AN ASYMPTOTIC CYLINDRICAL MODEL FOR WELL CEMENTING PROCESS

Frederico C. Gomes, fcgomes@puc-rio.br
Marcio S. Carvalho, msc@puc-rio.br
Department of Mechanical Engineering – Pontificia Universidade Catolica do Rio de Janeiro (PUC-Rio)

Abstract. *The cementing process is an important step in the construction of oil and gas wells. It provides zonal isolation and support for the well bore. During the cementing, it is necessary to displace the drilling mud by the cement slurry. To avoid mixing of these liquids, spacer fluids are usually used. Therefore, it is common to have three or more liquids flowing through the eccentric annular space. In some cases when problems occur during drilling, it is necessary to create a cement plug to abandon the well. A complete analysis of the flow in the annular space that occurs during cementing is extreme complex, because of the presence of different liquids, that often present non-Newtonian characteristic and the flow is three dimensional and transient. Thus, a complete model has a prohibitive high computational cost. Simplified models are available in the literature and are used by the oil industry in commercial simulation software for cementing. However, the strong simplifying assumptions, especially the use of Cartesian coordinates to describe the geometry of the annular space, limit the range of parameters at which these models are accurate. In this work, a 2D model that describes the annular space in cylindrical coordinates for this 3D transient flow was developed using lubrication theory. The results show that the use of Cartesian coordinates to represent the annular space may lead to inaccuracies that can compromise the operation design. This error is accentuated for operations of cement plug, at which the radii ratio is much smaller than unity.*

Keywords: *Cementing Process, Lubrication Model, Annular Space*

1. INTRODUCTION

The process of drilling a new well is divided in different at some stages. One of these stages is the cementing process. After positioning the steel casing into the well-bore, it is necessary to fill the space that it left between the casing and rock wall. The cement has the following objectives: fill the space between the casing and well, to promote adhesion between the wall (rock) and the casing; provide mechanical support for the coating; and to isolate the formation to prevent fluid loss.

For the success of the cementing operation it is necessary a complete remove the drilling fluid or spacer fluid and not allow mixing between them. Issues or failures that occur during cementing can affect the hydraulic isolation of the well leading to the migration of gases or liquids from rock to annular and can cause severe productivity problems, but also endanger the safety of operation and cause environmental damage.

There are many studies in the literature on cementing. Some factors strongly influence the final outcome of the cementing process, among them there is the rheology of the fluids pumped, the geometry of the wells, the flow rate and the pumped volume of each fluid.

Bittleston [5] developed a model that considers the eccentricity of well and use a Cartesian coordinate system to represent the geometry of the annular space. An asymptotic method the 2D problem as a sequence of 1D problems. As discussed in work, the main focus was to resolve the problem with low computational cost without a major concern with the accuracy of the solution.

The work of Pilipenko [4] was focused on viscoplastic fluids (Heschel-Bulkley). The model determines the areas where the stress is smaller than the yield stress and fluid does not flow. The model is tested for problems with steady-state fluid displacements in a reference frame attached to the interface. The eccentricity is taken to be constant and small.

Pina [2] developed a model using the lubrication theory and a cylindrical coordinate system. The effect of curvature of the straight section of the annular was not neglected. Thus, the model produces very accurate results, for any radii ratio R_i/R_0. However, the analysis was restricted to the flow of a Newtonian fluid.

Gomes and Carvalho [1] developed a 2D model for a non-Newtonian fluids using the lubrication theory and a Cartesian coordinate system. The effect of curvature of the straight section of the annular was disregards. Thus, the accuracy of the results is greater for radii ratio close to the unity.

The objective of this work is to develop an asymptotic model based on the theory of lubrication to study the displacement of different liquids through an annular space with variable eccentricity along well. The model, as well as the work discussed above, consider the effect of curvature in the annular and describe the annular space by a Cylindrical coordinate system. Thus, the accuracy of the results is greater for any radii ratio. The model considers that the inclination of the well can variable along the length. Unlike the work the literature, the model is not limited to small eccentricities.

2. MATHEMATICAL FORMULATION

To model the flow in the annular space formed by two cylinders with eccentricity varying along the axial direction, we chose to discard the curvature of the cylinder wall by adopting a Cylindrical system of coordinates (z, θ, r), where z is the coordinate in main flow direction (axial), r in the radial direction and θ tangential direction.

The origin of the coordinate system in each straight section of the well is located at the center of the smaller cylinder which has radius R_i. The position of the center of the outer cylinder is defined by the eccentricity $e(z) = (e_1^2 + e_2^2)^{0.5}$, where e_1 and e_2 are respectively, horizontal and vertical (orthogonal functions) eccentricity. The coordinate of the wall of the outer cylinder is defined in terms of the larger cylinder radius R_0 and the eccentricity:

$$R(z,x) = e(z)cos(\frac{x}{R_i} - \gamma) + \sqrt{R_0^2 - e^2(z)sin^2(\frac{x}{R_i} - \gamma)} \tag{1}$$

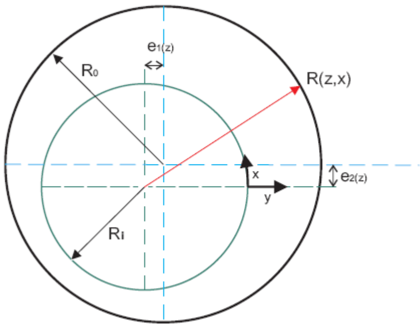

Figure 1. Geometry detail

2.1 Momentum Conservation

Using the Cylindrical system of coordinates, the momentum conservation equations of the flow in the annular space are:

$$\rho\left[u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} + \frac{w}{r}\frac{\partial u}{\partial \theta}\right] = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right] \tag{2}$$

$$\rho\left[u\frac{\partial w}{\partial z} + v\frac{\partial w}{\partial r} + \frac{w}{r}\frac{\partial w}{\partial \theta} + \frac{vw}{r}\right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{\partial^2 w}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rw)\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right] \tag{3}$$

$$\rho\left[u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} + \frac{w}{r}\frac{\partial v}{\partial \theta} + \frac{w^2}{r}\right] = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial w}{\partial \theta}\right] \tag{4}$$

where u, v and w are the axial, radial and tangential velocity components respectively.

The complete solution of this problem has a prohibitive high computational cost. This system can be simplified with using the dimensional analysis that eliminates some terms in the equations, this procedure is known as lubrication approximation (theory).

The length scale in the axial direction is much larger than the length on the radial and azimuthal direction:

$$\delta = R_o - R_i << L \tag{5}$$

Consequently, from the continuity equation, we can show that the velocity along the axial direction is much larger than the velocity components on the θ and z directions

$$\frac{\partial u}{\partial r} >> \frac{\partial u}{\partial z}, \frac{\partial u}{\partial \theta} \tag{6}$$

$$\frac{\partial^2 u}{\partial r^2} >> \frac{\partial^2 u}{\partial z^2}, \frac{\partial^2 u}{\partial \theta^2} \tag{7}$$

Furthermore, because the small annular space, the variation of the velocity components in the axial and tangential direction are much smaller than in the radial direction thus the second derivates are:

$$\frac{\partial w}{\partial r} >> \frac{\partial w}{\partial z}, \frac{\partial w}{\partial \theta} \tag{8}$$

$$\frac{\partial^2 w}{\partial r^2} >> \frac{\partial^2 w}{\partial z^2}, \frac{\partial^2 w}{\partial \theta^2} \tag{9}$$

For a better numerical stability an changed pressure was defined as:

$$p^* = p + \rho g \sin \alpha z \tag{10}$$

Use simplifications of the lubrication theory and changed pressure the equations can be written as:

$$-\frac{1}{\mu}\frac{\partial p^*}{\partial z} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + \frac{\rho g z}{\mu}\cos\alpha\frac{d\alpha}{dz} = 0 \tag{11}$$

$$-\frac{1}{\mu}\frac{1}{r}\frac{\partial p^*}{\partial x} + \frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial(rw)}{\partial r}\right)\right] - \frac{\rho g}{\mu}\cos\left(\frac{x}{R_i}\right)\cos\alpha = 0 \tag{12}$$

$$-\frac{\partial p^*}{\partial r} = 0 \tag{13}$$

Velocity profile in axial direction can be obtained integrating equation (11). It can be done because the pressure gradient its not function of y coordinate.

$$u(r) = \frac{1}{4\mu}\left[(r^2 - R_i^2) - (R_o^2 - R_i^2)\frac{\ln\frac{r}{R_i}}{\ln\frac{R_o}{R_i}}\right]\left(\frac{\partial p^*}{\partial z} - \frac{\rho g z}{4\mu}\cos\alpha\frac{d\alpha}{dz}\right) \tag{14}$$

For the tangential direction, the same procedure with equation (12) can be done.

$$w(r) = \frac{1}{2\mu}\frac{\partial p^*}{\partial \theta}\left\{\frac{r}{2}\left(\ln r - \frac{1}{2}\right) + \left(\frac{R_o^2}{R_o^2 - R_i^2}\ln\frac{R_i}{R_o} - \left(\ln R_i - \frac{1}{2}\right)\right)r - \left[\frac{R_i^2 R_o^2}{R_o^2 - R_i^2}\ln\frac{R_i}{R_o}\right]\frac{1}{r}\right\} + \frac{\rho g}{3\mu}\cos\alpha\cos\theta\left\{r^2 - \left[\frac{R_o^2}{R + R_i} + R_i\right]r + \frac{R - R_i}{r}\right\}$$

2.2 Mass Conservation

In Cylindrical coordinates the mass conservation equation is given by:

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial \theta} = 0 \tag{15}$$

This equation can be integrated in radial direction with limits R_i and R_o that inner cylinder radius and outer cylinder radius respectively.

$$\int_{R_i}^{R_o(z,\theta)}\left\{\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{1}{r}\frac{\partial w}{\partial \theta}\right\}dr = 0 \tag{16}$$

Integrating each of the terms separately using a Leibnitz's rule and considering the no slip boundary conditions on the walls, we obtain:

$$\int_{R_i}^{R_0} r u(r) dr = C_1\frac{\partial p^*}{\partial z} + C_2 \tag{17}$$

$$C_1 = \frac{1}{4\mu}\left[\frac{R_o^4 - R_i^4}{4} - \frac{(R_o^2 - R_i^2)(R_o^2 + R_i^2)}{2} + \frac{(R_o^2 - R_i^2)^2}{4\ln(R_o/R_i)}\right] \tag{18}$$

$$C_2 = -\frac{\rho g z}{4\mu}\cos\alpha\frac{d\alpha}{dz}\left[\frac{R_o^4 - R_i^4}{4} - \frac{(R_o^2 - R_i^2)(R_o^2 + R_i^2)}{2} + \frac{(R_o^2 - R_i^2)^2}{4\ln(R_o/R_i)}\right] \tag{19}$$

$$\int_{R_i}^{R_0} w(r)dr = C_3\frac{\partial p^*}{\partial \theta} + C_4 \tag{20}$$

$$C_3 = -\frac{1}{36\mu(R_o+R_i)}\left[2R_o^4 + 2R_o^3R_i + 12R_o^2R_i^2\ln\left(\frac{R_i}{R_o}\right) - 2R_oR_i^3 - 2R_i^4\right] \tag{21}$$

$$C_4 = -\frac{\rho g\cos\alpha\cos\theta}{36\mu(R_o+R_i)}\left[R_o^5 + R_o^4R_i - 8R_o^3R_i^2 + 8R_o^2R_i^3 - R_oR_i^4 - R_i^5\right] \tag{22}$$

Finally the equation of mass conservation is given by:

$$\frac{\partial}{\partial z}\left[C_1\frac{\partial p^*}{\partial z} + C_2\right] + \frac{\partial}{\partial \theta}\left[C_3\frac{\partial p^*}{\partial \theta} + C_4\right] = 0 \tag{23}$$

A solution of equation (23) gives the pressure field $p(z,\theta)$. With the pressure field, the velocity fields $u(\theta,z)$ and $w(\theta,z)$ can be obtained from equations (17) and (20). Its important to note that the coefficients $C_1(\theta,z)$, $C_2(\theta,z)$, $C_3(\theta,z)$, $C_4(\theta,z)$ depend on the geometry of annular space and the properties of liquid that occupies the point (z,x). Consequently, these coefficients vary with time as the fluid is replaced by another during the process of displacement.

To define the liquid properties at each point during the displacement process, we use a pure convective transport equation of a color function ϕ:

$$\phi\cdot\nabla\phi = 0 \tag{24}$$

each fluid has a different value of ϕ.

3. SOLUTION METHOD

The differential Eq. (23) was solve by finite difference method. To discretize the domain a rectangle grid is created with NZ nodes in axial direction and $N\theta$ in another direction.

$$\frac{\partial}{\partial z}\left[C_1\frac{\partial p}{\partial z}\right] = \frac{2}{\Delta z(i)+\Delta z(i-1)}\left\{C_1(i+1,j)\frac{p^*(i+1,j)-p^*(i,j)}{\Delta z(i)} - C_1(i,j)\frac{p^*(i,j)-p^*(i-1,j)}{\Delta z(i-1)}\right\} \tag{25}$$

$$\frac{\partial}{\partial \theta}\left[C_3\frac{\partial p}{\partial \theta}\right] = \frac{2}{\Delta\theta(j)+\Delta\theta(j-1)}\left\{C_3(i,j+1)\frac{p^*(i,j+1)-p^*(i,j)}{\Delta\theta(j)} - C_3(i,j)\frac{p^*(i,j)-p^*(i,j-1)}{\Delta\theta(j-1)}\right\} \tag{26}$$

$$\frac{\partial C_2}{\partial z} = \frac{C_2(i+1,j)-C_2(i,j)}{\frac{\Delta z(i)+\Delta z(i-1)}{2}} \tag{27}$$

$$\frac{\partial C_4}{\partial \theta} = \frac{C_4(i,j+1)-C_4(i,j)}{\frac{\Delta\theta(j)+\Delta\theta(j-1)}{2}} \tag{28}$$

Means velocities can be calculated for each face of the model.

$$\overline{U} = \frac{2}{H(i-1,j)+H(i,j)}\left[C_1\frac{p^*(i,j)-p^*(i-1,j)}{\Delta z(i-1)} + C_2(i,j)\right] \tag{29}$$

$$\overline{W} = \frac{2}{H(i,j-1)+H(i,j)}\left[C_3\frac{p^*(i,j)-p^*(i,j-1)}{\Delta x(j-1)} + C_4(i,j)\right] \tag{30}$$

To solve the problem, it is necessary to define the boundary conditions. For the well exit ($z = L$) we set a value of pressure P_s. Along left and right boundaries, $x = 0$ and $x = 2\pi$ a periodic conditions are considered.

Two different conditions of entry ($z = 0$) were considered. The first was to impose a pressure input P_e and the second, more used, was to define a input flow rate.

4. RESULTS

In the first example shown in this work was a comparison between the Cartesian model and the Cylindrical model. A vertical well with constant eccentricity ($e = 75\%$) and length of 1000 meters. The radii ratio between cylinders is 0.4. Three Herschel-Bulkley fluids are pumped sequentially with constant flow rate of 6 bpm and the properties of the table 1.

-	$K[Pa.s^n]$	$\rho[kg/m^3]$	$n[s^{-1}]$	$\tau_0[Pa]$
Fluid 1	1.77	967.86	0.40	1.36
Fluid 2	1.44	1097.58	0.40	1.39
Fluid 3	7.66	1646.37	0.30	9.24

Table 1. Fluids Data – Example Horizontal Well

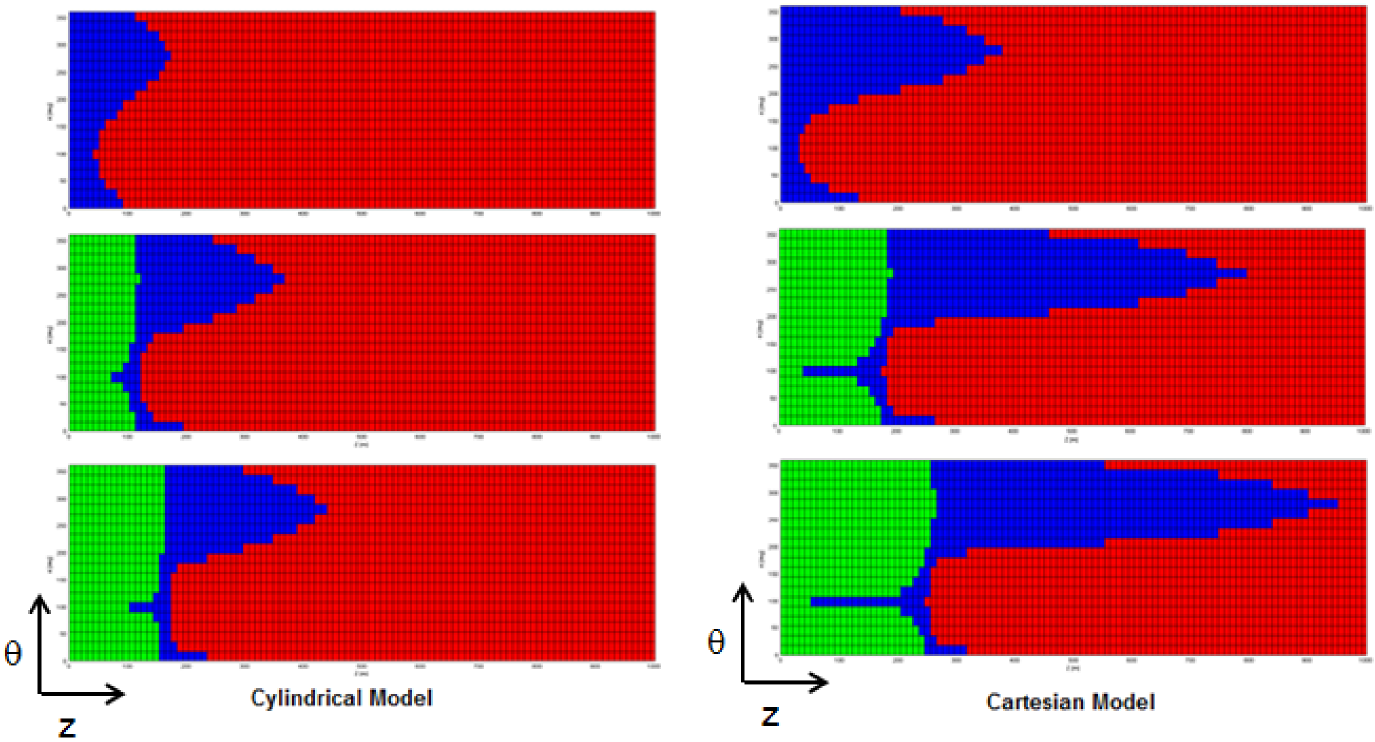

Figure 2. Example 1 – Evolution of fluid interface

The results presented in Figure 2 show that, as expected, in the Cartesian model of the interface fluid moves faster than in the Cylindrical model this is due to neglect of curvature terms.

5. ACKNOWLEDGEMENTS

F.C.G. was supported by FAPERJ and CNPq. This work was partial funded by Petrobras.

6. REFERENCES

F. C. Gomes and M. S. Carvalho, 2009, "Lubrication Model for the flow of several liquids through an annular space with varying eccentricity",Proceedings of COBEM 2009 - 19th International Congress of Mechanical Engineering, Gramado, RS, Brasil.

E. P. F. de Pina and M. S. Carvalho, 2006, "Three-Dimensional Flow of a Newtonian Liquid Through an Annular Space with Axially Varying Eccentricity", Journal of Fluids Engineering, Vol.128, pp. 223-231.

S Pelipenko, I A Frigaard, 2004, "Mud removal and cement placement during primary cementing of an oil well - Part 2; steady-state displacements", Journal of Engineering Mathematics, Vol.48, pp. 1-26.

S Pelipenko, I A Frigaard, 2004, "Two-dimensional computational simulation of eccentric annular cementing displacements", IMA Journal of Applied Mathematics, Vol.69, pp. 557-583.

S S.H. Bittleston, J. Ferguson and I.A. Frigaard, 2002, "Mud removal and cement placement during primary cementing of an oil well", Journal of Engineering Mathematics, Vol.43, pp. 229-253.

7. Responsibility notice

The author(s) is (are) the only responsible for the printed material included in this paper