PIPELINE HEATING METHOD BASED ON OPTIMAL CONTROL AND STATE ESTIMATION

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Abstract. *In production of oil and gas wells in deepwaters the flowing of hydrocarbon through pipeline is a challenging problem. This environment presents high hydrostatic pressures and low sea bed temperatures, which can favor the formation of solid deposits that in critical operating conditions, as unplanned shutdown conditions, may result in a pipeline blockage and consequently incur in large financial losses. There are different methods to protect the system, but nowadays thermal insulation and chemical injection are the standard solutions normally used. An alternative method of flow assurance is to heat the pipeline. This concept, which is known as active heating system, aims at heating the produced fluid temperature above a safe reference level in order to avoid the formation of solid deposits. The objective of this paper is to introduce a Bayesian statistical approach for the state estimation problem, in which the state variables are considered as the transient temperatures within a pipeline cross-section, and to use the optimal control theory as a design tool for a typical heating system during a simulated shutdown condition. An application example is presented to illustrate how Bayesian filters can be used to reconstruct the temperature field from temperature measurements supposedly available on the external surface of the pipeline. The temperatures predicted with the Bayesian filter are then utilized in a control approach for a heating system used to maintain the temperature within the pipeline above the critical temperature of formation of solid deposits. The physical problem consists of a pipeline cross section represented by a circular domain with four points over the pipe wall representing heating cables. The fluid is considered stagnant, homogeneous, isotropic and with constant thermo-physical properties. The mathematical formulation governing the direct problem was solved with the finite volume method and for the solution of the state estimation problem considered here, we used the Particle Filter. The optimal control was based on a linear quadratic controller and an associated quadratic cost functional, which was minimized through the solution of Riccati's equation.*

Keywords*: Flow Assurance, Pipeline Heating System, State Estimation Problem, Particle Filter, Optimal Control*

1. INTRODUCTION

The key ingredient to the success of flow assurance operations is the subsea thermal management. In most cases, thermal management determines the requirements to choose the best design in order to maintain the fluid temperature in the interior of the pipelines and in subsea production equipments above a minimum temperature. Thus, in deepwater fields the flowing of hydrocarbon through subsea pipelines is a challenging problem. This environment presents high hydrostatic pressures and low sea bed temperatures, which can favor the formation of solid deposits that in critical operating conditions, such as unplanned shutdown conditions, may result in a pipeline blockage and consequently incur in large financial losses (Jamaluddin *et al.*, 1991, Su and Cerqueira, 2001, Su, 2003). Figure 1 illustrates a typical hydrate blockage inside a production pipeline.

Thermal management includes both steady-state and transient studies for the different stages of the field's lifetime and must serve as a design tool for the selection of methods to avoid the formation of solid deposits. In steady state operations, the production fluid temperature decreases as it flows along the pipeline due to heat transfer through the pipe walls. This steady state temperature profile from the produced fluid is used to identify the flow rates and insulation systems that are needed to keep the system above the critical temperature during production. If at some moment the steady state flow conditions are interrupted, such as in shut-down conditions, a transient heat transfer analysis for the subsea system is necessary to ensure that the temperature of the fluid be above that of formation solid deposits. The main solid deposits formed inside subsea pipelines are wax and hydrates. For a given fluid, these solids deposits are formed at certain combinations of pressure and temperature. Wax deposits typically appear in temperatures ranging from 30 to 50^o C. Hydrate formation temperatures on the other hand, are typically around 20^o C at 100 bar (Su, 2003).

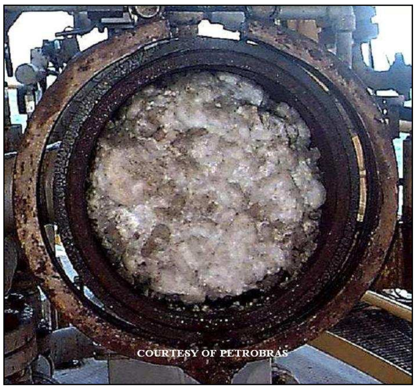


Figure 1 – Hydrate blockage in pipeline

There are different methods to protect the system and techniques to avoid and/or minimizing the formation of these solid deposits, which have been supported by an intensive research and field experience. The basic current strategies to avoid these problems are thermal insulation and chemical injection, but an alternative method is to heat the pipeline. This concept, which is known as active heating, aims at heating the produced fluid temperature above a safe reference level in order to avoid the formation of solid deposits.

The pipeline can be heated by several methods, but typical concepts are based on the so-called direct electrical heating system (DEH) (Hansen and Clasen, 1999) and indirect electrical heating system (IEH) (Denniel and Laouir, 2001). In the direct electrical heating system, electric current flows axially through the pipe wall causing Joule heating. On the other hand, in the indirect electrical heating system, the electric current flows through heating elements (e.g., one or more electrical cables) on the pipe surface.

The objective of this paper is to introduce a Bayesian statistical approach for the state estimation problem, in which the state variables are considered as the transient temperatures within a pipeline cross-section, and to use the optimal control theory as a design tool for a typical heating system during a simulated shutdown condition. Thus, an application example is presented to illustrate how Bayesian filters can be used to reconstruct the temperature field from temperature measurements supposedly available on the external surface of the pipeline. The temperatures predicted with the Bayesian filter is then utilized in a control approach for a heating system used to maintain the temperature within the pipeline above the critical temperature of formation of solid deposits. The physical problem consists of a pipeline cross section represented by a circular domain with four heating cables. The fluid is considered stagnant, homogeneous, isotropic and with constant thermo-physical properties. The optimal control was based on a linear quadratic controller and an associated quadratic cost functional was minimized through the solution of Riccati's equation.

2. STATE ESTIMATION PROBLEM

In state estimation problems (Maybeck, 1979, Kaipio and Somersalo, 2004, Scott and McCann, 2005, Orlande *et al*., 2008) observations obtained during the evolution of the system, are used together with prior knowledge about the physical phenomena and the measuring devices, in order to sequentially produce estimates of the desired dynamic variables. State estimation problems can be solved with the so-called Bayesian filters (Maybeck, 1979, Kaipio and Somersalo, 2004, Scott and McCann, 2005, Orlande *et al*., 2008).

In order to define the state estimation problem, consider a model the evolution of the state variables $\boldsymbol{x}$ in the form:

$$\boldsymbol{x}_k = \boldsymbol{f}_{k-1}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}, \boldsymbol{v}_{k-1}) \tag{1}$$

where $\boldsymbol{f}$ is, in the general case, a non-linear function of $\boldsymbol{x}$, of the control input to the system $\boldsymbol{u}$ and of the state noise or uncertainty vector given by $\boldsymbol{v} \in \boldsymbol{R}^{\boldsymbol{n}_v}$.

The vector $\boldsymbol{x}_k \in \boldsymbol{R}^{\boldsymbol{n}_x}$ is called the state vector and contains the variables to be dynamically estimated. This vector advances in time in accordance with the *state evolution model* (1). The subscript k =1, 2, 3, …, denotes a time instant t_k in a dynamic problem.

The observation model describes the dependence between the state variable $\boldsymbol{x}$ to be estimated and the measurements $\boldsymbol{z}$ through the general, possibly non-linear, function $\boldsymbol{h}$. This can be represented by

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{2}$$

where $\boldsymbol{z}_k \in \boldsymbol{R}^{\boldsymbol{n}_z}$ are available at times t_k, k=1, 2, 3,…. Eq. (2) is referred to as the *observation/measurement model*. The vector $\boldsymbol{n}_k \in \boldsymbol{R}^{\boldsymbol{n}_n}$ represents the measurement noise or uncertainty.

The *evolution and observation models*, given by Eqs. (1) and (2), respectively, are based on the following assumptions (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(a) The sequence $\boldsymbol{x}_k$ for k=1, 2, 3, …, is a Markovian process, that is,

$$\pi\left(\mathbf{x}_k \,|\, \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}\right) = \pi\left(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}\right) \tag{3.a}$$

(b) The sequence $\boldsymbol{z}_k$ for k=1, 2, 3, …, is a Markovian process with respect to the history of $\boldsymbol{x}_k$, that is,

$$\pi\left(\mathbf{z}_k \,|\, \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_k\right) = \pi\left(\mathbf{z}_k \,|\, \mathbf{x}_k\right) \tag{3.b}$$

(c) The sequence $\boldsymbol{x}_k$ depends on the past observations only through its own history, that is,

$$\pi\left(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{k-1}\right) = \pi\left(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}\right) \tag{3.c}$$

where $\pi\left(\mathbf{a}|\mathbf{b}\right)$ denotes the conditional probability of **a** when **b** is given.

For the state and observation noises, the following assumptions are made (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(a) For $i \neq j$, the noise vectors $\mathbf{v}_i$ and $\mathbf{v}_j$, as well as $\mathbf{n}_i$ and $\mathbf{n}_j$, are mutually independent and also mutually independent of the initial state $\boldsymbol{x}_0$.

(b) The noise vectors $\mathbf{v}_i$ and $\mathbf{n}_j$ are mutually independent for all i and j.

Different problems can be considered for the evolution-observation model described above, such as (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(i) The prediction problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \,|\, \mathbf{z}_{1:k-1}\right)$;

(ii) The filtering problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \,|\, \mathbf{z}_{1:k}\right)$;

(iii) The fixed-lag smoothing problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \,|\, \mathbf{z}_{1:k+p}\right)$, where $p \geq 1$ is the fixed lag.

(iv) The whole-domain smoothing problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \,|\, \mathbf{z}_{1:K}\right)$, where $\mathbf{z}_{1:K} = \left\{\mathbf{z}_i, i = 1, \cdots, K\right\}$ is the complete set of measurements.

3. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The idealized problem in this work considers a critical operational condition involving the cooling of the pipeline. The physical problem consists of a pipeline cross section represented by a circular domain filled with a stagnant fluid and bounded by a constant thickness pipe wall with four points over the external surface representing the heating cables (Denniel and Laouir, 2001, Denniel *et al.*, 2004). The fluid is considered as homogeneous, isotropic and with constant thermal properties. The idealized pipeline will be treated here with a transient heat conduction problem in a single medium, thus not taking into account the pipe wall. Figure 2 illustrates the hypothetic pipeline heating system applied on this work, where the indirect electrical heating system described above is assumed in this analysis. The heat flow rate resulting from Joule's effect is considered in the form of a transient heat flux appearing in the boundary condition of the fluid domain. Thus, for representing this physical problem it was proposed a simplified model involving two-dimensional transient heat conduction.

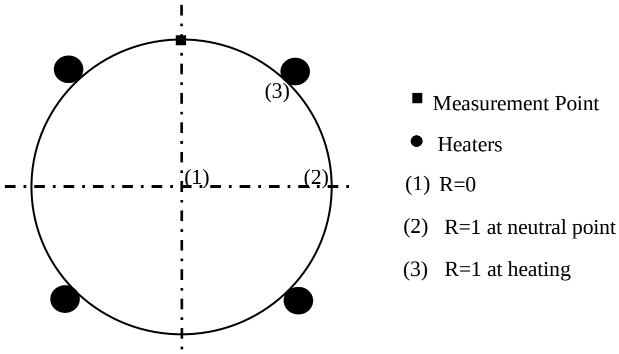


Figure 2 – Hypothetic heating system on a pipeline cross section

The dimensionless mathematical formulation for this problem in cylindrical coordinates is given by

$$\frac{\partial \theta(R,\emptyset,\tau)}{\partial \tau} = \frac{\partial^2 \theta(R,\emptyset,\tau)}{\partial R^2} + \frac{1}{R}\frac{\partial \theta(R,\emptyset,\tau)}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \theta(R,\emptyset,\tau)}{\partial \emptyset^2} \qquad 0 \le R < 1\,,\ 0 \le \emptyset < 2\pi,\ \tau > 0 \qquad (4.a)$$

where $\theta(R,\emptyset,\tau)$ is the dimensionless temperature distribution into the pipeline. This equation was solved subjected to the following boundary and initial conditions:

$$\frac{\partial \theta(R,\emptyset,\tau)}{\partial R} + Bi\,\theta(R,\emptyset,\tau) = Q(\emptyset,\tau) \qquad R = 1\,,\ 0 \le \emptyset < 2\pi \quad \tau > 0 \qquad (4.b)$$

$$\theta(R,\emptyset,\tau) = 1 \qquad 0 \le R < 1\,,\ 0 \le \emptyset < 2\pi,\ \tau = 0 \qquad (4.c)$$

where the following dimensionless groups were defined:

$$\theta(R,\emptyset,\tau) = \frac{T(r,\phi,t) - T_\infty}{T(r,\phi,0) - T_\infty} \qquad (5.a)$$

$$\tau = \frac{\alpha t}{r^{*2}} \qquad (5.b)$$

$$R = \frac{r}{r^*} \qquad (5.c)$$

$$Bi = \frac{h\,r^*}{k} \qquad (5.d)$$

$$Q = \frac{q(\emptyset,\tau)\,r^*}{k\,(T_0 - T_\infty)} \qquad (5.e)$$

Here, T_∞ is the surrounding environment temperature, h is the convective heat transfer coefficient, k and α are the fluid thermal conductivity and diffusivity, respectively, r^* is the external radius, *Bi* is the *Biot* number and $q(\emptyset,\tau)$ is the heat flux imposed on the external surface resulting from the heating cable.

The mathematical formulation governing the heat conduction problem given by eqs. (4.a-c), was solved with the finite-volume method. The computer code developed for this purpose was verified by using an analytical solution obtained with the Classical Integral Transform Technique.

4. OPTIMAL CONTROL BASED ON PARTICLE FILTER OBSERVER

The state space representation of a dynamical system consists of the specification of the evolution model for the state variables and observation model that links the observations/measurements to the state variables. Thus, for the classical linear time-invariant discrete state estimation problem, the evolution model may be written in the form

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1} + \boldsymbol{v}_{k-1} \tag{4}$$

where $\boldsymbol{F}$ is the linear evolution matrix of the state variables $\boldsymbol{x}_{k-1}$ and $\boldsymbol{G}$ is the input matrix. The state uncertainty or noise $\boldsymbol{v}_{k-1}$ is assumed to be a Gaussian random variable with zero mean and known covariance Ω_v.

The linear observation equation is given in the form

$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{n}_k \tag{5}$$

where $\boldsymbol{z}_k$ is the measurement vector and $\boldsymbol{H}$ is the linear observation matrix. The observation noise $\boldsymbol{n}_k$ is assumed to be a Gaussian random variable with zero-mean and known covariance Ω_n. The state and observation noises are assumed to be mutually independent.

In the application under study, the evolution model is given by the finite-volume representation of eqs. (4.a-c). The state vector $\boldsymbol{x}_k$ contains the values of the temperatures at each of the volumes and the control variable $\boldsymbol{u}$ is given by the heat flux imposed on the external boundary.

Uncertainties in the evolution model come from the fact that different quantities in the formulation are not exactly known, such as the *Biot* number.

The main objective of the pipeline heating system is to keep the fluid temperature above the critical temperature. Such critical temperature is approached by the fluid during cooling periods. Thus, for the application of the control strategy in accordance with the optimal control theory for linear problems, the case under analysis in this work is to find the control inputs $\boldsymbol{u}$ (the boundary heat flux) that minimizes the difference between the fluid temperature field and a desired profile $\boldsymbol{r}$.

For the implementation of the control strategy we consider (Scott and McCann, 2005):

$$\bar{\boldsymbol{u}}_{\boldsymbol{k}} = \boldsymbol{u}_{\boldsymbol{k}}^* - \boldsymbol{u}_{\boldsymbol{d}} \tag{6.a}$$

$$\bar{\boldsymbol{x}}_{\boldsymbol{k}} = \boldsymbol{x}_{\boldsymbol{k}}^* - \boldsymbol{x}_{\boldsymbol{d}} \tag{6.b}$$

where $\boldsymbol{u}_{\boldsymbol{d}}$ and $\boldsymbol{x}_{\boldsymbol{d}}$ refer to the steady values of the control input and state variables, respectively. Hence, $\bar{\boldsymbol{u}}_{\boldsymbol{k}}$ and $\bar{\boldsymbol{x}}_{\boldsymbol{k}}$ are considered as deviations from their steady state values.

In terms of the linear quadratic regulator problem, the optimal values of the control input $\bar{\boldsymbol{u}}_{\boldsymbol{k}}$ are obtained by minimizing the following quadratic cost functional (Scott and McCann, 2005),

$$J = \lim_{t_k \to \infty} \frac{1}{t_k} \sum_{t=0}^{t_k} \left[(\bar{\boldsymbol{x}}_{\boldsymbol{k}})^T \boldsymbol{Q} (\bar{\boldsymbol{x}}_{\boldsymbol{k}}) + \bar{\boldsymbol{u}}_{\boldsymbol{k}}{}^T \boldsymbol{R} \bar{\boldsymbol{u}}_{\boldsymbol{k}} \right] \tag{7}$$

where the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are symmetric positive definite.

The solution to the optimal control problem is the state feedback control law [6, 11]

$$\bar{\boldsymbol{u}}_{\boldsymbol{k}} = -\mathrm{K}\bar{\boldsymbol{x}}_{\boldsymbol{k}} \tag{8}$$

where the discrete-time state feedback gain K is of the form

$$\mathrm{K} = (\mathrm{R} + \mathrm{G}^{\mathrm{T}}\,\mathrm{S}\,\mathrm{G})^{-1}\,\mathrm{G}^{\mathrm{T}}\,\mathrm{S}\,\mathrm{F} \tag{9}$$

The matrix S is the steady state solution to the discrete-time Riccati equation

$$\mathrm{F^TSF - S + Q - F^TSG(R + G^TSG)^{-1}G^TSF} = 0 \tag{10}$$

Thus, the control input $\boldsymbol{u}_{\boldsymbol{k}}^*$ can be calculated from the control law (8) as:

$$\boldsymbol{u}_{\boldsymbol{k}}^* = \boldsymbol{u}_{\boldsymbol{d}} - \boldsymbol{K}(\boldsymbol{x}_{\boldsymbol{k}}^* - \boldsymbol{x}_{\boldsymbol{d}}) \tag{11}$$

However, when state variables are not directly available for control, an observer must be built to estimate the state variables from the input and output variables of the system. For this case, the solution of the state estimation problem considered here, which involves the estimation of the transient temperature field in the medium from temperature measurements taken at the surface of the pipe (see figure 2), is obtained with the Particle Filter method (Arulampalam *et al.*, 2001, Andrieu *et al.*, 2004, Scott and McCann, 2005).

The particle filter is a Monte Carlo technique used for the solution of state estimation problems, where the main idea is to represent the required posterior density function by a set of random samples with associated weights and to compute the estimates based on these samples and weights. Let $\{\boldsymbol{x}_{0:k}^{i},\ i=0,\ldots,I\}$ be the particles with associated weights $\{\boldsymbol{w}_{k}^{i},\ i=0,\ldots,I\}$ and $\boldsymbol{x}_{0:k}=\{\boldsymbol{x}_{j},\ j=0,\ldots,k\}$ be the set of all states up to t_k, where I is the number of particles. The weights are normalized, so that $\sum_{i=1}^{I} w_k^i = 1$. Then, the posterior density at t_k can be discretely approximated by:

$$\pi(\mathbf{x}_{0:k}|\mathbf{z}_{1:k-1}) \approx \sum_{i=1}^{I} w_k^i\, \delta(\mathbf{x}_{0:k}-\mathbf{x}^{i}{}_{0:k}) \tag{12}$$

where $\delta(.)$ is the Dirac delta function. By taking hypotheses (3.a-c) into account, the posterior density in Eq. (12) can be written as $\pi(\boldsymbol{x}_{k}|\mathbf{z}_{1:k-1}) \approx \sum_{i=1}^{I} w_k^i\, \delta(\boldsymbol{x}_{k}-\boldsymbol{x}^{i}{}_{k})$.

A common problem with the Particle Filter method is the degeneracy phenomenon, where after a few states all but one particle may have negligible weight. The degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation of the posterior density function is almost zero. This problem can be overcome by increasing the number of particles, or more efficiently by appropriately selecting the importance density as the prior density $\pi(\boldsymbol{x}_{k}|\boldsymbol{x}^{i}{}_{k-1})$. In addition, the use of the resampling technique is recommended to avoid the degeneracy of the particles (Arulampalam *et al.*, 2001, Andrieu *et al.*, 2004, Scott and McCann, 2005).

Resampling involves a mapping of the random measure $\{\boldsymbol{x}^{i}{}_{k},\ w_k^i\}$ into a random measure $\{\boldsymbol{x}^{i}{}_{k},\ N^{-1}\}$ with uniform weights. It can be performed if the number of effective particles with large weights falls below a certain threshold number. Alternatively, resampling can also be applied indistinctively at every instant t_k, as in the Sampling Importance Resampling (SIR) algorithm used here (Arulampalam *et al.*, 2001, Andrieu *et al.*, 2004, Scott and McCann, 2005). This algorithm can be summarized in the steps presented in Table 1, as applied to the system evolution from t_{k-1} to t_k.

Table 1 – Sampling Importance Resampling Algorithm

Step 1
For $i=1,\cdots,N$ draw new particles $\mathbf{x}_k^i$ from the prior density $\pi\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}^i\right)$ and then use the likelihood density to calculate the correspondent weights $w_k^i=\pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)$.
Step 2
Calculate the total weight $T_w=\sum_{i=1}^{N} w_k^i$ and then normalize the particle weights, that is, for $i=1,\cdots,N$ let $w_k^i=T_w^{-1}\, w_k^i$
Step 3
Resample the particles as follows : Construct the cumulative sum of weights (CSW) by computing $c_i=c_{i-1}+w_k^i$ for $i=1,\cdots,N$, with $c_0=0$. Let $i=1$ and draw a starting point u_1 from the uniform distribution $U\left[0,N^{-1}\right]$ For $j=1,\cdots,N$ Move along the CSW by making $u_j=u_1+N^{-1}\left(j-1\right)$ While $u_j>c_i$ make $i=i+1$. Assign sample $x_k^j=x_k^i$ Assign sample $w_k^j=N^{-1}$

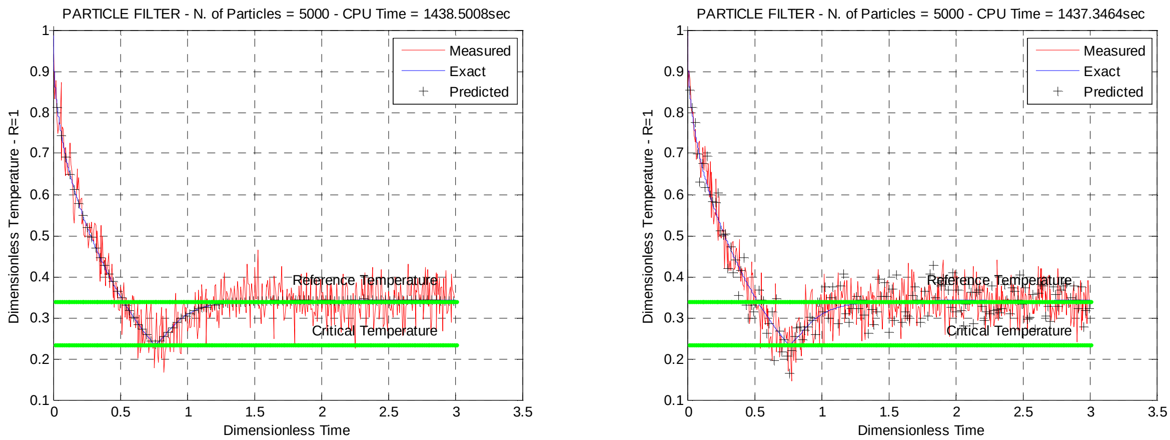
Although the resampling step reduces the effects of the degeneracy problem, it may lead to a loss of diversity and the resultant sample can contain many repeated particles. This problem, known as sample impoverishment, can be severe in the case of small evolution model noise. In this case, all particles collapse to a single particle within few instants. Another drawback of the particle filter is related to the large computational cost due to the Monte Carlo method, which may limit its application only to fast computing problems.

5. RESULTS AND DISCUSSION

In order to examine a test case involving typical conditions resulting from a shut-down of the flow through the pipeline, a hypothetical situation was simulated where the stagnant fluid was assumed to be initially at the uniform temperature of 80°C in a circular domain with external diameter of 0.1682 m (6”). The surrounding temperature was considered of T_∞ = 4°C. The thermophysical properties were assumed constant and given by k = 12.54 W m^{-1} °C^{-1}, ρ = 933.59 kg m^{-3} and c_p = 1826.80 J kg^{-1} °C^{-1}. The objective of the heating system was to drive the stagnant fluid temperature to a reference value of 30°C. The heating system was turned on when the lowest predicted temperature in the domain reached the critical value of formation of solid deposits, which was assumed to be 20°C. For the results presented below, the *Biot* number was taken as 1.

For the prediction of the state variables, one single sensor was considered available, located at the surface of the circular domain (see figure 2). The simulated measurements contain additive, uncorrelated, Gaussian errors, with zero mean and a constant standard deviation of 3°C. It corresponds to 3.75% of the maximum temperature in the region, that is, the initial temperature of the stagnant fluid (80°C). Errors in the evolution model are also supposed to be additive, Gaussian, uncorrelated, with zero mean and constant standard deviation. The effects of the errors in the evolution model, on the prediction of the temperature field in the region, are examined below by considering two different standard deviations for such errors, namely 0.1°C and 3°C. For the results presented below, 5000 particles were used in the Particle Filter method. Numerical experiments revealed that such number of particles would be sufficient to represent the posterior distribution of the predicted states.

Figures 3.a,b present the simulated measured temperatures, both during the cooling and heating periods, for standard deviations in the evolution model of 0.1°C and 3°C, respectively.



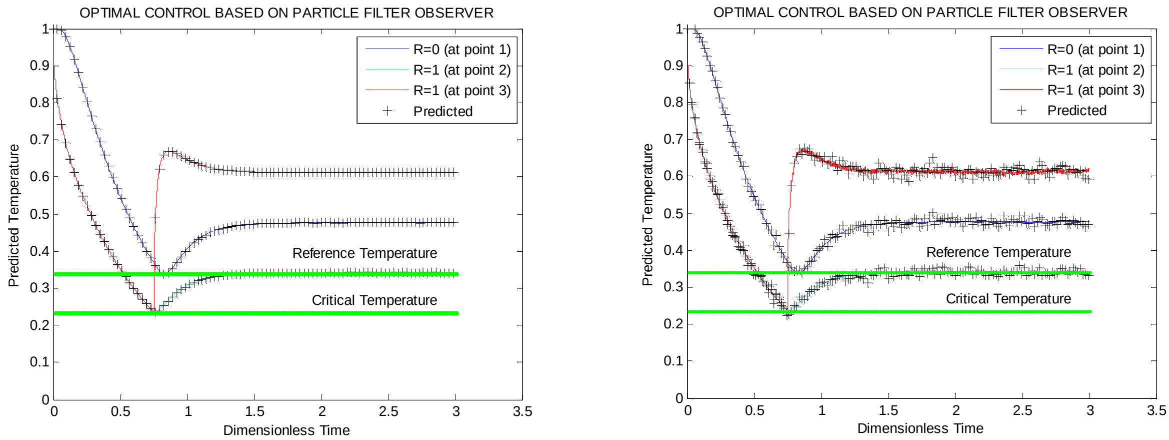
(a) (b)

Figure 3 – Exact normalized temperatures, simulated measurements and predicted normalized temperatures for a standard deviation in the evolution model errors: (a) 0.1°C and (b) 3°C

We now present the results obtained for the state estimation problem and optimal control under analysis, by using simulated experiments. For test case 1 the simulated transient measured temperatures contain Gaussian errors with standard deviation of 3°C and the standard deviation in the evolution model error is of 0.1°C. For test case 2 the same Gaussian errors are used for the measured temperatures, but with standard deviation in the evolution model error of 3°C. For both cases, we compare the exact temperature (obtained with the numerical solution with finite volumes) and predicted temperatures at three positions at R = 0 and R = 1.0 (see figure 2).

The temperatures predicted by the particle filter (implemented in accordance with the SIR algorithm) in the whole domain were used in the control strategy described above. The control strategy was applied with the weighting matrices **Q** = **R** = **I** (identity matrix). Figure 4 shows the time evolution of the predicted temperatures at three positions in the domain (R = 0 and R = 1), for test cases 1 and 2, respectively. One can clearly see that the heating is turned on when the lowest temperature in the domain (at R = 1) reaches the critical value.

It is possible to notice in figure 4 different temperature levels inside the domain, where the aim of heating was to drive the lowest fluid temperature to the reference temperature. Then, the temperature at point 2 gradually approaches the reference value through the action of the control system on the boundary heat flux. It is also observed that the temperature at point 3 (see figure 2) reaches larger values due to the heater position.

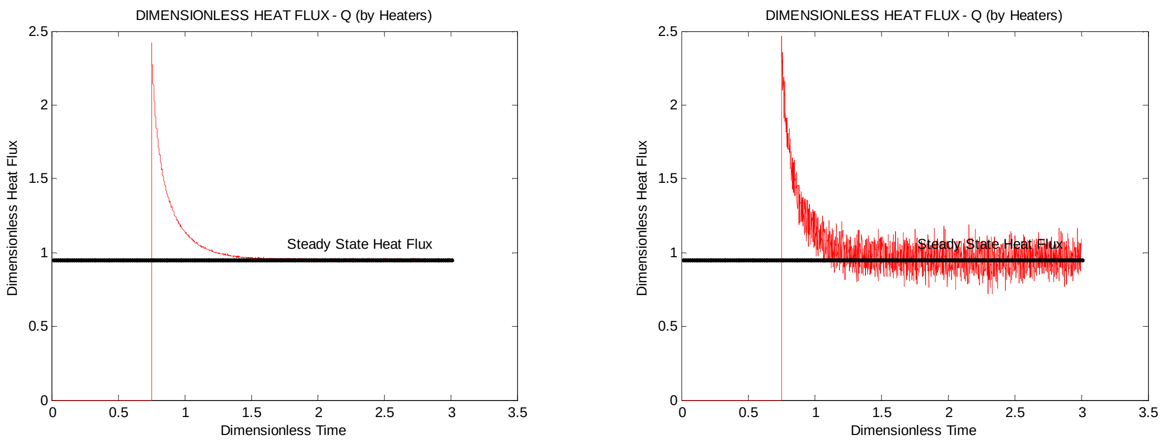


(a) (b)

Figure 4 – Evolution of the predicted temperatures compared with exact ones under the action of the optimal control with a standard deviation in the evolution model errors of: (a) 0.1^{o}C and (b) 3^{o}C.

Figures 4.a-b clearly reveal an excellent agreement between exact and predicted temperatures, even for the large standard deviation of the evolution errors of 3^{o}C.

The optimal heat flux obtained through the control strategy described above is presented in figures 5.a,b, for test-cases 1 and 2, respectively. This figure shows that the heat flux attains large values when the heating is turned on, but gradually tends to a constant value that provides the required minimum temperature in the medium within the time range of interest.



(a) (b)

Figure 5 – Optimal heat flux on the boundary surface under the action of the optimal control with a standard deviation in the evolution model errors of: (a) 0.1^{o}C and (b) 3^{o}C

A comparison of figures 3 and 4 shows the effect of the Particle filter on the temperature at the position R = 1. It is also important to note that a completely erratic heat flux would be obtained if the measurements shown in figure 2 were directly used in the control approach.

7. CONCLUSIONS

The objective of this paper was to apply an optimal control strategy to a heating system, in order to avoid the formation of solid deposits in pipelines. The optimal control input was determined with a linear quadratic regulator, where a quadratic cost functional was minimized through the solution Riccati's equation. Predicted temperatures in the whole domain, obtained with the Particle filter, were used in the control strategy instead of the direct measurements. The Particle filter was capable of providing accurate estimates for the temperature field in the region, even for large errors in the observation model. With the present approach, the control strategy could be effectively applied and the temperature in the region was maintained above the critical one during the time range of interest.

8. ACKNOWLEDGEMENTS

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