# A SIMPLE CORRELATION, INCLUDING SIDE WALL EFFECTS, FOR PRESSURE DROP IN PACKED BEDS OF MONOSIZED SPHERES

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***Abstract.*** *Based on pressure drop measurements for water flowing across packed beds of monosized spheres, an empirical correlation was developed to predict the pressure gradient. The developed equation accounts for the influence of the ratio between the column and particle diameters, being valid for* $3 < D/d_p < 17$ *and for particle Reynolds numbers in the range of* $3 < Re_p < 379$. *The mean deviation obtained between the experimental and the calculated pressure drop, was 9.8%.*

***Keywords***: *packed beds of spheres, pressure drop, wall effect*

## 1. INTRODUCTION

One of the problems when working with packed beds is the pressure drop of the fluid across the bed. A large pressure drop means that high operating costs will be encountered. In addition, pressure drop affects the heat and mass transfer processes occurring in the system. Many studies have been performed to evaluate, either experimentally or theoretically, the pressure drop of the fluid in packed beds. Packed beds have a wide range of industrial applications, including absorption towers, catalytic fixed bed reactors, water filters and grain dryers. The pressure drop of the fluids across packed beds has been the subject of much study for several decades. These investigations revealed that pressure drop is dependent on the fluid velocity, the physical properties of the fluid (viscosity and density), the average bed porosity, the shape and the surface of the particles, the influence of the ratio between the container and the particle diameters $\left(D/d_p\right)$, and the bed height (Ergun, 1952; Foumeny et al., 1993; Eisfeld and Schnitzlein, 2001; Rangel et al., 2001; Di Felice and Gibilaro, 2004; Montillet et al.; 2006).

Based on the equations of Blake-Kozeny (for viscous flow) and Burke-Plumber (for turbulent flow), Ergun (1952) proposed one of the most accepted correlations for calculating pressure drop in packed beds:

$$\Delta P = 150\ \mu \frac{(1-\varepsilon)^2 L}{\varepsilon^3 d_p^2} v_s + 1{,}75 \frac{(1-\varepsilon) L \rho}{\varepsilon^3 d_p} v_s^2 \tag{1}$$

where $d_p$ is the effective diameter of the particles, $L$ is the height of the bed, $v_s$ is the superficial velocity of the fluid, $\varepsilon$ is the average porosity of the bed, and $\mu$ and $\rho$ are the viscosity and the density of the fluid respectively. Equation (1) was developed based on a data bank of 640 experiments involving systems of various sizes of spheres, sand, pulverized coke and the following gases: $CO_2$, $N_2$, $CH_4$ and $H_2$. However, Ergun's equation does not account for the influence of the side wall, since the data used was obtained for conditions where $D/d_p > 10$, which is valid only for infinite beds.

According to Eisfeld and Schnitzlein (2001), the flow regime for a fluid through a packed bed is dependent on the particle Reynolds number, $Re_p$, defined as:

$$\mathrm{Re}_p = \frac{\rho v_s d_p}{\mu} \tag{2}$$

which can be subdivided into: laminar $\left(Re_p < 10\right)$, transitional $\left(10 \le Re_p \le 300\right)$ and turbulent $\left(Re_p > 300\right)$.

The influence of the ratio $D/d_p$ on pressure drop (side wall effect) has been studied by several authors (Eisfeld and Schnitzlein, 2001; Di Felice and Gilibaro, 2004; Montillet et al., 2007; among others). Eisfeld and Schnitzlein (2001) analysed an extensive experimental pressure drop data bank consisting of 2300 points taken from the literature. They concluded that the existence of the side wall can introduce an additional resistance due to the wall friction. On the other hand, the presence of the wall causes the particles to be ordered in such a way that a region of higher void fraction is formed. This region extends approximately half particle diameter from the wall into the bed. These authors also studied the performance of several existing correlations by fitting experimental data points from the data bank.

Di Felice and Gibilaro (2004) presented a model to calculate pressure drop in beds of spherical particles that accounts for the side wall effect, for cases where $D/d_p$ is down to about 5. Their model considers that the fluid flows in parallel through two regions: the bulk zone and the wall region, so the observed pressure drop is $\Delta P = \Delta P_b = \Delta P_w$ (pressure drop in the bulk zone, $\Delta P_b$, equals pressure drop in the wall zone, $\Delta P_w$). These authors showed that their model accounts for the effect of $D/d_p$ on pressure drop already discussed by Eisfeld and Schnitzlein (2001): an increase in pressure drop (over that estimated by Ergun's equation), with a decrease of $D/d_p$ at the low Reynolds numbers, and a decrease in pressure drop with a decrease of $D/d_p$ at high Reynolds numbers.

Montillet et al. (2006) presented a correlation for calculating pressure drop for packed beds of spheres, based on data obtained for water and aqueous solutions of glycerol, for $3.8 < D/d_p < 40 - 50$ and $10 < Re_p < 2500$. The equation proposed is valid for loose and dense packings.

The present investigation is concerned with the development of an empirical correlation to predict the pressure gradient $(\Delta p/L)$ in packed beds of monosized spheres, and which also takes into account the side wall effect. The correlation was based on a data bank of 454 experimental points and is valid for $3 < Re_p < 379$ and $3 < D/d_p < 17$.

## 2. EXPERIMENTAL PART

The schematic diagram of the experimental set-up to measure pressure drop in packed beds is shown in Fig. 1. The experiments were performed in a 32 mm ID Perspex column with a height of 760 mm, where the liquid entered at the bottom of the column (upward flow). The experiments were carried out with glass beads with nominal sizes of 2, 4, 6 and 10 mm. The characterization of the particles in terms of their size and density, as well as the determination of the packed bed's porosity, is reported in Ribeiro et al. (2010).

The pressure drop was measured in a central part of the column, with the pressure tappings being 0.501 m apart. The height of the bed used was sufficiently large to prevent any influence of the bed height on the pressure drop measurements. One of the pressure tappings was located 0.04 m from the base of the bed and the other at 0.2 m below the top of the bed, minimizing end effects. The column was packed with the desired spheres and subsequently tapped, resulting in a random dense packing, as described by Klerk (2003). Pressure drop was measured using a U tube manometer provided with a scale whose precision was ±0.5 mm. The liquid was stored in a 5 L plastic container and it circulated through the system by means of a centrifugal pump. The flow rate of the liquid was determined by collecting a given mass of the liquid after it flowed through the column and by taking note of the collecting time.

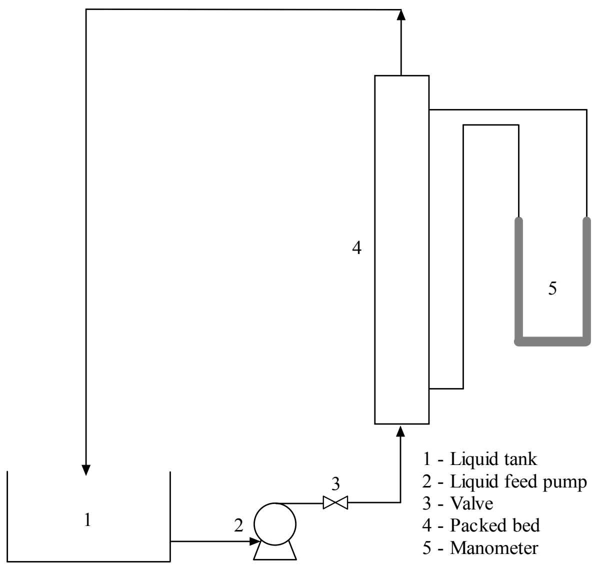


Figure 1 – Schematic diagram of the experimental apparatus.

Pressure drop measurements were performed at ambient temperature, with water being used as the flowing fluid in the packed bed. Six different runs were carried out for each particle size and for each run the column was emptied and refilled. Pressure drop was measured using carbon tetrachloride as the manometric fluid.

The water flow rate was varied in the range from $9.3 \times 10^{-4}$ to $3.0 \times 10^{-2}$ kg/s for all of the experiments. These flow rates were conditioned by the capacity of the pump, the height of the U tube manometer, the manometric fluid properties and the bed particle size.

## 3. RESULTS AND DISCUSSION

Pressure drop measurements were conducted for water upflow through packed beds of monosized spheres with nominal sizes of 2, 4, 6 and 10 mm, in a circular column 32 mm ID. The experimental conditions under study are summarized in Tab. 1. The bed height, $L$, is not expected to influence the pressure drop measurements because the values of $L/d_p$ used were sufficiently large for all experiments. Most of the experimental data falls into the laminar and the transitional flow regimes.

Details about the experimental data can be found in Ribeiro et al. (2010). From the conditions under study, it was observed that the pressure gradient increased both with the particle Reynolds number and with the ratio between the bed and the particle diameters, $D/d_p$ .

Table 1 – Experimental conditions used in the pressure drop measurements with water flowing through the packed beds

| $d_p$ (mm) | $D/d_p$ | Bed porosity | $L/d_p$ | $Re_p$ | Number of experimental points |
|---|---|---|---|---|---|
| 1.92 | 16.5 | 0,373 | 396 | $3 < Re_p < 28$ | 128 |
| 3.90 | 8.09 | 0,374 | 195 | $25 < Re_p < 126$ | 134 |
| 5.85 | 5.40 | 0,384 | 130 | $7 < Re_p < 205$ | 118 |
| 10.01 | 3.15 | 0,441 | 76 | $41 < Re_p < 379$ | 74 |

The correlation proposed by Ergun was tested against the experimental data. The mean deviation, *Dm*, proposed by Wen and Chen (1982) was calculated in order to determine the performance of the equation to fit the pressure drop data. This mean deviation is defined as:

$$D_m = \sqrt{\frac{\sum_{i=1}^{N}\left(\frac{y_{cal,i} - y_{\exp,i}}{y_{\exp,i}}\right)}{N}} \tag{3}$$

where $y_{cal,i}$ and $y_{exp,i}$ represent the calculated and the experimental values of the variable respectively (in this case the pressure drop) and $N$ is the number of data points. A $D_m$ of 41% was obtained showing that Ergun's equation does not give a good fit. This is expected, as this correlation does not take into account the side wall effect.

As the present results were obtained for conditions where the side wall has a significant effect on pressure drop, an empirical correlation was developed to predict the pressure gradient. After several attempts, it was found that the following equation provides a good correlation of the experimental results:

$$\frac{\Delta p}{L} = \left(0.00761\,Re_p + 0.000178\,Re_p^2\right)\left(\frac{D}{d_p}\right)^{3.5} \tag{4}$$

Equation (4) is valid for $3 < D/d_p < 17$ and for $3 < Re_p < 379$ , and $(\Delta p/L)$ is defined in Pa/m.

In order to evaluate the ability of Eq. (4) to fit the experimental data, the corresponding mean deviation calculated by Eq. (3) was again determined. The value of $D_m$ obtained was 9.8%, confirming that the correlation developed is suitable for representing the pressure drop of the data. Figure 2 shows the comparison between the experimental and the calculated pressure gradient as a function of the particle Reynolds number, for constant values of $D/d_p$ .

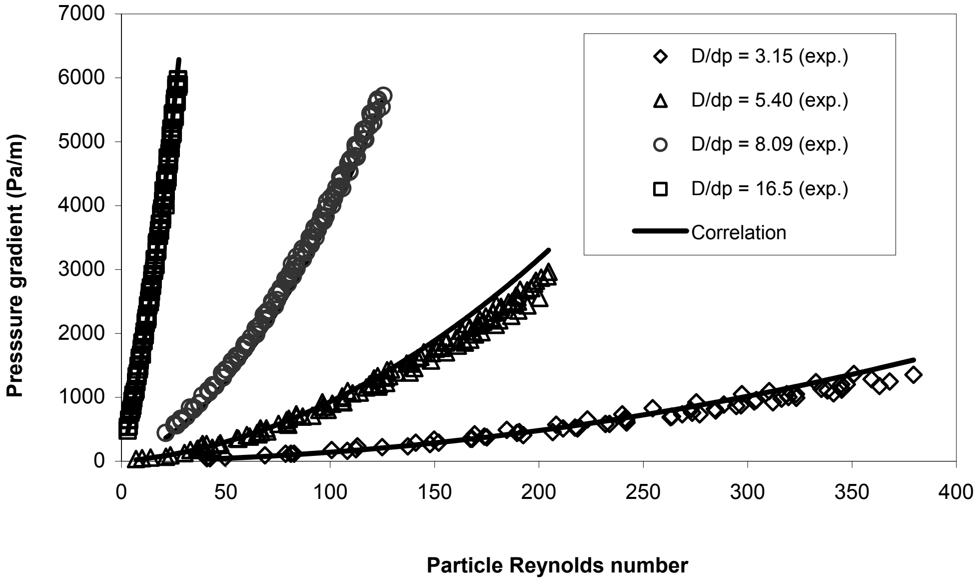


Figure 2 - Experimental and calculated pressure gradient versus particle Reynolds number.

## 4. CONCLUSIONS

In the present work, the experimental data on the pressure drop for water flowing across beds of monosized spheres presented by Ribeiro et al. (2010) was tested against the correlation of Ergun (1952). The mean deviation obtained between the experimental and the calculated pressure drop was 41%. This equation is not suitable for fitting the data, as it does not take into account the side wall effect.

An empirical correlation was developed to predict the pressure gradient, which does take into account the side wall effect. The correlation is valid for $3 < Re_p < 379$ and $3 < D/d_p < 17$, and the mean deviation obtained was 9.8%, showing a good performance for representing the experimental results.

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