HYBRID SOLUTION OF ABLATION IN THERMAL PROTECTION MATERIALS WITH DEGRADATION

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Abstract. *A hybrid numerical-analytical solution has been developed for heat transfer with ablation and pyrolysis in thermal protection of space vehicles undergoing atmospheric reentry, aimed at contributing to the development of a design and optimization methodology of ablative thermal protection systems. A theoretical model of sufficient complexity has been proposed and implemented within the symbolic computation platform Mathematica, and solved through the Generalized Integral Transform Technique (GITT). The Integral Balance Approach is employed for accelerating convergence of the eigenfunction expansions. The hybrid solution is verified with literature results and covalidated with simpler models previously implemented within the present project. A typical ballistic atmospheric reentry of the SARA vehicle is considered more closely to illustrate the utilization of the developed methodologies in an actual pre-design situation.*

Keywords*: Ablation, Hybrid methods, Thermal protection system, Integral Transforms, Heat conduction*

1. INTRODUCTION

The aerodynamic heating delivered to a space vehicle surface and its interaction with the thermal protection system (TPS) are an essential step in the design of recoverable orbital platforms. The analysis of the heat transfer problem during the atmospheric reentry phase of the flight is in general aimed at optimizing the TPS weight (Bouilly, et al., 1998; Guoliang & Guiqing, 2004), while warranting the integrity of the satellite structure and satisfaction of the payload thermal environment restrictions. The Institute for Aeronautics and Space, IAE/CTA, has been leading the design and construction of an orbital platform for microgravity experiments, the vehicle SARA - "*Satélite de Reentrada Atmosférica*" (Moraes Jr., 1998), and has been devoting substantial effort towards the comprehension of the involved thermal phenomena and adequate selection of thermal protection materials (Costa & Moraes Jr., 2002; Gregori et al., 2007; Machado and Pessoa-Filho, 2007). This technological development in such a sensible area has required a close collaboration with Universities and other Research Centers throughout the country, under the sponsorship of the Brazilian Space Agency, AEB. The collaborative research with the Mechanical Engineering Dept. of COPPE/UFRJ is part of these initiatives (Cotta et al., 2002), associated with the thermo-mechanical analysis of ablative thermal protections. Besides the accurate characterization of thermophysical properties of candidate materials (Rey Silva, 2001), this collaboration has resulted in the construction of a computer simulation tool (Cotta et al., 2004), written within the *Mathematica* system (Wolfram, 2005), which provides a robust and precise mixed symbolic-numerical approach to handle actual reentry flight conditions and general surface heat flux excitations, but also permitting the recurrent solution of the problem to achieve an optimized form of the thermal protection system. The methodology behind this so-called TPS-Nose code is the Coupled Integral Equations Approach, CIEA (Cotta & Mikhailov, 1997; Ruperti Jr. & Cotta, 2000), which is essentially a problem reformulation approach, which reduces the order of differential formulations by averaging in one or more space coordinates. The TPS-Nose code was initially developed to handle the thermal analysis of a TPS that undergoes ablation with negligible pyrolysis (Cotta et al., 2004). More recently, the analysis was substantially enhanced by the incorporation of a local solution based on the Generalized Integral Transform Technique, GITT (Cotta, 1993), still for the case of ablation without degradation (Sias et al., 2009a) and then complemented by the consideration of a simplified pyrolysis model (Hogge & Guerrekens, 1982), again handled in local partial differential formulation via GITT (Sias et al., 2009b).

In this work, hybrid numerical-analytical solutions based on integral transforms (Diniz et al., 1990; Cotta, 1993; Cotta & Mikhailov, 1997; Gomes et al., 2006; Sias et al., 2009a) have been developed for a more general heat transfer model of ablation including materials that undergo pyrolysis, aimed at further contributing with the development of a design and optimization methodology of ablative thermal protection systems. A theoretical model of reasonable complexity has been proposed and implemented within the symbolic computation platform *Mathematica* (Wolfram, 2005), and solved through the Generalized Integral Transform Technique (GITT), invoking the Integral Balance Approach for convergence acceleration of the eigenfunction expansions (Sias et al., 2009a,b). Here we thus focus on modeling ablation within materials that result in thermal degradation governed by an Arrhenius-type relation (Macedo et al., 2000; Rey Silva and Orlande, 2002).

2. PROBLEM FORMULATION

The mathematical model here considered involves one-dimensional transient heat conduction, for both the pre-ablation and ablation periods, with constant thermophysical properties in each subregion (Lin & Yang, 2005), as depicted in Figure 1 below. The pre-ablation period is defined as a period of heat conduction with pyrolysis, where both the heat sink and the transpiration cooling effects due to, respectively, the degradation and the gases flow, are accounted for. An insulated boundary is modeled at $x = 0$ and a prescribed transient heat flux is considered at $x = L$. The problem formulation is then written as:

$$\rho_{\mathrm{v}} c_{\mathrm{Pv}} \frac{\partial T_{\mathrm{v}}(x,t)}{\partial t} = k_{\mathrm{v}} \frac{\partial^2 T_{\mathrm{v}}(x,t)}{\partial x^2} - c_{\mathrm{pg}} \dot{m}_{\mathrm{p}}(t) \frac{\partial T_{\mathrm{v}}(x,t)}{\partial x} + \Delta H_{\mathrm{p}} \frac{\partial \rho(x,t)}{\partial t}, \qquad 0 < x < L;\ \ 0 < t < t_{\mathrm{ab}} \tag{1a}$$

$$T_{\mathrm{v}}(x,t) = T_0 \qquad t = 0;\ \ 0 \le x \le L \tag{1b}$$

$$-k_{\mathrm{v}} \frac{\partial T_{\mathrm{v}}(x,t)}{\partial x} = 0 \qquad x = 0;\ \ 0 < t < t_{\mathrm{ab}} \tag{1c}$$

$$k_{\mathrm{v}} \frac{\partial T_{\mathrm{v}}(x,t)}{\partial x} = q_{\mathrm{w}}(\mathrm{t}) \qquad x = L;\ \ 0 < t < t_{\mathrm{ab}} \tag{1d}$$

The mass flow rate of the pyrolysis gases is computed from:

$$\dot{m}_{\mathrm{p}}(t) = -\int_0^L \frac{\partial \rho(x,t)}{\partial t}\, dx \tag{1e}$$

while the density variation is governed by an Arrhenius-type relation as:

$$\frac{\partial \rho(x,t)}{\partial t} = -\bar{\kappa} e^{-E/RT(x,t)} \rho_{\mathrm{v}}^{1\text{-n}} \rho^{\mathrm{n}}(x,t) \qquad 0 < x < L;\ \ 0 < t < t_{\mathrm{ab}} \tag{1f}$$

$$\rho(x,t) = \rho_{\mathrm{v}} - \rho_p \qquad t = 0;\ \ 0 \le x \le L \tag{1g}$$

where

$$\rho(x,t) = \rho^*(x,t) - \rho_{\mathrm{p}} \tag{1h}$$

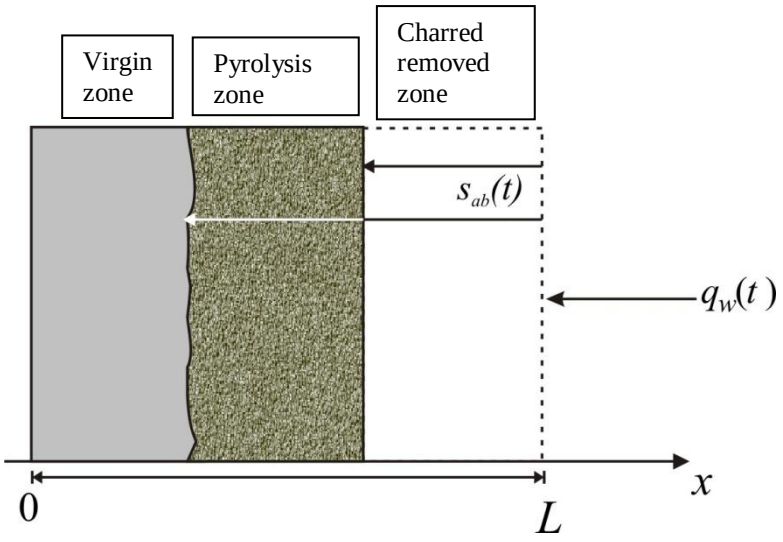

Figure 1 – Representation of the ablation process with degradation, including the position of the ablative boundary.

The problem formulation for the TPS layer during the ablation period can then be written as:

$$\rho_{\mathrm{v}} c_{\mathrm{Pv}} \frac{\partial T_{\mathrm{v}}}{\partial t} = \frac{\partial^2 T_{\mathrm{v}}}{\partial x^2} - c_{\mathrm{Pg}} \dot{m}_{\mathrm{p}}(t) \frac{\partial T_{\mathrm{v}}}{\partial x} + \Delta H_{\mathrm{p}} \frac{\partial \rho(x,t)}{\partial t}, \quad 0 < x < s_{\mathrm{ab}}(t);\ t > t_{\mathrm{ab}} \tag{2.a}$$

$$T_{\mathrm{v}}(x,t) = \tilde{T}_{\mathrm{ab}}(x), \qquad t = t_{\mathrm{ab}};\ 0 \le x \le s_{\mathrm{ab}}(t) \tag{2.b}$$

$$-k_{\mathrm{v}} \frac{\partial T_{\mathrm{v}}(x,t)}{\partial x} = 0, \qquad x = 0;\ t > t_{\mathrm{ab}} \tag{2.c}$$

$$T_{\mathrm{v}}(x,t) = T_{\mathrm{ab}}^{*}, \qquad x = s_{\mathrm{ab}}(t);\ t > t_{\mathrm{ab}} \tag{2.d}$$

where $\tilde{T}_{\mathrm{ab}}(x) = T_{\mathrm{v}}(x, t_{\mathrm{ab}})$ and $\rho(x,t) = \rho^{*}(x,t) - \rho_{\mathrm{p}}$. Also, the energy balance at the ablating boundary is:

$$\rho_{\mathrm{p}} H_{\mathrm{ab}} \frac{ds_{\mathrm{ab}}(t)}{dt} = -q_{\mathrm{w}}(t) + k_{\mathrm{p}} \frac{\partial T_{\mathrm{v}}(x,t)}{\partial x}, \qquad x = s_{\mathrm{ab}}(t);\ t > t_{\mathrm{ab}} \tag{3.a}$$

$$s_{\mathrm{ab}}(t) = L, \qquad t = t_{\mathrm{ab}} \tag{3.b}$$

The mass flow rate of the pyrolysis gases is obtained along the ablation period by:

$$\dot{m}_{\mathrm{p}}(t) = -\int_{0}^{s_{\mathrm{ab}}(t)} \frac{\partial \rho(x,t)}{\partial t} dx \tag{4}$$

while the material degradation is evaluated from the Arrhenius relation applied to the moving domain:

$$\frac{\partial \rho(x,t)}{\partial t} = -\bar{\kappa} e^{-E/RT_{\mathrm{v}}(x,t)} \rho_{\mathrm{v}}^{1-n} \rho^{n}(x,t), \qquad 0 < x < s_{\mathrm{ab}}(t);\ t > t_{\mathrm{ab}} \tag{5.a}$$

$$\rho(x,t) = \tilde{\rho}_{\mathrm{ab}}(x), \qquad t = t_{\mathrm{ab}};\ 0 \le x \le s_{\mathrm{ab}}(t) \tag{5.b}$$

3. SOLUTION METHODOLOGY

The solution of this heat conduction problem with a moving boundary is divided in the two periods of pre-ablation and ablation. To avoid repetition from previous developments, only the detailed solution methodology of the ablation period is here presented. The solution of the pre-ablation period essentially provides the initial condition of the ablation period defined in Eq. (2.b) as $\tilde{T}_{\mathrm{ab}}(x)$, as well as the time of the ablation initialization, t_{ab}. The Arrhenius equation, Eq.(1.f), is considered all along the pre-ablation period, continuously evaluating the thermal degradation of the material. Parameters as the pre-exponential factor ($\bar{\kappa}$), energy of activation (E) and reaction coefficient (n) are expected to be known and are extremely important to evaluate within reasonable precision the thermal degradation and, therefore, the ablative boundary recession.

Following the formalism in the application of the Generalized Integral Transform Technique (GITT), (Cotta, 1993; Cotta & Mikhailov, 1997), the transform-inverse pair is defined:

$$\bar{T}_i(t) = \int_{0}^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t) T_{\mathrm{v}}(x,t) dx \qquad \textit{Transform} \tag{6.a}$$

$$T_{\mathrm{v}}(x,t) = \sum_{i=1}^{\infty} \tilde{\Psi}_i(x,t) \bar{T}_i(t) \qquad \textit{Inverse} \tag{6.b}$$

where $\tilde{\Psi}_i(x,t)$ are the normalized eigenfunctions for the eigenvalues, $\beta_i(t)$, with norm, $N_i(t)$, defined as :

$$\tilde{\Psi}_i(x,t) = \frac{\varphi_{\mathrm{ab},i}(x,t)}{\sqrt{N_i(t)}} \tag{7}$$

The eigenfunctions $\varphi_{ab,i}(x,t)$ are obtained by solution of the corresponding auxiliary Sturm-Liouville problem for the energy equation, given by:

$$\begin{aligned} &\frac{d^2\varphi_{ab,i}(x,t)}{dx^2}+\beta_{ab,i}^2\varphi_{ab,i}(x,t)=0 && 0<x<s_{ab}(t)\\ &\frac{d\varphi_{ab,i}(x,t)}{dx}=0 && x=0\\ &\varphi_{ab,i}(x,t)=0 && x=s_{ab}(t) \end{aligned} \tag{8.a-c}$$

where

$$\varphi_{ab,i}(x,t)=Cos\left(\beta_{ab,i}(t)x\right),\ \beta_i(t)=\frac{(2i-1)\pi}{2s_{ab}(t)},\ N_i(t)=\frac{s_{ab}(t)}{2} \tag{8.d-f}$$

The density is also expanded in terms of eigenfunctions, due to its space variation, through the following integral transform pair:

$$\bar{\rho}_n(t)=\int_0^{s_{ab}(t)}\tilde{\Omega}_n(x,t)\rho(x,t)dx \qquad \textit{Transform} \tag{9.a}$$

$$\rho(x,t)=\sum_{n=1}^{\infty}\tilde{\Omega}_n(x,t)\bar{\rho}_n(t) \qquad \textit{Inverse} \tag{9.b}$$

where $\tilde{\Omega}_n(x,t)$ are the normalized eigenfunctions for the eigenvalues, $\lambda_n(t)$, and norm, $M_n(t)$, given by:

$$\tilde{\Omega}_n(x,t)=\frac{\Omega_{ab,n}(x,t)}{\sqrt{M_n(t)}} \tag{10}$$

The eigenfunctions $\Omega_{ab,n}(x,t)$ are obtained from solution of the Sturm-Liouville problem for the mass balance equation, given by:

$$\begin{aligned} &\frac{d^2\Omega_{ab,n}(x,t)}{dx^2}+\lambda_{ab,n}^2(t)\Omega_{ab,n}(x,t)=0 && 0<x<s_{ab}(t)\\ &\frac{d\Omega_{ab,n}(x,t)}{dx}=0 && x=0\\ &\Omega_{ab,n}(x,t)=0 && x=s_{ab}(t) \end{aligned} \tag{11.a-c}$$

where

$$\Omega_{ab,n}(x,t)=Cos\left(\lambda_n(t)x\right),\ \lambda_n(t)=\frac{(2n-1)\pi}{2s_{ab}(t)},\ M_n(t)=\frac{s_{ab}(t)}{2} \tag{11.d-f}$$

Applying the integral operator $\int_0^{s_{ab}(t)}\tilde{\Omega}_n(x,t)<.>dx$ on Eq. (5.a) yields the transformed equation for density given as:

$$\frac{d\bar{\rho}_n(t)}{dt}=-\sum_{m=1}^{\infty}A_{nm}(t)\bar{\rho}_m(t)+\bar{P}_{\rho,n}(t) \tag{12.a}$$

where

$$A_{nm}(t)=\int_0^{s_{ab}(t)}\tilde{\Omega}_n(x,t)\frac{\partial\tilde{\Omega}_m(x,t)}{\partial t}dx \tag{12.b}$$

and $\bar{P}_{\rho,n}(t)$ is evaluated by semi-analytical integration as presented in (Cotta & Mikhailov, 2005; Sphaier et al., 2009):

$$\bar{P}_{\rho,n}(t) = \sum_{k=1}^{k\max} \int_{x_k}^{x_{k+1}} \tilde{\Omega}_n(x,t) P_\rho^k(x,t)\, dx \tag{12.c}$$

where it has been considered a linear approximation for $P_\rho^k(x,t)$ in each sub-region of the semi-analytical integrations procedure.

The initial condition, Eq.(5.b), is also integral transformed to yield:

$$\bar{\rho}_n(t_{\mathrm{ab}}) = \int_0^{s_{\mathrm{ab}}(t_{\mathrm{ab}})} \tilde{\Omega}_n(x,t_{\mathrm{ab}})\, \rho(x,t_{\mathrm{ab}})\, dx = \tilde{f}_{\rho,n}(t_{\mathrm{ab}}) \tag{12.d}$$

Applying the integral operator, $\int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_{\mathrm{i}}(x,t) <.> dx$ on the energy equation, Eq.(2.a), and on the initial condition, Eq.(2.b), and after substituting the boundary conditions as well as the inverse formulae, Eqs. (9.a), one obtains the transformed temperatures problem as:

$$\frac{d\bar{T}_i(t)}{dt} = -\sum_{j=1}^{\infty}\left(\delta_{ij}\alpha_v\beta_i^2(t) + B_{ij}(t)\right)\bar{T}_j(t) + \sum_{n=1}^{\infty}\left(F_{in}(t)\bar{\rho}_n(t) + G_{in}(t)\frac{d\bar{\rho}_n(t)}{dt}\right) \tag{13.a}$$

$$\bar{T}_i(t_{\mathrm{ab}}) = \int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t)\left(\tilde{T}_{\mathrm{ab}}(x) - T_{\mathrm{ab}}^*\right) dx = \tilde{f}_{T,i}(t_{\mathrm{ab}}) \tag{13.b}$$

where,

$$B_{ij}(t) = \int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t)\frac{\partial\tilde{\Psi}_j(x,t)}{\partial t}\, dx \tag{13c}$$

$$F_{in}(t) = \frac{\alpha_{\mathrm{v}}}{k_{\mathrm{v}}}\left(\Delta H_{\mathrm{p}} D_{in}(t) + c_{\mathrm{pg}}\frac{d\bar{f}_{\rho,n}(t)}{dt}\sum_{j=1}^{\infty} C_{ij}(t)\bar{T}_j(t)\right) \tag{13d}$$

$$G_{in}(t) = \frac{\alpha_{\mathrm{v}}}{k_{\mathrm{v}}}\left(\Delta H_{\mathrm{p}} E_{in}(t) + c_{\mathrm{pg}}\bar{f}_{\rho,n}(t)\sum_{j=1}^{\infty} C_{ij}(t)\bar{T}_j(t)\right) \tag{13e}$$

$$C_{ij}(t) = \int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t)\frac{\partial\tilde{\Psi}_j(x,t)}{\partial x}\, dx \tag{13f}$$

$$D_{in}(t) = \int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t)\frac{\partial\tilde{\Omega}_n(x,t)}{\partial t}\, dx \tag{13g}$$

$$E_{in}(t) = \int_0^{s_{\mathrm{ab}}(t)} \tilde{\Psi}_i(x,t)\tilde{\Omega}_n(x,t)\, dx \tag{13h}$$

Substituting the inverse formula, Eq.(9.b), into Eq.(4) one obtains the working relation for the mass flow rate of the pyrolysis gases, in the form:

$$\dot{m}_{\mathrm{p}}(t) = -\sum_{n=1}^{\infty}\left(\frac{d\bar{f}_{\rho,n}(t)}{dt}\bar{\rho}_n(t) + \bar{f}_{\rho,n}(t)\frac{d\bar{\rho}_n(t)}{dt}\right) \tag{14a}$$

where

$$\bar{f}_{\rho,n}(t) = \int_0^{s_{ab}(t)} \tilde{\Omega}_{\mathrm{n}}(x,t)\, dx \tag{14b}$$

As convergence rates of the eigenfunction expansion in the numerical-analytical solution may be slowed down by the source terms effects, the convergence acceleration technique called Integral Balance is applied (Cotta & Mikhailov, 1997; Sias et al., 2009a). This technique consists of integrating the original partial differential equation over the spatial domain and finding a more appropriate relation for the temperature derivative at $x = s_{\mathrm{ab}}(t)$ that is required in the heat balance at the moving boundary, Eq. (3.a). Thus, Eq. (3.a) is rewritten in the following form:

$$\rho_{\mathrm{p}} H_{\mathrm{ab}} \frac{ds_{\mathrm{ab}}(t)}{dt} = -q_{\mathrm{w}}(t) + k_{\mathrm{p}} \left(\sum_{j=1}^{\infty} \left(\frac{\bar{f}_{\rho,n}(t)}{\alpha_{\mathrm{v}}} \frac{d\bar{T}_j(t)}{dt} + \left(\frac{1}{\alpha_{\mathrm{v}}} \frac{d\bar{f}_{\rho,n}(t)}{dt} - \frac{\dot{m}_{\mathrm{p}}(t)}{k_{\mathrm{v}}} c_{\mathrm{pg}} \tilde{\Psi}_j(0,t) \right) \bar{T}_j(t) \right) - \frac{\dot{m}_{\mathrm{p}}(t)}{k_{\mathrm{v}}} \Delta H_{\mathrm{p}} \right) \tag{15}$$

Therefore, the solution of the ablation period is obtained through numerical solution of the ordinary differential system formed by Eqs.(12.a), (13.a) and (15), with the initial conditions defined by Eqs. (12.b), (13.b), and (3.b). Reliable algorithms are readily available to numerically handle this ODE system, after truncation to a sufficiently large finite order. The *Mathematica* system (Wolfram, 2005) provides the routine NDSolve for solving stiff ODE systems such as the one here considered, under automatic absolute and relative error controls. Once the transformed potentials have been numerically computed, the *Mathematica* routine automatically provides an interpolating function object that approximates the *t* variable behavior of the solution in a continuous form. Then, the inversion formula can be recalled to yield the density and temperature fields representation at any desired position *x* and time *t*.

4. RESULTS AND DISCUSSION

The constructed hybrid solution was first validated against results of an ablation model without pyrolysis (Sias et al., 2009a) and then thoroughly analyzed in terms of the eigenfunction expansions convergence. This analysis has been undertaken considering the resulting net heat flux of a typical ballistic atmospheric reentry of the SARA vehicle (Cotta et al., 2004), Figure 2, where $Q(\tau) = (q(t)L)/\left(k_{\mathrm{v}}(T^*_{\mathrm{ab}} - T_0)\right)$ is the dimensionless prescribed heat flux.

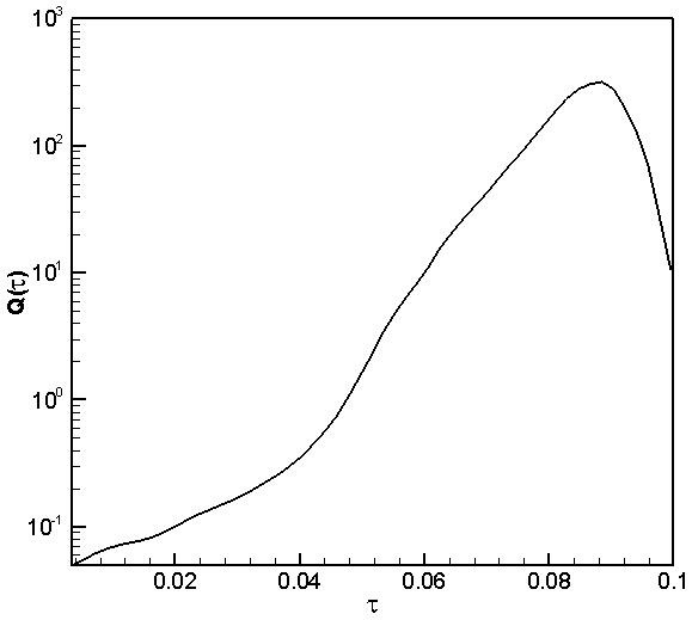

Figure 2. Dimensionless prescribed heat flux typical of a atmospheric reentry flight (Cotta et al., 2004).

In the model with thermal degradation the values for latent heat of pyrolysis and specific heat of the pyrolysis gases are forced to be null for validation purposes, i.e., $\Delta H_{\mathrm{p}} = 0$ and $c_{\mathrm{pg}} = 0$. The material data employed in this validation are the same ones as used in (Ruperti Jr. and Cotta, 2000):

$$L = 0.0065\text{m};\ k = 0.22\frac{\text{W}}{\text{mK}};\ \rho = 1922\frac{\text{kg}}{\text{m}^3};\ c_{\text{P}} = 1256\frac{\text{J}}{\text{kgK}};\ H_{\text{ab}} = 2326x10^3\frac{\text{J}}{\text{kg}};\ T_{\text{ab}}^{*} = 833\text{K}$$

Figure 3.a,b present a set of validation results between the models without and with degradation, first by providing the comparison of the ablative front displacement obtained through both models, with $N = 120$ terms in the eigenfunction expansions for the present simulation. Figure 3.b presents the comparison for the temperature distribution during the ablation period, again with $N = 120$ in the eigenfunction expansions for the present analysis. In both cases a full agreement to the graph scale is achieved between these two independent implementations of the integral transform method.

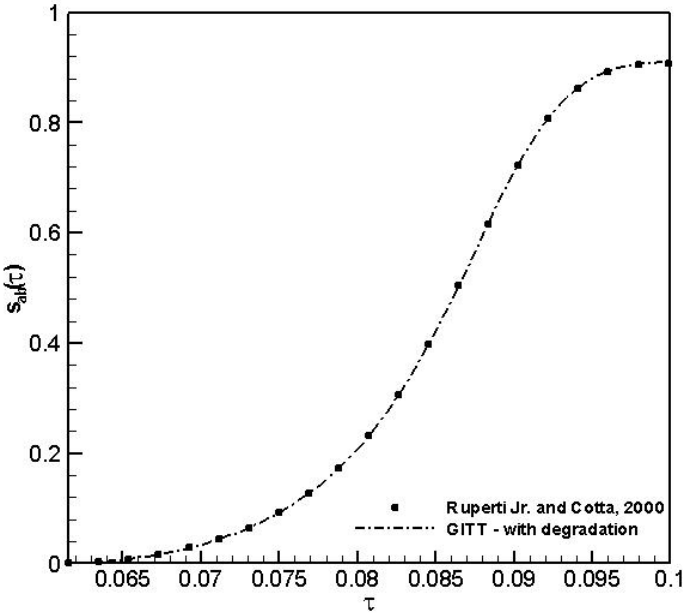

Figure 3.a. Validation of the dimensionless ablative front displacement as compared to the results of (Ruperti Jr. and Cotta, 2000), with N=120 terms in the eigenfunction expansions.

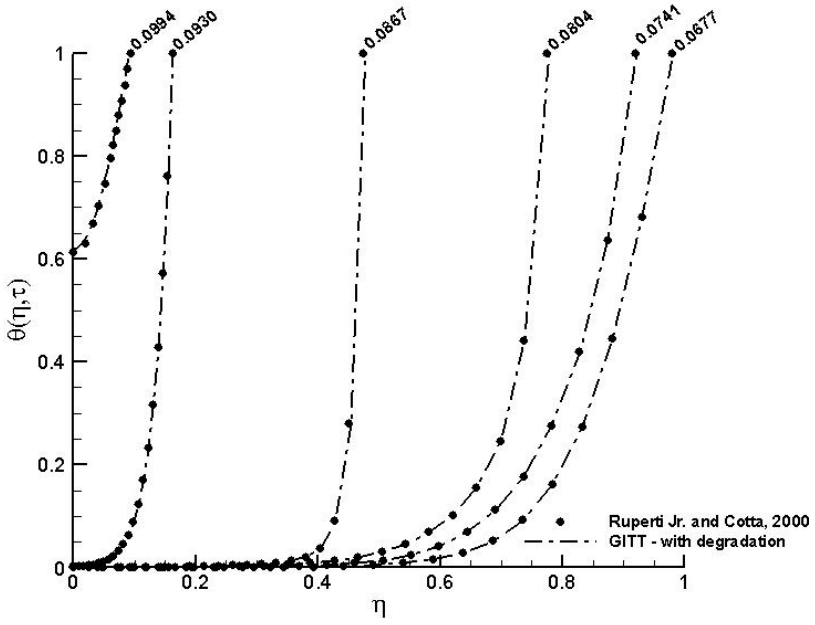

Figure 3.b. Validation of the dimensionless temperature distributions as compared to the results of (Ruperti Jr. and Cotta, 2000), with N=120 terms in the eigenfunction expansions.

A few representative results were also obtained for an ablation case with material degradation (Hogge and Guerrekens, 1982). In order to allow for direct comparison with the results in (Hogge & Guerrekens, 1982), the following dimensionless variables have been computed and reported:

$$\Theta(\eta,\tau)=\frac{T(x,t)-T_0}{T_p^*-T_0};\quad \eta=1-\frac{x}{L};\quad \tau=\frac{\alpha t}{L^2};\quad Q_v(\tau)=\frac{L\,q_w(t)}{k_v(T_p^*-T_0)};\quad \eta_p(\tau)=1-\frac{s_p(t)}{L};\quad \eta_{ab}(\tau)=1-\frac{s_{ab}(t)}{L} \qquad (16a\text{-}f)$$

The physical data adopted in (Hogge & Guerrekens, 1982), although with some questionable numerical values, have been here employed for comparison purposes, as summarized below:

$$L=0.4\,cm;\ \rho_v=0.00165\,{}^{kg}\!/_{cm^3};\ \rho_p=0.00133\,{}^{kg}\!/_{cm^3}\ \ \rho_v c_{Pv}=0.64\,{}^{J}\!/_{cm^3K};\ \rho_p c_{Pp}=0.424\,{}^{J}\!/_{cm^3K}$$
$$k_v=2.13x10^{-3}\,{}^{W}\!/_{cm\,K};\ k_p=3.20x10^{-3}\,{}^{W}\!/_{cm\,K};\ \rho_v H_p=711\,{}^{J}\!/_{cm^3};\ \rho_p H_{ab}=4090\,{}^{J}\!/_{cm^3};$$
$$T_p=2355\,K;\ T_{ab}=8367\,K;\ q_{w0}=500\,{}^{kW}\!/_{cm^2}$$

Also, in light of the lack of numerical values for the pyrolysis parameters, since the simplified model in (Hogge & Guerrekens, 1982) does not explicitly account for an Arrhenius-type relation, some literature values were collected for the quartz-phenolics material, as required by Eq.(5.a), namely (Rey Silva, 2001):

$$c_{Pg}=1873.88\frac{J}{kgK};\Delta H_p=430909\frac{J}{kg};\ \bar{\kappa}=3x10^2\,s^{-1};\ E=71138.6\,J/mol$$

We have first carried an analysis of the influence on the ablation behavior of the reaction coefficient in the material degradation relation, Eq.(5.a), for three different reaction coefficients, n. Figure 4 shows the displacement of the dimensionless ablation front for the three values, $n = 1, 2\ and\ 3$. These results have been obtained with N=35 terms in the truncation of the eigenfunction expansions of both temperature, Eq.(6.b), and density, Eq.(9.b). Clearly, there is a more marked accumulated effect as the ablation process progresses, as observable for the larger values of the dimensionless time, with a faster boundary recession for larger values of the reaction coefficient.

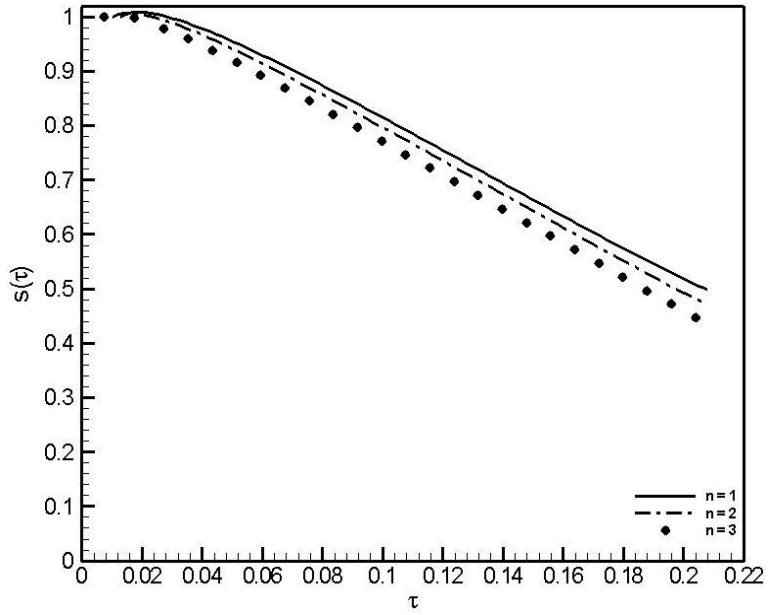

Figure 4. Influence of the reaction coefficient, *n*, on the ablation front displacement for the case with degradation

Figure 5 also illustrates the influence of the reaction coefficient but for the dimensionless temperature distribution across the medium, where the faster movement of the ablation front can be more clearly observed, with the increase of the reaction coefficient.

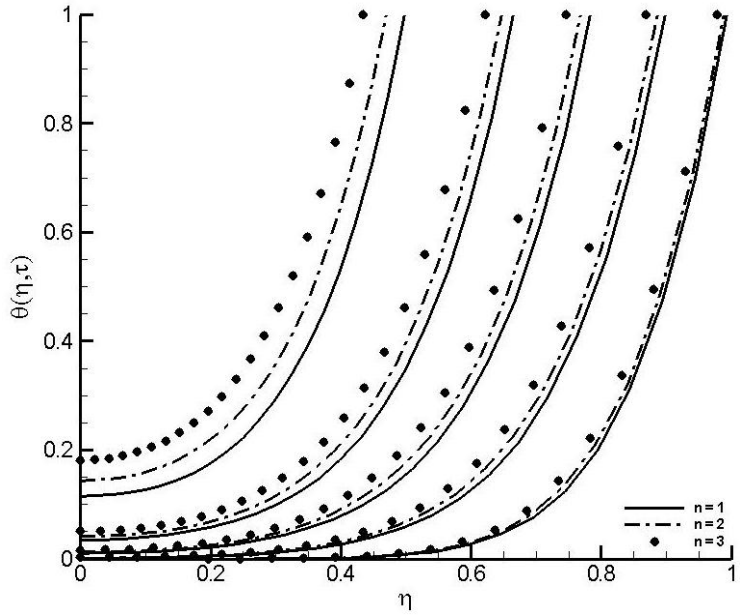

Figure 5. Influence of the reaction coefficient, *n*, on temperature profiles for the case with degradation

Figure 6 shows the influence of the reaction coefficient on the density profiles, that ultimately express the degree of degradation of the material. Clearly, it is observed that the material more quickly degrades as the reaction coefficient decreases, while the reaction zone is clearly wider for the larger value of *n*.

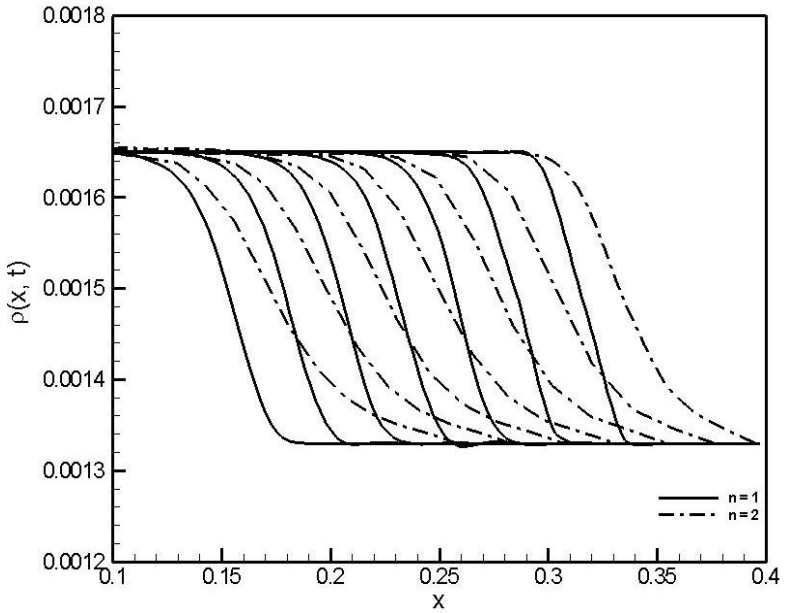

Figure 6. Influence of the reaction coefficient, *n*, on the density profiles for the case with degradation

According to the observations in Figure 6, the flow of the pyrolysis gases which are more readily initiated for the smaller values of *n*, provides a transpiration cooling effect that retards the beginning of the ablation as compared to the other cases with larger values of *n*. This conclusion is confirmed by the values of the time for the onset of ablation that have been obtained in each case, i.e., n=1 (t_{ab}=0.6187), n=2 (t_{ab}=0.4936), and n=3 (t_{ab}=0.3515).

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