to those under steady-state conditions (Funazukuri *et al.*, 2004). The theoretical treatment of the method was made by Taylor (1953), and further developed by many authors, especially by Aris (1956) and by Alizadeh, Castro and Wakeham (1980). This technique has been used for measuring binary diffusion coefficients in gaseous, liquid and supercritical fluids due to its simplicity and accuracy. We intend to use it to determine the binary diffusion coefficient of salts in liquids.

In essence, the experimental technique makes use of the velocity profile characteristic of laminar flow to enhance the dispersion of the pulse brought about by molecular diffusion alone (Alizadeh *et al.*, 1980). Coupling the forced dispersion by the laminar flow to the molecular diffusion, a significant dispersion of the initial concentrated pulse is obtained in a short time.

So using this principle in the experiment, we consider a homogeneous liquid mixture of solute and solvent flowing in laminar flow through an infinitely long, isothermal, straight tube of uniform, circular cross section with impermeable walls. At time $t = 0$, a sample of liquid mixture of the same two components of a different composition is introduced into the tube at $z = 0$ (Alizadeh *et al.*, 1980). We measure then the radial average of the concentration in the output of the tube along the time. In the method, the diffusion coefficient D is determined by the analysis of this dispersion (Fig. 4).

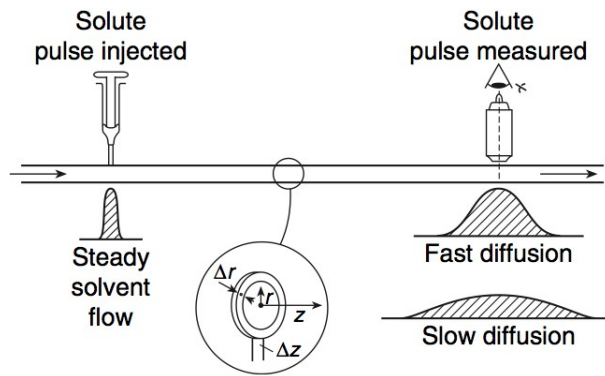


Figure 4. Simple scheme of the Taylor Dispersion Method : solvent is passing in steady laminar flow through a long, thin tube. A pulse of solute is injected near the tube's entrance. This pulse is dispersed by the solvent flow (Cussler, 2009)

Taylor (1954) better explained the physical basis of his experiment by first explaining dispersion by convection alone. Notice that the following analysis is done considering a Newtonian fluid, but it could be also done considering a non-Newtonian fluid, the difference would be the velocity profile.

Imagine a straight pipe through which a viscous fluid is streaming in laminar flow, and suppose that at time $t = 0$ the color of the stream entering at $x = 0$ is changed (by adding a dye). The initial condition is indicated in Fig. 5(a). The distribution of velocity is parabolic and the mean velocity U is half the maximum, so that after time t the forward edge of the color lies in a paraboloid and its vertex is at $x = 2Ut$. This condition is shown in Fig. 5(b). If c_0 is the concentration of the dissolved dye as it enters the tube, the mean concentration c at any section is:

$$c = c_0 \frac{\text{paraboloid cross-section area}}{\text{tube cross-section area}} \quad (4)$$

The areas of the sections of a paraboloid are proportional to their distance from their vertex. Thus the concentration decreases linearly from $c = c_0$ at $x = 0$ to $c = 0$ at $x = 2Ut$. Under the representations of the flow in Fig. 5, the distribution of concentration c is shown.

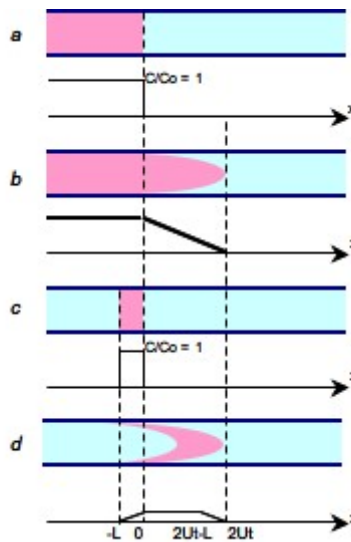


Figure 5. Dispersion by convection alone: - (a) and (c) are initial distributions - (b) and (d) distributions at time t . To each case there is the corresponding distribution along the tube of mean concentration over cross sections (Taylor, 1954)

Now consider the case when the dye is initially confined to a short length of the tube from $x = -L$ to $x = 0$, the fluid in the front and behind being pure solvent. The condition is represented in Fig. 5(c). After the time t both the front and back surfaces of separation are paraboloids (Fig. 5(d)). The dye has penetrated to $x = 2Ut$. From $x = 2Ut - L$ to $x = 2Ut$, c decreases exactly as in Fig. 5(b). c is constant from $x = 0$ to $x = 2Ut - L$, and from $x = -L$ to $x = 0$ it increases uniformly. The distributions of c at times 0 and t are shown in Figs 5(b) and 5(d).

To obtain these distributions of concentration experimentally it is necessary to carry out the experiment so quickly that transverse diffusion does not have time to modify the dispersion produced by pure convection. Taylor found no difficulty in doing this in his work (Taylor, 1954).

Now we can analyze the effects of molecular diffusion. The dispersion by molecular diffusion depends on the concentration gradient of the solute. To understand what happens in the case of the Taylor dispersion, we can divide the diffusion of the solute in the tube in both axial and radial diffusions (Fig. 6). If we could stop convection at the instant represented in Fig. 6(a) and wait for the effect of the diffusion, we would have the scattering of the dye. If we divided the diffusion in axial and radial we would see Fig.6(b) and Fig.6(c) respectively. Since diffusion is a very slow process, the axial diffusion is very slow compared to the modification of the axial dispersion due to convection, being then negligible.

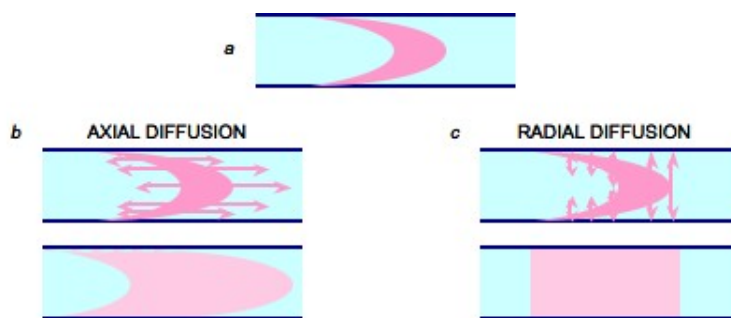


Figure 6. Axial and Radial Diffusion

When the transverse diffusion is not sufficiently large to wipe the effect of axial convection, the transverse distribution of concentration will not be uniform unless the axial distribution is also uniform. In other words, if there is no radial diffusion and there is a modification on the axial concentration (for example by putting a dye in a part of the flow, like we just did to visualize the effect of convection), the radial distribution will not be uniform: therefore along the radius we will see different concentrations like in Fig. 7. So, without radial diffusion, the radial distribution will only be uniform if the axial distribution is also uniform (it would be the case of having the same concentration of dye along the tube: even with the velocity profile, the concentration wouldn't change).

This last explanation contains the key to the solution of the problem: if the axial distribution is uniform, the rate at which the dissolved substance passes a section which moves with the mean velocity of the flow, is zero. On the other hand, if there is a small axial gradient of concentration along the tube, convection will cause a small transverse variation of concentration, which will cause a small transport of the solute across a section which moves with the mean velocity. It is not a long step then to see that this small transport and the small axial concentration gradient must be proportional to one another. Thus the combined effect of axial convection and transverse diffusion is the dispersion of the solute axially relatively to a frame moving at the mean speed of flow by a mechanism which obeys the same law as ordinary one-dimensional diffusion relative to a fluid at rest (shown in Fig. 1 and 2).

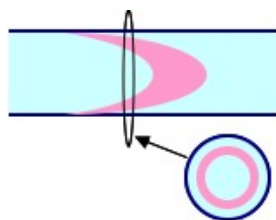


Figure 7. Non-uniform radial distribution

In other words, the solute is dispersed by a combination of two processes: molecular diffusion, caused by the concentration gradient, and the parabolic velocity profile of the flow. These processes act in opposition: the laminar flow quickly distorts the initial pulse of solute, tending to disperse it. If there is no diffusion, the distortion continues unabated, and the pulse is widely dispersed. If, instead, there is rapid diffusion, the material in the center of the tube tends to diffuse outward, into a region of solvent that is moving more slowly. Simultaneously, the material that is left behind near the tube walls tends to diffuse towards the center, into a region of faster flow. This radial diffusion thus inhibits the dispersion induced by axial convection (Cussler, 2009). As a consequence, even though it seems paradoxical, solutes with higher diffusion coefficients present less dispersion (Loh, 1997).

By the comprehension of the physical phenomena, it is evident that a fundamental condition for the experiment is that the tube be capillary. Since diffusion is a slow process, the tube radius must be small enough in order for the effects of the radial diffusion not to be negligible.

We can make a mathematical formulation to find an equation that supplies the concentration of solvent in one determined area of the tube in a determined instant of time as a function of different known parameters (as the tube radius, mean velocity, among others) and of the diffusion coefficient. So, measuring the dispersion of the solute we can find the diffusion coefficient.

Aris (1956) proved that after a certain time, the axial distribution of concentration in the interior of the tube (that is, the distribution of the solute) assumes an almost Gaussian form. As the dispersion has a normal distribution, the diffusion coefficient D can be obtained from the parameters of this curve. Another option to obtain D is to make a curve fitting.

2. MATHEMATICAL MODEL

The following analysis allows the use of the Taylor Dispersion Method to non-Newtonian fluids. Let us consider the steady, hydrodynamically developed flow of a non-Newtonian power-law fluid in a pipe. The axial velocity is given by:

$$v_z(r) = \frac{1/n + 3}{1/n + 1} \bar{v}_z \left[1 - \left(\frac{r}{R} \right)^{1/n+1} \right] \equiv a(n) \bar{v}_z \left[1 - \left(\frac{r}{R} \right)^{b(n)} \right] \quad (5)$$

where n is the power-law index, R is the pipe radius, r is the radial coordinate, and \bar{v}_z is the average axial velocity, defined as

$$\bar{v}_z = \frac{1}{\pi R^2} \int_0^R v_z(r) 2\pi r dr = \frac{2}{R^2} \int_0^R v_z(r) r dr = 2 \int_0^1 v_z \left(\frac{r}{R} \right) \left(\frac{r}{R} \right) d \left(\frac{r}{R} \right) \quad (6)$$

Suppose now that at time $t = 0$ a patch of concentrated solution of negligible axial length is injected at $z = 0$. The evolution of the salt concentration $c(r, z, t)$ is described by

$$\frac{\partial c}{\partial t} + v_z(r) \frac{\partial c}{\partial z} = D \left[\frac{\partial^2 c}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) \right] \quad (7)$$

where z is the axial coordinate and D is the diffusion coefficient.

The initial and boundary conditions are

$$t = 0, \quad \forall z, \quad c = \frac{M}{\pi R^2} \delta(z) \quad (8)$$

$$t > 0, \quad r = 0, \quad \frac{\partial c}{\partial r} = 0 \quad (9)$$

$$r = R, \quad \frac{\partial c}{\partial r} = 0 \quad (10)$$

$$z \rightarrow \infty, \quad \forall t, \quad c = 0 \quad (11)$$

$$z \rightarrow -\infty, \quad \forall t, \quad c = 0 \quad (12)$$

where M is the amount of mass of salt injected and $\delta(z)$ is Dirac's delta function.

Suppose now that c can be decomposed into its cross-sectional average \bar{c} and a deviation c' :

$$c(r, z, t) = \bar{c}(z, t) + c'(r, z, t) \quad \text{where} \quad \bar{c} = \frac{2}{R^2} \int_0^R c r dr \quad (13)$$

Thus (note that, by definition, $\bar{c}' = 0$):

$$\frac{\partial \bar{c}}{\partial t} + \bar{v}_z \frac{\partial \bar{c}}{\partial z} + \overline{v_z(r) \frac{\partial c'}{\partial z}} = D \frac{\partial^2 \bar{c}}{\partial z^2} \quad (14)$$

Manipulating the previous equations we arrive at:

$$\frac{\partial c'}{\partial t} + [v_z(r) - \bar{v}_z] \frac{\partial \bar{c}}{\partial z} + v_z(r) \frac{\partial c'}{\partial z} - \overline{v_z(r) \frac{\partial c'}{\partial z}} = D \left[\frac{\partial^2 c'}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c'}{\partial r} \right) \right] \quad (15)$$

For large times ($t \sim O(R^2/D)$), the radial diffusion has smoothed out the initially sharp gradients. Under this condition, $\bar{c} \gg c'$, but the radial gradients are still much larger than the axial ones, $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c'}{\partial r} \right) \gg \frac{\partial^2 c'}{\partial z^2}$. Thus, we find

$$c'(r, z, t) = c'(1, z, t) - \frac{\bar{v}_z R^2}{D} \frac{\partial \bar{c}}{\partial z} \left[\frac{(a-1)}{4} \left(1 - \left(\frac{r}{R} \right)^2 \right) - \frac{a}{(b+2)^2} \left(1 - \left(\frac{r}{R} \right)^{b+2} \right) \right] \quad (16)$$

where $a = \frac{3n+1}{n+1}$ e $b = 1 + \frac{1}{n}$.

Since $\bar{c}' = 0$, we can determine $c'(1, z, t)$ by taking the average of Eq. (16) and find $c'(r, z, t)$:

$$c'(r, z, t) = \frac{\bar{v}_z R^2}{D} \frac{\partial \bar{c}}{\partial z} \left[\frac{a}{(b+2)^2} - \frac{(a-1)}{8} - \frac{a}{(b+2)(b+4)} + \frac{(a-1)}{4} \left(\frac{r}{R}\right)^2 - \frac{a}{(b+2)^2} \left(\frac{r}{R}\right)^{b+2} \right] \quad (17)$$

Writing a and b in terms of n ,

$$c'(r, z, t) = \frac{\bar{v}_z R^2}{D} \frac{\partial \bar{c}}{\partial z} \left[\frac{1}{(1/n+1)} \left(\frac{1}{(1/n+3)} - \frac{1}{4} - \frac{1}{(1/n+5)} \right) + \frac{1}{2(1/n+1)} \left(\frac{r}{R}\right)^2 - \frac{1}{(1/n+1)(1/n+3)} \left(\frac{r}{R}\right)^{1/n+3} \right] \quad (18)$$

or, defining

$$d(n) = \frac{6}{(1/n+1)} \left(\frac{4}{(1/n+3)} - 1 - \frac{4}{(1/n+5)} \right) \quad (19)$$

$$e(n) = \frac{12}{(1/n+1)}; \quad g(n) = \frac{24}{(1/n+1)(1/n+3)}; \quad h(n) = b+2 = 1/n+3$$

$$c'(r, z, t) = \frac{\bar{v}_z R^2}{24D} \frac{\partial \bar{c}}{\partial z} \left[d + e \left(\frac{r}{R}\right)^2 - g \left(\frac{r}{R}\right)^h \right] \quad (20)$$

Differentiation with respect to z yields, we can calculate the term $\overline{v_z(r) \frac{\partial c'}{\partial z}}$ by:

$$\overline{v_z(r) \frac{\partial c'}{\partial z}} = -\frac{\bar{v}_z^2 R^2}{48D} F(n) \frac{\partial^2 \bar{c}}{\partial z^2} \quad (21)$$

where

$$F(n) = \frac{24}{(1/n+3)(1/n+5)} \quad (22)$$

Substituting Eq. (21) into Eq. (14), we get the transport equation for $\bar{c}(z, t)$:

$$\frac{\partial \bar{c}}{\partial t} + \bar{v}_z \frac{\partial \bar{c}}{\partial z} = \left(D + \frac{\bar{v}_z^2 R^2}{48D} F(n) \right) \frac{\partial^2 \bar{c}}{\partial z^2} \quad (23)$$

Let us now write the above equation in terms of a new pair of dimensionless independent variables (ζ, τ) , defined as $\zeta \equiv (z - \bar{v}_z t)/R$ and $\tau \equiv \bar{v}_z t/R$. Note that ζ is an axial coordinate measured from a reference frame traveling with the average axial velocity \bar{v}_z . We get:

$$\frac{\partial \bar{c}}{\partial \tau} = \left(\frac{D}{\bar{v}_z R} + \frac{\bar{v}_z R}{48D} F(n) \right) \frac{\partial^2 \bar{c}}{\partial \zeta^2} \quad (24)$$

Introducing the Péclet number $Pe \equiv \bar{v}_z R/D$ we get

$$\frac{\partial \bar{c}}{\partial \tau} = \left(\frac{1}{Pe} + \frac{Pe}{48} F(n) \right) \frac{\partial^2 \bar{c}}{\partial \zeta^2} \quad (25)$$

or, defining the dimensionless dispersion coefficient D^* as

$$D^* = \left(\frac{1}{Pe} + \frac{Pe}{48} F(n) \right) \quad (26)$$

we get our final form of the transport equation for \bar{c} , which should be accurate for $t > R^2 D$ or $\tau > Pe$:

$$\frac{\partial \bar{c}}{\partial \tau} = D^* \frac{\partial^2 \bar{c}}{\partial \zeta^2} \quad (27)$$

The initial and boundary conditions are:

$$\tau = 0, \quad \forall \zeta, \quad \bar{c} = \frac{M}{\pi R^3} \delta(\zeta) \quad (28)$$

$$\zeta = 0, \quad \forall \tau, \quad \frac{\partial \bar{c}}{\partial \zeta} = 0 \quad (29)$$

$$\zeta \rightarrow \infty, \quad \forall \tau, \quad \bar{c} = 0 \quad (30)$$

Note that Eq. (30) is the transient diffusion equation, which means that, in the reference frame traveling with velocity \bar{v}_z , this situation is completely analogous to a purely diffusion problem. This equation is readily solved using Laplace transforms, and the solution is

$$\bar{c} = \frac{M/\pi R^3}{\sqrt{4\pi D^* \tau}} \exp\left[-\frac{\zeta^2}{4D^* \tau}\right], \quad \text{for } \tau > Pe \quad (31)$$

3. EXPERIMENTAL METHOD

3.1 Experimental requirements for the application of the Taylor Dispersion Method

In order for the experiment to occur as foreseen, we must have a homogeneous liquid mixture of the involved species draining in a laminar flow in an infinitely long isothermal tube of a circular uniform section of radius R and with impermeable walls. Moreover, the mean velocity has to be constant and equal to \bar{v}_z ; at time $t = 0$, a sample of the liquid mixture of a different concentration of the same components of the initial fluid, must be introduced in the tube in $z = 0$ (as a δ pulse). The density and viscosity must be constant. The sample must fill the transversal section of the tube at $z = 0$. The ideal experiment is represented in Fig.8. This idealized experiment cannot be done in practice. Nevertheless, Alizadeh *et al.* (1980) showed that a practical instrument can be designed so that its departures from the ideal are small or negligible. The authors studied the influence of these departures in the results and they determined some conditions to be met to assure an accurate result. We followed their instructions to design our experiment.

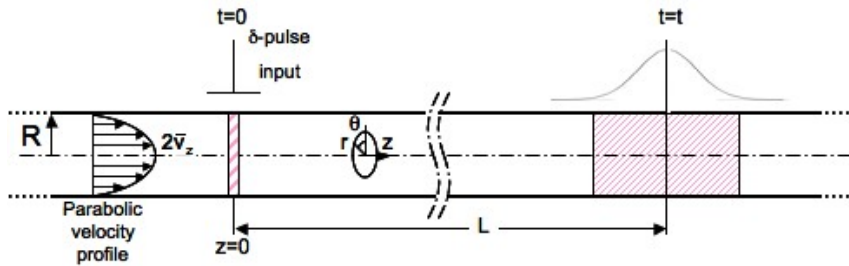


Figure 8. The ideal Taylor dispersion experiment (Alizadeh *et al.*, 1980)

To satisfy the requirements of laminar flow and to assure the correct magnitudes of axial and radial diffusion, and also to have enough time to arrive to a Gaussian form of dispersion, we should obey the three following conditions:

$$Re = \frac{2\bar{v}_z R \rho}{\eta} < 2000 \quad (32)$$

$$\frac{Dt}{R^2} \geq 10 \quad (33)$$

$$\frac{R\bar{v}_z}{D} \geq 700 \quad (34)$$

where Re is the Reynolds number. We use a very long and thin tube, in the form of a coil. To eliminate the coiling effects, conditions must be chosen so that:

$$w \leq 100 \quad (35)$$

and

$$De^2 Sc \leq 20 \quad (36)$$

where De is the Dean number, Sc the Schmidt number and w the ratio of the coil radius R_s to the tube radius R .

The coil radius R_s , the tube radius R , the length L of the tube, the flow rate (and consequently the mean velocity \bar{v}_z) were chosen to fulfill all of the above-mentioned conditions. Thus R is 0.87mm, R_s is 0.16m and L is 10m. The flow rate should be around 0.15mL/min (the pump flow rate range is 0.001 to 10.000 mL/min with a default setting of 100 μ L with a percent error no greater than $\pm 1\%$). With this dimension, each experiment should last around two and a half hours.

3.2 The experiments

The experiment is represented by the following scheme (Fig. 9).

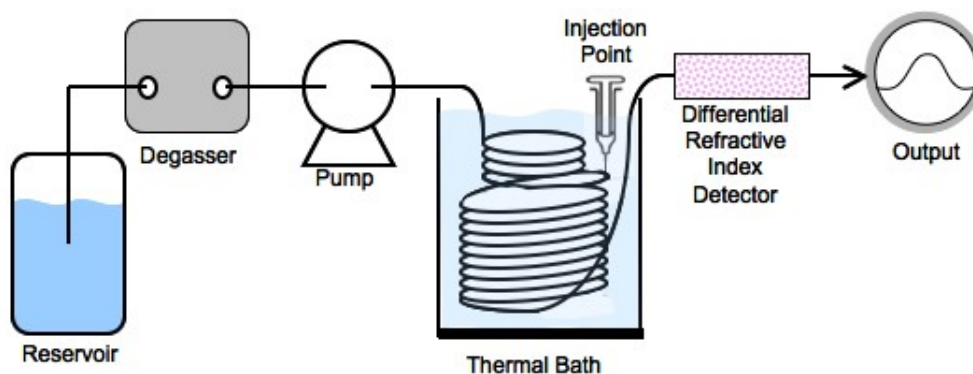


Figure 9. Schematics of the experimental apparatus

The fluid (pure solvent or solvent with a low concentration of solute) is stored in a reservoir. We should use a low concentration instead of the pure solvent in order not to have a big difference in the viscosity and density between the initial fluid and the injected fluid (with a higher concentration of solute). It's not a problem to have a small difference between both concentrations, since the sensibility of the equipment used to determine the concentration is very refined.

The fluid passes by a degasser and then it is pumped into a coil inside a thermal bath. The thermal bath assures a homogeneous temperature, which is important because the diffusion coefficient changes with temperature. In addition, the thermal bath allows us to do tests with higher temperatures than the ambient temperature. Since our experiment consists on measuring the diffusion coefficient of salts in drilling fluids, it is important to measure it at temperatures which are similar to the ones in the oil wells (around 80° C).

The sample of concentrated solution is injected into the coil. In practice it is impossible to provide a delta function pulse, but it is relatively straightforward to introduce an approximately rectangular pulse of a sample into the tube at $z = 0$, uniform across the tube. If the volume of the sample doesn't exceed 1% of the tube volume, the error associated is negligible (Alizadeh *et al.*, 1980). In our apparatus, the sample is injected by a microsyringe through a reducing union Tee. Since the maximum volume injected by the micro-syringe is 10 μ L, the sample volume is only 0,04% of the tube volume (smaller than 1%).

After the injection, the fluid will pass by a 10 meter coil inside the thermal bath. At the end of the coil a device will measure the concentration dispersion. We chose a differential refractive index detector. Refraction is a physical property of gases, liquids and solids. In solutions, the refractive index changes linearly with the concentration. A high precision optical equipment, a differential refractive index detector needs only small volumes and it is easy to use. So at $z = L$, the differential refractive index detector measures the concentration as a function of time and the appropriated software saves the data to be analyzed later. Notice that the degasser at the inlet of the apparatus is necessary to assure that there won't be bubbles in the fluid, because the refractive index can change with them (and we want it to depend on the concentration only).

3.3 Determination of the Diffusion Coefficient

In the second section, Mathematical Model, we found the Eq. (31), that gives the concentration as a function of ζ and τ (that are dimensionless variables to position z and time t). In the apparatus, the differential refractive index detector is installed at a position L , so the experimental curve gives the concentration as a function of time: $\bar{c}(t)$. Then, if we replace z for L in the definition of the variable ζ in the Eq. (31), the equation gives the concentration in a position L as a function of the time. This equation is the link between the experimental data and the diffusion coefficient D .

Observing the Eq. (31), we notice that it depends on M : the amount of injected salt. We can calculate this amount while preparing the concentrated solution, but we don't have much precision in the true value (it is in moles). So it is not desirable to depend on the value of M to obtain the diffusion coefficient. For this reason only the exponential factor will

be used to determine D (this way we can reduce the error).

Aris has shown that in the conditions chosen for the experiment, the concentration curve has the form of a normal distribution (Aris, 1956). As we know, a normal distribution has the following equation:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad (37)$$

where parameters μ and σ^2 are the mean and the variance. The similarity between Eqs. (31) and (37) is evident.

There has been a long time since the Taylor Dispersion Method has been used (its first article dates of 1953 -Taylor, 1953). Most of the authors that studied the Taylor Method used the parameters of the curve (mean and variance) to determine the coefficient D . It's a practical way to calculate D when you don't have advanced computational resources, for this reason it was the most common way. To find D by the parameters of the curve, we only have to manipulate the equation of the concentration to find the exponent as it is done in the normal distribution equation $\left(\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\right)$, and so we get:

$$\sigma^2 = \frac{2RD^*t}{v_z} = \frac{2Dt}{v_z^2} + \frac{R^2tF(n)}{24D} \quad (38)$$

where t is the time when the peak of concentration occurs (t is actually the mean of the curve). So finding the mean and the variance we have D .

Nowadays the computational resources are much more developed, so we can easily find D doing a curve fitting. In this way the result is more accurate than with the parameters method. Like explained previously, we don't have an accurate value of M , so once again we will use only the exponent factor to determine D .

We intend to use the values of D obtained by the curve fitting method, but we will also calculate it by the parameters of the curve, to be able to compare both results.

4. FINAL REMARKS

Currently we are in the phase of assembling the experiment. When it becomes operational, we will initiate the tests using known systems, such as NaCl in water. Since we know the diffusion coefficient of well known systems, we will be able to verify the feasibility of our experiment and then to validate the experimental apparatus. Then the experiment will be ready to be used to measure the diffusion coefficient of different salts in different drilling fluids at a wide range of temperatures.

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