COMPUTACIONAL SIMULATION OF TWO-DIMENSIONAL TRANSIENT NATURAL CONVECTION IN VOLUMETRICALLY HEATED SQUARE ENCLOSURE

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Abstract. *Natural convection is a physical phenomenon that has been investigated in nuclear engineering so as to provide information about heat transfer in severe accident conditions involving nuclear reactors. This research reported transient natural convection of fluids with uniformly distributed volumetrically heat generation in square cavity with isothermal side walls and adiabatic top/bottom walls. Two Prandtl numbers were considered,* 0.0321 *and* 0.71*. Direct numerical simulations (DNS) were applied in order to obtain results about the velocities of the fluid in directions* x *and* y*. These results were used in Fast Fourier Transform (FFT), which showed the periodic, quasi-chaotic and chaotic behavior of transient laminar flow.*

Keywords: *heat transfer, DNS, FFT, severe accident, natural convection*

1. INTRODUCTION

The study of natural convection in nuclear engineering is covered by the scenario of severe accidents in nuclear reactors. Since 1960 many studies focused on this physical phenomenon have been developed in this area, however, a significant increase of researches with this approach after the accident in the unit 2 of nuclear power plant Three Mile Island (TMI), in 1979, is observed.

The problem of transient natural convection in an enclosure of aspect ratios under or equal to unity containing fluid with Prandtl number (Pr) equal to 2.0 and 7.0 was studied by Patterson and Imberger (1980). The main conclusion obtained by Patterson and Imberger (1980) was the transient flows strongly depended on Prandtl numbers differently of the steady state flows.

Many numerical studies were performed on natural convection heat transfer in cavities which provided information about some behavior's influences of field flow like aspect ratio, properties of material and boundary conditions of the domain.

Fusegi et al. (1992) analyzed natural convection in diferentially heated retangular cavity with distributed volumetrically heat and filled with a fluid which Prandtl number was 5.85. The simulations were performed at ratios aspects $0.3 - 5.0$, external Rayleigh numbers (Ra_e) 4×10^5-5×10^7 and internal Rayleigh numbers (Ra_i) 4×10^7-5×10^9. The main observation of Fusegi et al. (1992) was that the stationary laminar flows were observed in all investigated cases.

Churbanov et al. (1994) studied natural convection in a volumetrically heated square enclosure with the four isothermal walls. The utilised fluid had Prandtl number 7.0 and Rayleigh internal numbers ranged from 10^5 to 10^8. Churbanov et al. (1994) observed the initial of randomic flutuations and transition of laminar to turbulent flow when Ra_i was 10^8. By using a differentially heated square cavity with adiabatic top and bottom walls containing a fluid with $Pr = 0.71$, Le Quéré and Behnia (1998) observed that the onset of unstediness flow occured in external Rayleigh numbers between 1.81×10^8 and 1.83×10^8.

The natural convection caused by internal heat source in square cavity was also studied by Arcidiacono et al. (2001), however the applied boundary conditions were isothermal side walls and adiabatic horizontal walls. The analyzed fluid had low Prandtl number ($Pr = 0.0321$) and the flow's behavior was investigated using Grashof numbers ($Gr = Ra/Pr$) in the range of 10^5 to 10^9. Arcidiacono et al. (2001) concluded that the initial periodic transition occured in $Gr = 5.4 \times 10^7$ and in the $Gr = 10^9$ the flow becamed completely chaotic.

The majority of studies available in literature investigated the natural convection with differentially heated walls, while fewer, however not less available were directed to the investigation of this phenomenon caused by volumetrically heat generation. This paper aims at analyzing the passage from laminar stationary to transient flow of natural convection in a volumetrically heated square cavity. The same conditions applied by Arcidiacono et al. (2001) were used but the Rayleigh internal numbers were in the range of 5×10^2 to 2.5×10^{12}. Due to the lack of knowledge of melt material's properties in the nuclear reactor in severe accident conditions, two Prandtl numbers were investigated, $Pr = 0.0321 and 0.71$, each of which respectively represent liquid metal and air.

2. TECHNIQUE APPROACH

2.1 Governing equation

The governing equations of natural convection are continuity, momentum and energy transport equations. This research used these two-dimensional equations under the Boussinesq approximation, which can be written in the dimensionless form as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = (\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}) - \frac{\partial P}{\partial X} \tag{2}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} - \frac{\partial P}{\partial Y} + \frac{Ra_i}{Pr}\theta \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}) + \frac{1}{Pr} \tag{4}$$

in which $X = x/H, U = (uH)/\nu, V = (vH)/\nu, \tau = t\nu/H^2, P = (P_d H^2)/(\rho\nu^2), \theta = (T - T_0)k/(q_v H^2)$.

The Prandtl, internal Rayleigh and the average Nusselt (Nu_1) numbers were defined as follows:

$$Pr = \frac{c_p \mu}{k} \tag{5}$$

$$Ra = \frac{g\beta q^{'''} H^5}{\alpha\nu k} \tag{6}$$

$$Nu_1 = \frac{q_w H}{k(T_{ave} - T_0)} \tag{7}$$

2.2 Physics aspects and solution procedure

A square volumetrically generation heat rate cavity, illustrated in Fig. 1, was used in all the analyzed cases. The applied boundary conditions were:

$$U = V = 0, \quad \theta = \theta_0 \quad \text{em} \quad X = 0 \tag{8}$$

$$U = V = 0, \quad \theta = \theta_0 \quad \text{em} \quad X = 1 \tag{9}$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{em} \quad Y = 0 \tag{10}$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{em} \quad Y = 1 \tag{11}$$

The numerical simulations were performed in ANSYS CFX 12.0, as well as the geometry and grid generation. This program uses numerical method of finite volume as solution (Element Based Finite Volume Method - EBFVM), which allows the resolution of problems even in unstructured grids. Therefore, it is possible to obtain a numerical solution of discretized momentum and mass balance equations. All the analyzed cases were done with double precision, high resolution of transients therms discretization and Second Order Implicit Euler of advection therms discretization. The time steps used were 10^{-2} seconds and the convergence criterion of numerical solutions were medium residues (RMS) equal or under to 10^{-6}.

All the calculations were performed using 200×200 uniform grid, with 80420 nodes and 40000 elements.

The grid independence study was done in the case of $Pr = 0.0321$ and $Ra_i = 10^5$ comparing the calculated non-dimensional temperature obtained in a central line (along y and $x = H/2$) with 100×100 and 200×200 grids. This location's domain was chosen because it is where the maximum value of the temperature in dimensionless form occurs as shown in Fig. 2. The results of grid analysis are introduced in Fig. 3 and as depicted, the spacial convergence occurred.

3. RESULTS AND DISCUSSION

3.1 Temporal convergence

The temporal convergence was verified in the case of $Pr = 0.0321$ and $Ra_i = 7.5 \times 10^5$. This procedure was executed applying 0.6 and 0.3 seconds as time step. According to Fig. 4 the onset of oscillations occurred later using $0.6s$ than $0.3s$ as time step due to the discretization errors associated to high time steps.

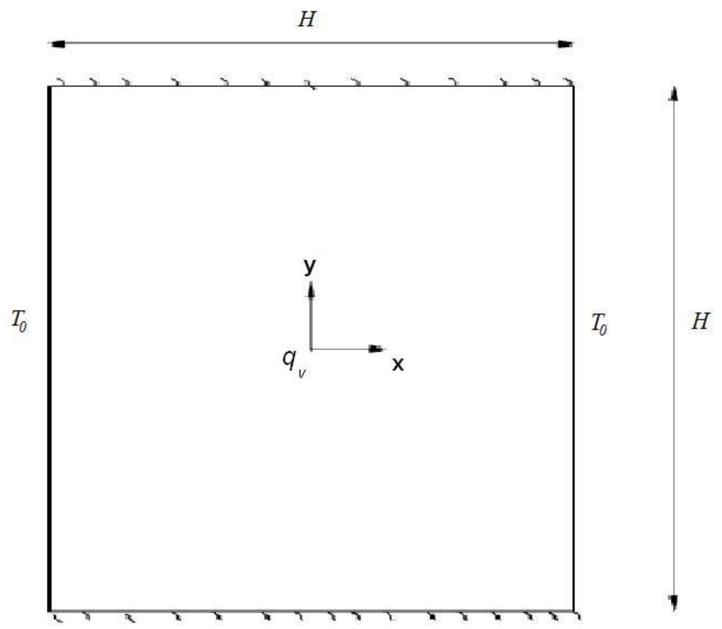


Figure 1. Sketch of the square cavity model with isothermal vertical walls and adiabatic horizontal walls.

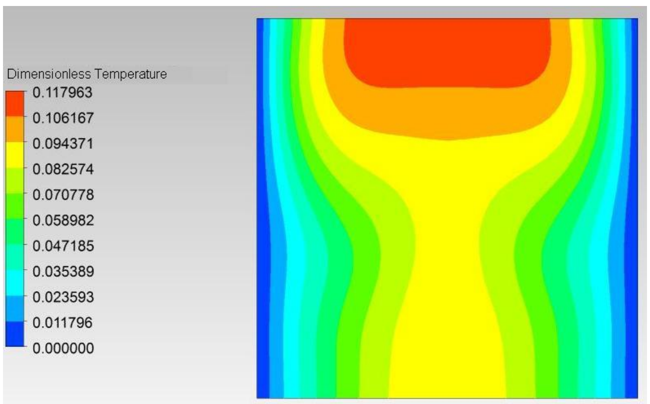


Figure 2. Non-dimensional temperature field

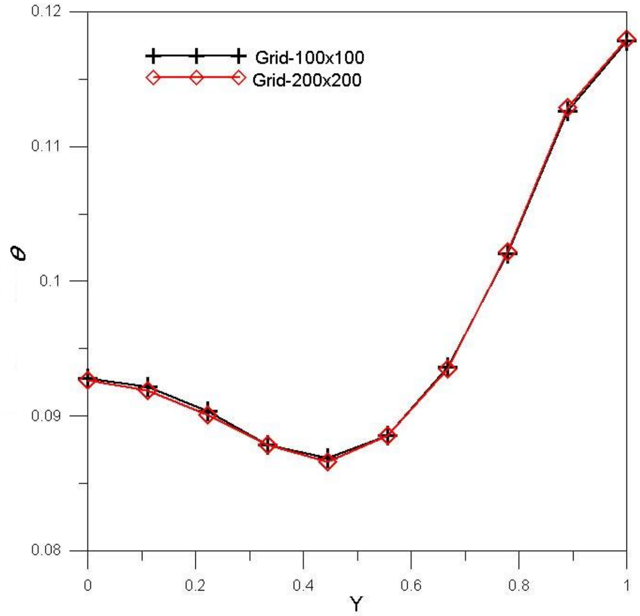
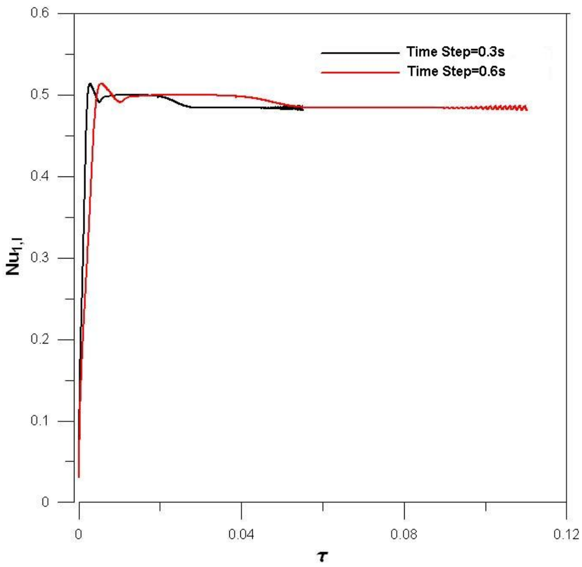
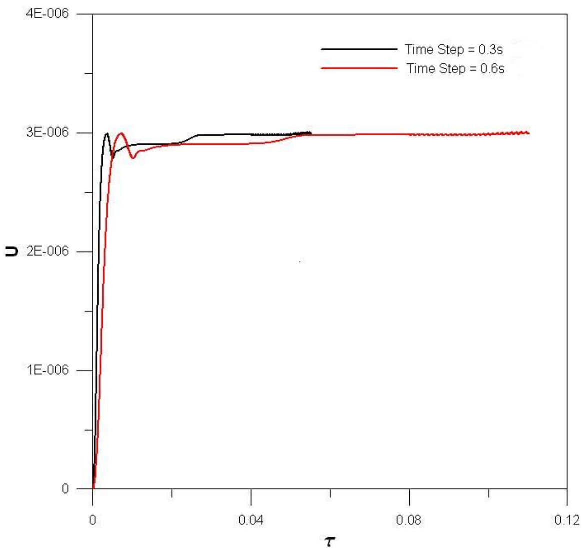


Figure 3. Non-dimensional temperature along y and $x = H/2$ to 100×100 and 200×200 grids, for $Pr = 0.0321$ and $Ra_i = 10^5$



(a)



(b)

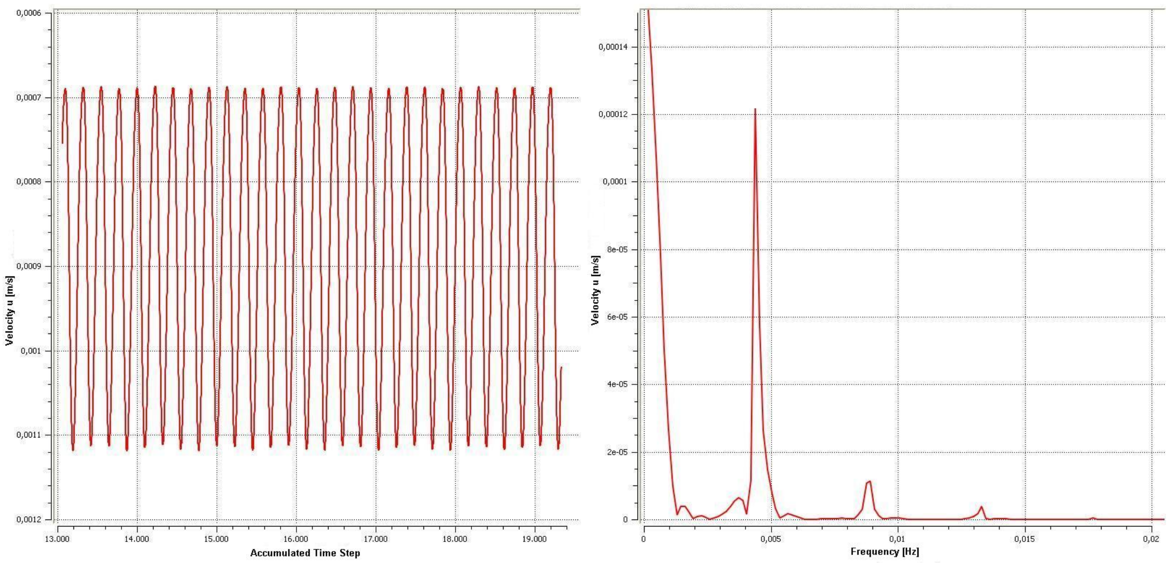
Figure 4. Non-dimensional thermal flux ($Nu_{1,l}$) at isothermal left wall (a) and Average non-dimensional velocity in all domain (b), for $Pr = 0.0321$ and $Ra_i = 7.5 \times 10^5$

3.2 Critical internal Rayleigh number (Ra_{ic}) and FFT analysis

The Rayleigh internal numbers in the range of 1.6×10^2 to 7.5×10^5 were investigated to analyzed Prandtl numbers ($Pr = 0.0321$ and 0.71). In the case of $Pr = 0.0321$ and $Ra_i = 10^5$, using stationary laminar simulation, the numerical residues were under the stipulated convergence criterion (RMS$\leq 10^{-6}$) differently to the case of $Ra_i = 7.5 \times 10^5$, when fluctuations were observed in all parameters analyzed besides high residues. Therefore the Ra_i in the range of 10^5 to 7.5×10^5 was analyzed to obtain the most accurate value of Ra_{ic} where the passage from laminar to transient flow occurs. The best aproximated value to Ra_{ic} in the case of $Pr = 0.0321$ was 5.88×10^5.

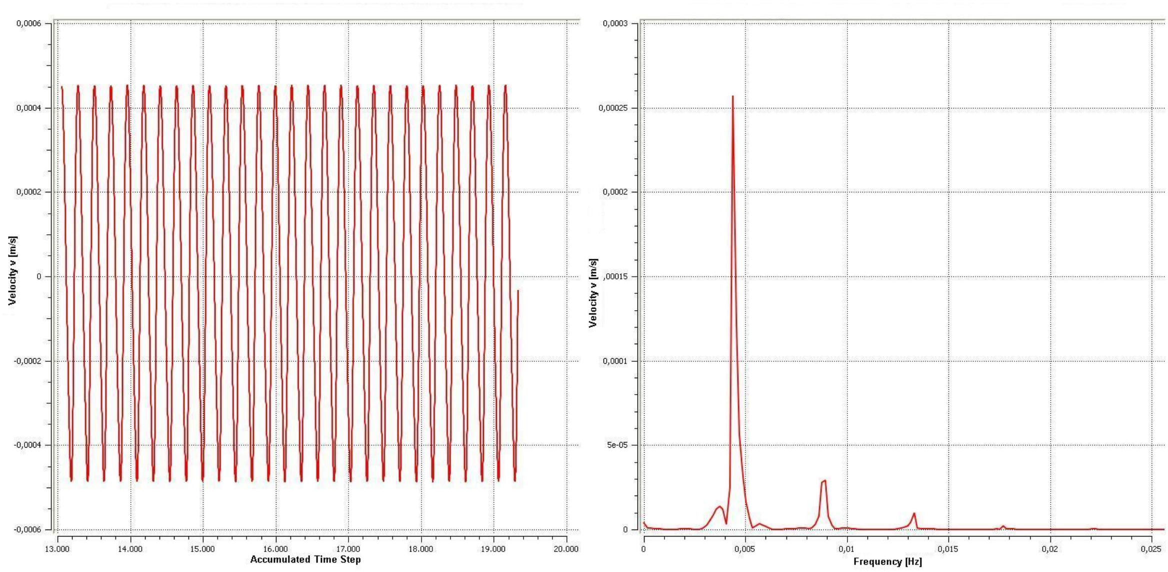
The Ra_{ic} obtained to $Pr = 0.71$ was 1.05×10^9, using the same criterions and procedures in the case of $Pr = 0.0321$.

Different power spectra show the periodic transient laminar flow to $Ra_i = 10^6$ (5 and 6) and quasi-chaotic to $Ra_i = 10^8$(7 as well as is depicted in Fig. 8) in the case of $Pr = 0.0321$. As shown in figure 8 the spectrum indicates the same fundamental frequency of $0.005hz$, characterizing the quasi-chaotic transient flow. Figures 9, 10, 11 and 12 show the chaotic transient regime to $Pr = 0.71$, which are marked by lower frequencies and cannot be described by a small number of well-defined characteristic frequencies. By this observation, is possible that the change from stationary to transient flow occurs with Ra_i in the range of 10^9 and 1.05×10^9. Paolucci and Chenoweth (1989) also analyzed the behavior of natural convection in fluid with $Pr = 0.71$, but in a differently square cavity, and observed that the transition from periodic to chaotic flow occured in $Ra_e = 4 \times 10^8$, a different value obtained by this work due to the boundary conditions used.



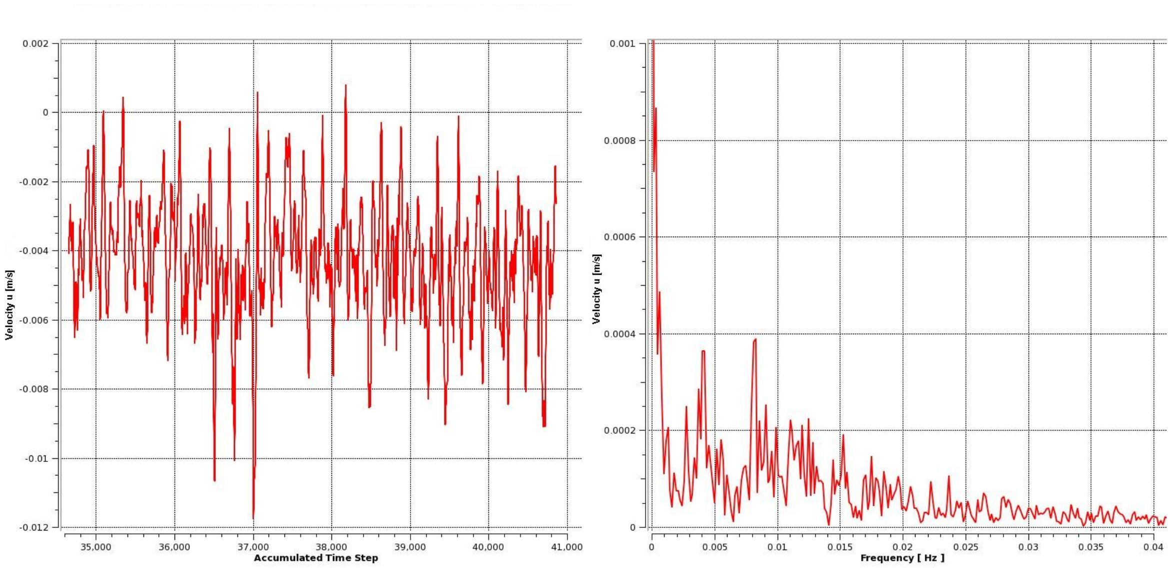
(a) (b)

Figure 5. Velocity u (a) and power spectrum of velocity u (b), in a monitor point ($0,008$,-$0,018$) to $Pr = 0,0321$ and $Ra_i = 10^6$



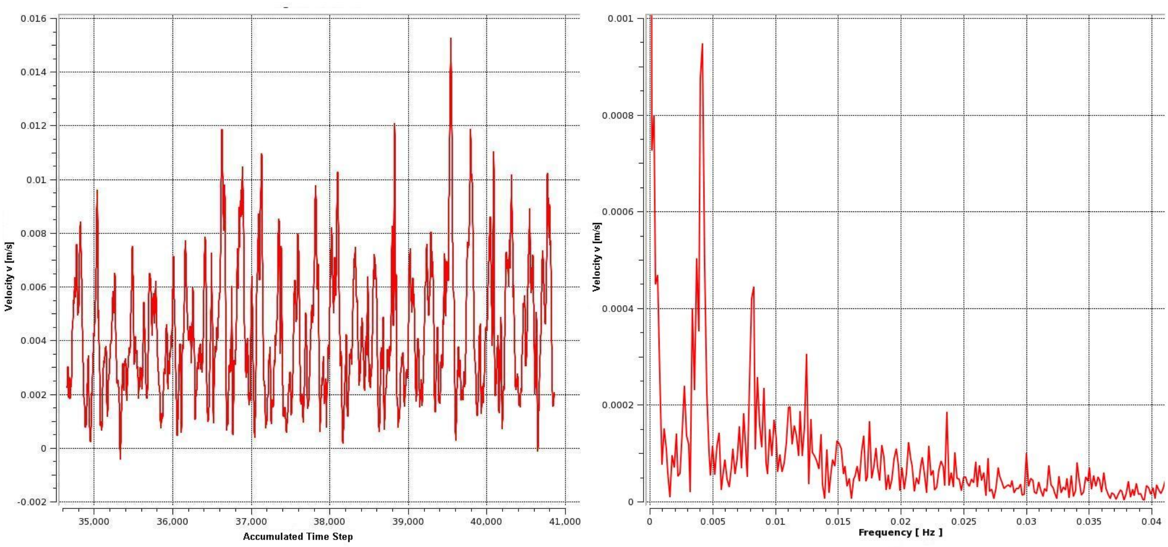
(a) (b)

Figure 6. Velocity v (a) and power spectrum of velocity v (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,0321$ and $Ra_i = 10^6$



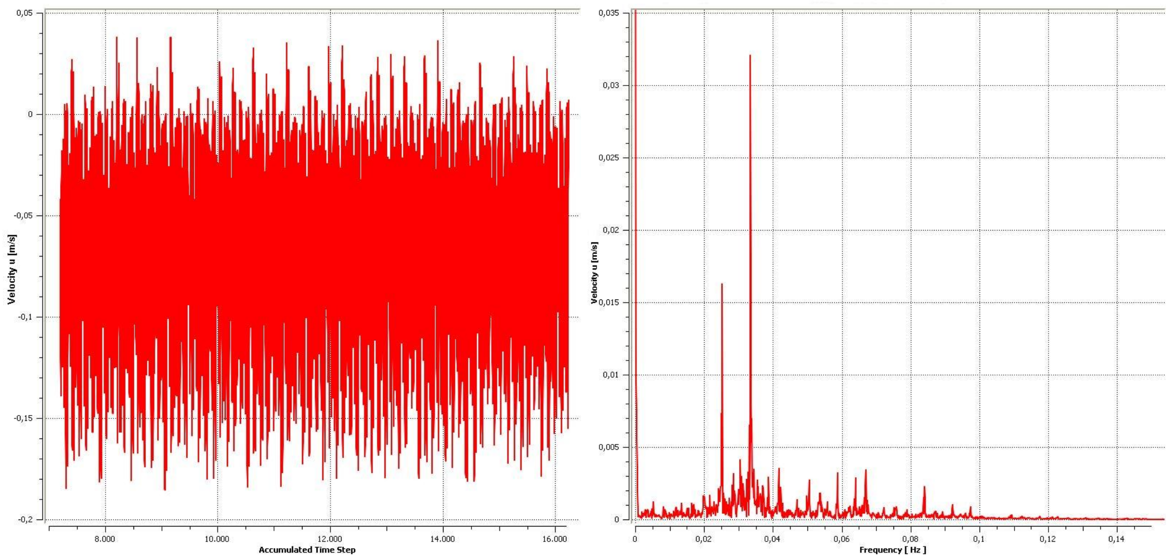
(a) (b)

Figure 7. Velocity u (a) and power spectrum of velocity u (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,0321$ and $Ra_i = 10^8$



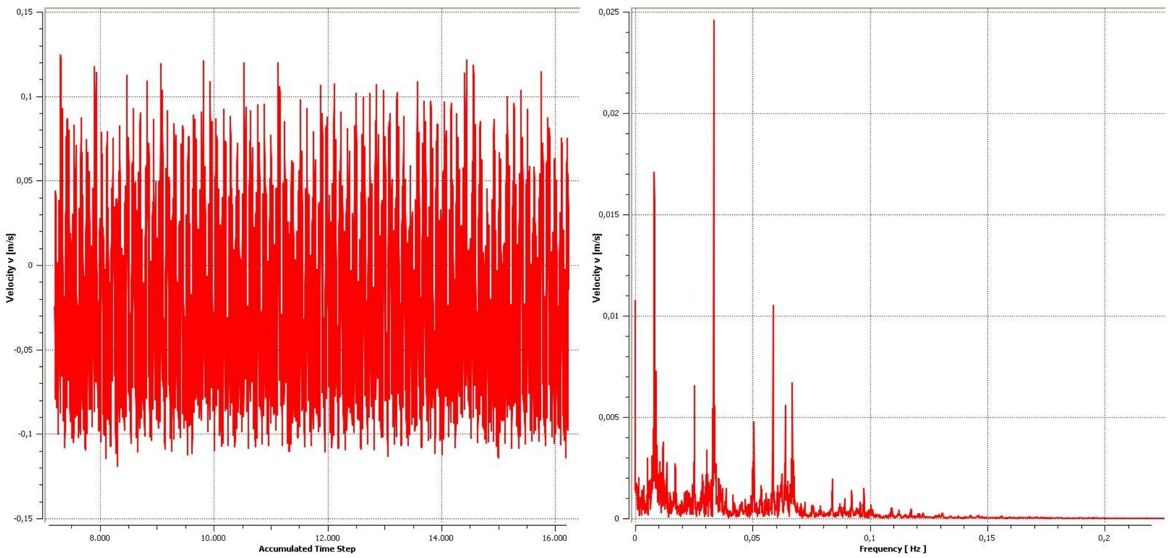
(a) (b)

Figure 8. Velocity v (a) and power spectrum of velocity v (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,0321$ and $Ra_i = 10^8$



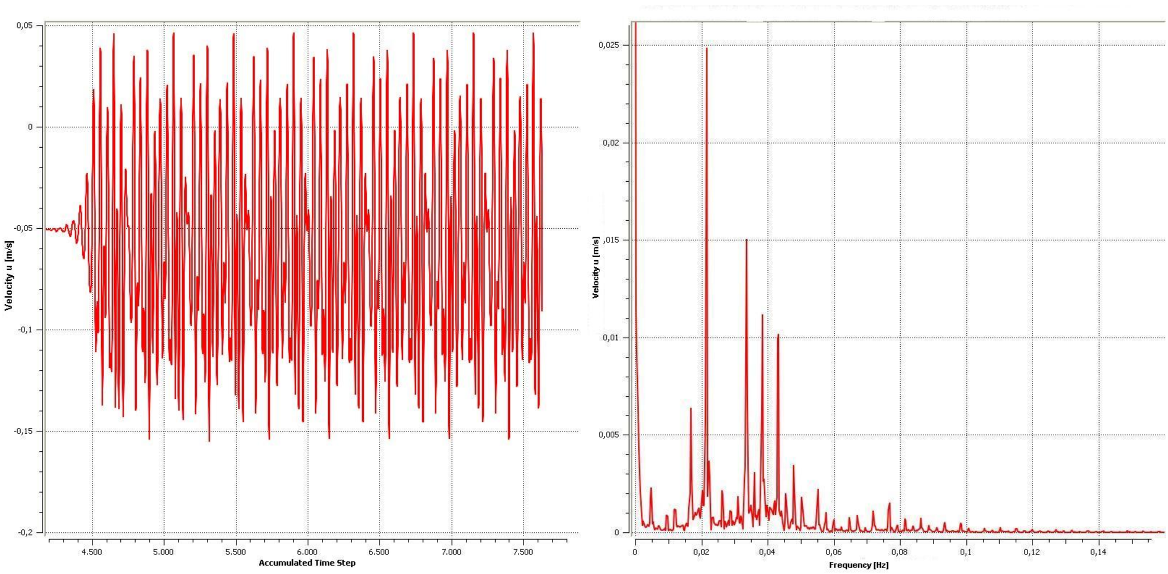
(a) (b)

Figure 9. Velocity u (a) and power spectrum of velocity u (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,71$ and $Ra_i = 1,5 \times 10^9$



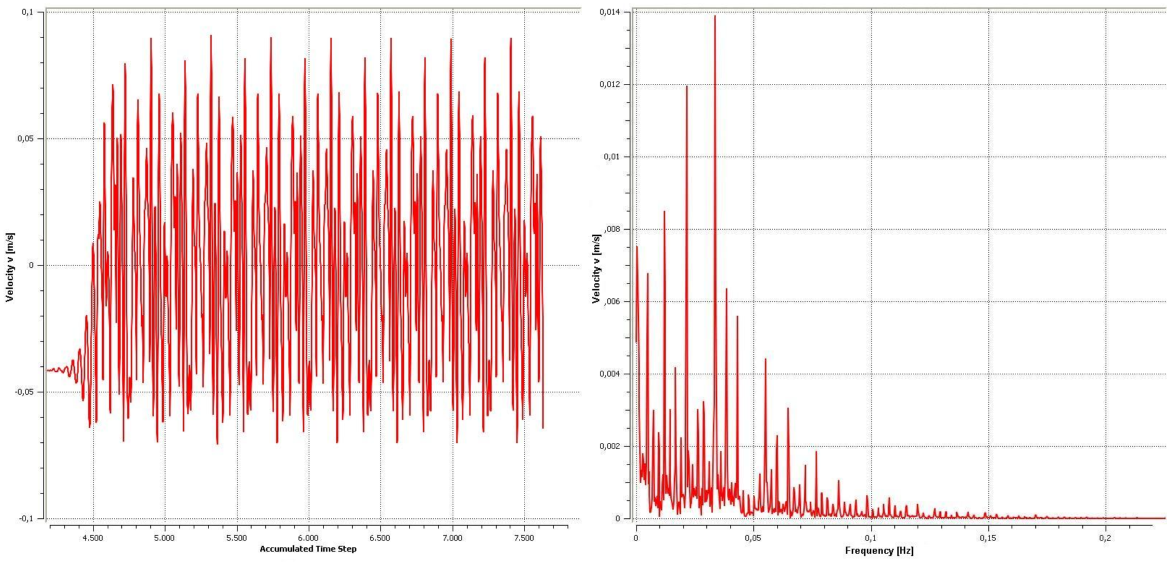
(a) (b)

Figure 10. Velocity v (a) and power spectrum of velocity v (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,71$ and $Ra_i = 1,5 \times 10^9$



(a) (b)

Figure 11. Velocity u (a) and power spectrum of velocity u (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,71$ and $Ra_i = 1,05 \times 10^9$



(a) (b)

Figure 12. Velocity u (a) and power spectrum of velocity u (b), in a monitor point $(0,008$,-$0,018)$ to $Pr = 0,71$ and $Ra_i = 1,05 \times 10^9$

4. CONCLUSIONS

The independence grid study showed that the numeric results of both analyzed grids were in good agreement, and because of the low effort computational the more refined grid was used. The investigation of critical Rayleigh internal number showed that the higher Prandtl number the higher was the Rayleigh internal number when occured the passage from stationary to transient flow, and the fast Fourier transform could be used to expose the periodic ($Pr = 0.0321$ and $Ra_i = 10^6$), quasi-chaotic ($Pr = 0.0321$ and $Ra_i = 10^8$) and chaotic ($Pr = 0.71$ and $Ra_i = 1.05 \times 10^9$ and 1.5×10^9) behavior of the transition flow. This research could presents the influence of internal heat generation in the behavior of transient laminar flow in fluids with natural convection, besides the Prandlt number's influence in this heat transfer phenomenon.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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