THE TRANSIENT RESPONSE OF A COMPACT PHOTOBIOREACTOR FOR MICROALGAE CULTIVATION

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Abstract. *Biofuels from microalgae are currently the subject of funded scientific research in many countries due to their high productivity of oil when compared with other crops. Microalgae can also be used in many important applications such as to obtain compounds of interest for food, chemicals, and pharmaceuticals. The high productivity of microalgae when compared with other crops is achieved because agricultural land is not mandatory for their cultivation, since they can be grown in open ponds, sea or vertical photobioreactors. In this paper, a mathematical model is introduced for assessing the transient microalgae growth as a function of variable light intensity, temperature and environmental conditions in the daily cycle. Photobioreactor geometry is considered as well. Light intensity is obtained from sun position, photobioreactor geometry, and the installation location in the world. The photobioreactor was discretized in space by the the volume element method, VEM. Balances of energy and species together with thermodynamics, heat transfer and chemistry empirical and theoretical correlations are applied to each volume element. Therefore, a system of ordinary differential equations with respect to time only is capable of delivering temperatures and concentrations as functions of space and time, even with a coarse mesh. The numerical results are capable of predicting the transient and steady state photobioreactor biomass production with low computational time. Microalgae specific growth rate as a function of average light intensity inside the tubes and time was calculated. As a result, the model is expected to be a useful tool for simulation, design, and optimization of compact photobioreactors.*

Keywords*: Numerical Simulation, Microalgal Growth, Light Intensity, Volume Element Method, Heat Transfer, Sun Radiation, View Factor.*

1. INTRODUCTION

Microalgae are presented as an energy alternative to petroleum oil, capable of producing energy in the form of biofuel. Through this, industries are increasingly investing resources so that researchers are able to produce the most biofuel from microalgae. In general, the microalgae offer a satisfactory return in a short time due to its rich biodiversity and also for having high-density lipid in its structure (Xu *et al.*, 2006). In comparison with the production of oilseeds, with regard to biodiesel production, the cultivation of microalgae present a series of advantages such as a doubling of biomass in a very short time interval, the use of a smaller physical space, the ability to be grown in areas unsuitable for agriculture and less waste generation (Perez, 2007; Lawrence, 2006).

The present study attempts to mathematically and computationally model a physical system capable of cultivating microalgae. With current technology, there are many possible approaches to obtain the numerical solution of the problem. Generally, the numerical representation of a solution to a problem is a table of solution values, which may be exact or approximate. This model is based on a synthesis of mathematics that allows us to identify several theories of Biology, Thermodynamics, Heat and Mass Transfer and Chemical Kinetics. In this form, biological and chemical information conceptually feed the mathematical model whose purpose is to describe the temporal evolution of the system of cultivation of microalgae for a compact photobioreactor from known initial data.

Microalgae are an extremely diverse group of microorganisms that can be found in virtually all aquatic systems, including locations which present a great variation of physico-chemical parameters of development. The cultural diversity of these organisms represents an important technological feature, allowing the cultivation of different genera and species in a wide range of operating conditions (Thajuddin and Subramanian, 2005).

Microalgae are strongly affected by factors such as temperature, light intensity, pH and the nutrient composition of the culture medium. All of these factors have a direct influence on their cellular composition. When these conditions can be controlled by engineering and architecture of the photobioreactor, it is possible to obtain higher yields of microalgal biomass. The pH and composition of nutrients in the culture medium can be controlled by devices installed in the photo bioreactor. The light intensity and temperature are more difficult to be controlled in external photobioreactors, because they depend on the sun and the wind speed and temperature at the site.

With respect to light availability, Richmond (1992) notes that each cell in a photoautotrophic culture is a function of intensity and duration of the incident light and the cell concentration, or density, which affects the growth process through mutual shading. According to Serenotti *et al.* (2004), under appropriate lighting conditions, cells can store energy and produce intermediate products (such as ATP) that are used for the fixation of carbon dioxide and biomass synthesis regardless of whether the cells are in conditions of lightness or darkness.

In the case of temperature, when working with the cultivation of microalgae, as with any microorganism, when grown at temperatures outside of the optimum temperature, microalgae experiences a decrease in the rate of cell growth (at low temperatures) or inhibition and death (at high temperatures). However, microalgae can adapt to different growing conditions from which the increase or decrease in temperature occurs gradually. Likewise, sudden increases in temperature will invariably provide cell death.

The mathematical model presented here incorporates the phenomenon of microalgal growth under the influence of the geometry of the photobioreactor, environment temperature, and installation position on the globe. This model uses a mathematical equation, based on the equation proposed by Molina Grima *et al.* (1994), to represent the kinetics of microalgae growth. The phenomena were described using the method of volume elements (Vargas *et al.*, 2001) which discretizes the domain (in this case the photobioreactor) into small control volumes. The principles of Thermodynamics, Heat and Mass Transfer, Fluid Mechanics, empirical relationships, Computer Graphics and Analytical Geometry were applied to each element. The result of this work was a computer program that evaluates the influence of design and operating conditions in the productivity of algal biomass.

2. MATERIALS AND METHODS

In this model, the Law of the Conservation of Energy is used to calculate the temperature of the walls of the photobioreactor tubes and the temperature of the culture medium. The Law of the Conservation of Species (Continuity) is used to calculate the concentration of microalgae as a function of time in the photobioreactor. The photobioreactor was discretized in space using the Volume Element Method, VEM, proposed by Vargas *et al.* (2001). This procedure resulted in a system of ordinary differential equations which was solved numerically using the classical Runge-Kutta method with fourth order accuracy and adaptive stepping.

Initially, the system shown in Fig 1 was divided into four different types: a) Type 1 (tank), Type 2 (bundle of transparent tubes), Type 3 (pump) and Type 4 (opaque tubes). Since it is of interest to study the variation of the concentration of microalgae as a function of time, this study only considers Type 1 (reservoir) and Type 2 (bundle of plastic tubes) sub-systems where these effects are apparent. The sub-system type 3 (pump) is responsible only for generating a pressure difference sufficient to create flow in the system. The transient time of microalgae by the sub-system Type 4 (opaque tubes) is negligible compared to the transient time for other sub-system, thus, its effects are not considered.

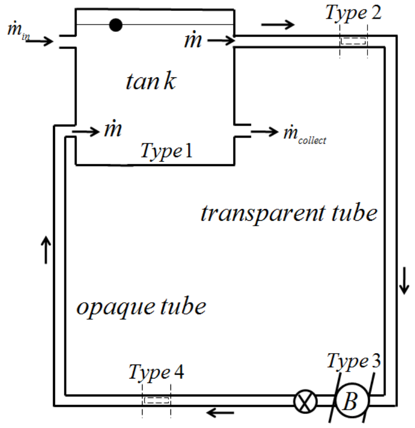

Figure 1. Photobioreactor system and sub-systems

2.1. Modeling of sub-system Type 1 (reservoir)

The tank was modeled as a single volume element, as seen in Fig. 2. This sub-system is considered thermally insulated (no heat transfer between the contents and the outside). Due to this, only a mass balance is performed. Two species were considered, algae and other components. The output, $\dot{m}_{collet}$, represents the mass flow rate of fluid collection containing microalgae and the entrance mass flow rate, $\dot{m}_{in}$, is for the new medium composed of water and nutrients introduced into the system. The input and output are connected to the system by transparent photobioreactor tubes, sub-system type 2. The symbol $\dot{m}$ represents the flow supplied by the pump from sub-system type 3 which keeps the fluid circulating in the reactor. It is assumed that the entire volume of the reservoir is filled with liquid medium. The variation in concentration of microalgae in the tank is not calculated. Therefore, the residence time is much smaller here than in the sub-system type 2 (bundle of transparent tubes).

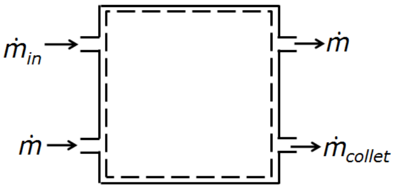

Figure 2. Volume element of the reservoir.

2.2. Modeling sub-system Type 2 (bundle of transparent tubes)

The sub-system type 2 is used for modeling bundle of transparent tubes. The growth of microalgae occurs in this bundle of tubes. During the day, sunlight powers the photosynthesis process and consequently increases biomass. Apart from photosynthetically active radiation (PAR), the tubes receive heat via solar radiation and also thermal interaction with the atmospheric air. The photosynthetically active radiation (PAR) is electromagnetic radiation contained in the range 400-700 µm in wavelength, which is directly responsible for the process of photosynthesis.

The bundle of tubes was divided into smaller elements by creating a volume element model (VEM). Each element is formed by the tube wall and the contents of the tube. The principles of thermodynamics, heat and mass transfer and empirical relationships were applied to each part of the generic volume element. The objective was to obtain a system of ordinary differential equations that capture the system behavior.

2.2.1. Side of the fluid volume element (culture medium)

The transparent tubes were divided into a volume element, Fig. 3, for the modeling of the fluid which consists of water, nutrients, and microalgae. For each volume element, the equations of the conservation of species and the conservation of energy were applied.

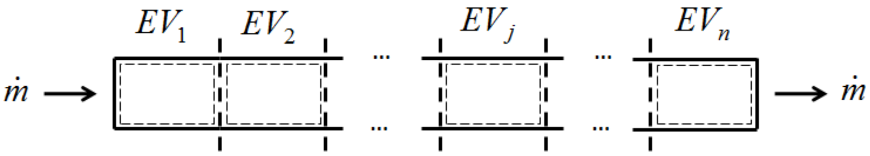

Figure 3. Volume elements relating to fluid seeping through the transparent tubes

Applying the law of the conservation of species in the volume element of Fig. 4 gives Equation (1).This equation corresponds to the increase in mass of species $i = 1$, microalgae. Where: V_e is the volume of the volume element (m^3), Y_1 is the concentration of the species $i = 1$, microalgae, (kg/m^3), t is time (s), $\dot{m}$ is the mass flow rate (kg / s), ρ_l is the density (kg/m^3), µ, the specific growth rate (s^{-1}), and α is the maintenance fee (s^{-1}).

$$V_e^{(j)} \frac{dY_1^{(j)}}{dt} = \frac{\dot{m}}{\rho_l}\left(Y_1^{(j-1)} - Y_1^{(j)}\right) + V_e^{(j)} Y_1^{(j)} (\mu - \alpha) \quad (1)$$

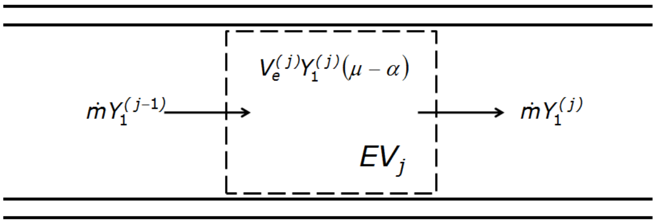

Figure 4. Volume element j and application of the equation of the conservation of species

Microalgal growth rate is influenced by temperature and solar radiation. Each species has different rates of irradiation and appropriate temperatures. Inadequate lighting and temperatures inhibit growth and may even lead to cell death. Recently, Sanchez *et al.* (2008) proposed Eq. (2) to attempt to correlate the specific growth rate, μ as a function of temperature T, the average light intensity inside the tubes, $I_{médio}$ and immediate response to the light intensity on the surface of the medium, I_0. These intensities represent the fraction of photosynthetic active radiation. The constants A_1 and A_2 are frequency factors or pre-exponential value (h^{-1}), E_a and E_b represent the activation energy (kJ / mol), b is a dimensionless parameter and c is a parameter with dimensions of (μE / (m2.s)). These are all characteristics of each species of alga. The symbol R is the general gas constant (kJ / mol), T is absolute temperature (K) and T_0 is the reference temperature (K)

$$\mu=\frac{\left(A_1e^{\left(\frac{E_a}{RT}\frac{T-T_0}{T_0}\right)}-A_2e^{\left(\frac{E_b}{RT}\frac{T-T_0}{T_0}\right)}\right)I_{médio}^{b+\frac{c}{I_0}}}{I_{médio}^{b+\frac{c}{I_0}}+\left(I_K\left(1+\left(\frac{I_0}{K_i}\right)^a\right)\right)^{b+\frac{c}{I_0}}} \tag{2}$$

The other species are evaluated as a single species only. Concentration of species $i=2$ (others) is Y_2 (kg/m^3), Eq (3).

$$Y_2^j=1-Y_1^j \tag{3}$$

Applying the law of the conservation of energy in the volume element of Fig. 5 gives Eq. (4). The methods of heat transfer that occur are: the heat flow originating in the solid wall of the tube $\dot{Q}^{(j)}$ (W), and heat which is transported along with the flow of fluid between elements. The variable $h_{int}^{(j-1)}$ is the enthalpy of the fluid that enters the volume element (W / kg) and $h_{\text{int}}^{(j)}$ is the enthalpy of the fluid leaving the volume element with units of (W/kg). The specific heat of the culture medium with the microalgae is c_l (J/(kgK)) and $dT^{(j)}/dt$ is the derivative of the temperature of the medium (K) with respect to time (s).

$$\dot{Q}^{(j)}+\dot{m}h_{int}^{(j-1)}=m_l^{(j)}c_l\frac{dT^{(j)}}{dt}+\dot{m}h_{int}^{(j)} \tag{4}$$

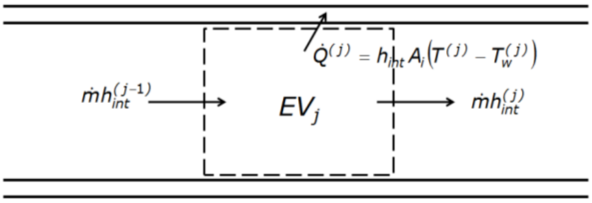

Figure 5. Volume element *j* and application of the equation of the conservation of energy on the fluid

The plot of heat transfer between the middle and the tube wall $\dot{Q}^{(j)}$ (W), is calculated by Eq. (5). The wall temperature is $T_w^{(j)}$ (K), A_i is the area of the wall (m_2) and h is the heat transfer coefficient for convection (W/(m^2K)).

$$\dot{Q}^{(j)} = h\ A_i\left(T^{(j)} - T_w^{(j)}\right) \tag{5}$$

2.2.2. Side wall of the volume element (tube wall)

The solid part of the volume elements, the tube wall is shown in Fig. 6. At the wall there is no mass transport, thus on the law of the conservation of energy was applied. The tube wall experiences three modes of heat transfer, Fig 7: heat transfer by radiation in the light from the sun, heat transfer by convection with the outside air and heat transfer by convection with the liquid medium contained in the tube. The latter was explained earlier in Eq. (5).

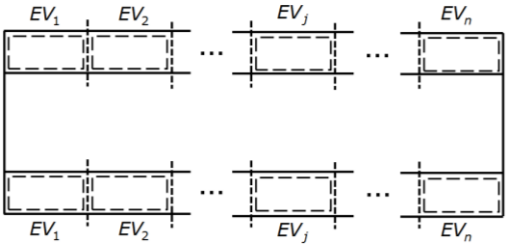

Figure 6. Volume elements concerning the wall of the transparent tubes

The application of energy balance on the wall of the transparent tubes leads to Eq. (5). The effect of solar radiation is represented by $\dot{Q}_{rad}^{(j)}$ (W). The heat transfer with the outside air is $\dot{Q}_{air}^{(j)}$ (W). The mass of transparent tube contained in the volume element is $m_w^{(j)}$ (kg) and c_w is the specific heat of tube material ((J/(kg.K)). Finally, $T_w^{(j)}$ is the temperature of the tube (K).

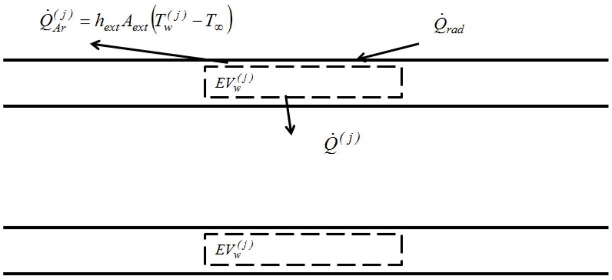

Figure 7. Volume element *j* and the equation of the conservation of energy at the wall of transparent tubes

$$\dot{Q}_{rad}^{(j)} - \dot{Q}^{(j)} - \dot{Q}_{air}^{(j)} = m_w^{(j)} c_w \frac{dT_w^{(j)}}{dt} \tag{5}$$

Heat transfer at the wall with outdoor air, $\dot{Q}_{air}$ (W) is calculated by Eq (6):

$$\dot{Q}_{air}^{(j)} = h_e A_e\left(T_w^{(j)} - T_\infty\right) \tag{6}$$

where h is the heat transfer coefficient by convection between the outside wall and the ambient air (W/m^2K), A_e is the area of outside wall (m^2) and T_∞ is the temperature of the ambient air (K).

2.2.3. Interior lighting of the volume element (culture medium)

The lighting inside of a volume element containing algae depends on the concentration of algae C (kg/m^3), the distance d (m) that the beam travels within the medium and an extinction coefficient K_a (m^2kg^{-1}). This extinction coefficient is characteristic to each species of alga. The equation relating all the factors is the Lambert-Beer Law. This law states that a ray of light of intensity I_0 (μE/(m^2.s)), passing through any medium suffers an exponential decay. The average illuminance, I_{av}, (μE/(m^2.s)) is obtained by integrating the field of illumination and dividing it by the total volume of the volume element, V (m^3), Eq. (7). The values of I_0 take into account only the fraction of photosynthetically active radiation.

$$I_{av} = \frac{1}{V}\int_V I_0 . e^{-d.C.K_a} dV \tag{7}$$

The extinction coefficient K_a was proposed by Molina Grima *et al.* (1994) in the form of Eq. (8).The dimensionless coefficients b_0, b_1 and b_2, are specific to each alga. The other coefficients, Y_p and Y_b (m^{2}kg^{-1}), are also specific to each alga.

$$K_a = Y_p.\left(b_0 - b_1.C + b_3.C^2\right) + Y_b \tag{8}$$

2.2.4. Solar Irradiation

The radiant energy that reaches a surface perpendicular to the rays of the sun, G_n can be estimated by Eq. (9) where G_0 is the solar constant, τ_a is the transfer coefficient for the air mass, and m is the relative air mass, defined as the ratio of actual path length to the length of shortest path possible.

$$G_n = G_0 \tau_a^m \tag{9}$$

When the sun is seen from the earth, the zenith angle varies with the latitude of the location, time of day and solar declination; Equation (10) connects these variables (Kreith, 1977). In this equation z (rad) is the zenith angle, δ_s (rad) is the solar declination, ϕ (rad) is the latitude of the observing site, or the installation and h is the hour angle. Since the zenith angle depends on the latitude of the installation site, it is possible to evaluate the influence of location of the plant on the system performance.

$$z = cos^{-1}\left(sin\phi . sin\delta_s + cos\,\phi . cos\,\delta_s . cosh\right) \tag{10}$$

Another important angle is the azimuth angle. This angle is formed between the north-south axis and the projection of the sun at the earth's surface, Eq. (11). With the zenith and azimuth angles, view factors can be calculated which affect the phenomenon of thermal radiation when photosynthetically active.

$$A = sin^{-1}\left(cos\,\delta_s \frac{sin\,h}{cos\left(\pi/2 - z\right)}\right) \tag{11}$$

2.2.5. Irradiation on the bundle of tubes

The need to assess the amount of solar radiation, thermal or photosynthetically active, which reaches the surface of each tube arrangement was satisfied by using the method of ray tracing. This method was first developed to study the trajectory of subatomic particles and later adapted for use in graphic computing. This method essentially follows the reverse path of the beam. The point of origin of the ray is the center of the tube. This point of origin in conjunction with the vector at the sun represents a ray. Each tube can be represented by a circle with radius and geometrical coordinates of the center, both known. Through analytic geometry, it can be determined whether a ray intersects the circle or not. This determines whether the tube is illuminated by the sun or is in the shade at that moment. The method was based on work by Lopes (2009).

A ray is a line, $\vec{P}(t)$ originating from point $\mathbf{C}_0$ and with a direction given by the vector $\mathbf{V}_d$, Fig. 8. The line equation that represents the light ray, in parametric form, corresponds to Eq. (12):

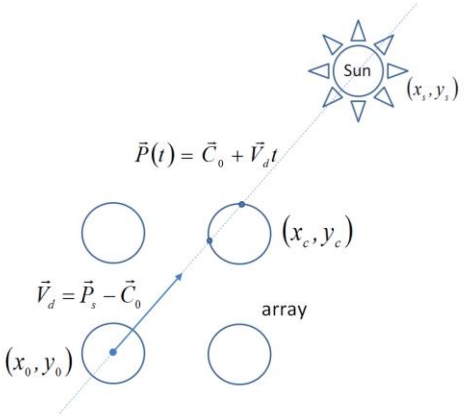

Figure 8. Determination of illumination on the surface of a transparent tube arrangement

$$\vec{P}(t) = \vec{C}_0 + \vec{V}_d t \tag{12}$$

2.3. Numerical Methods

The system of ordinary differential equations (ODE) resulting from the mathematical model was solved using the Runge-Kutta method with fourth order accuracy and adaptive stepping. This method was chosen to enable the evaluation of the transient nature of the equipment. The method was programmed using the FORTRAN language. The results were post-processed to analyze the results of interest such as the rate of production of microalgae and other parameters typically used for comparison.

3. RESULTS AND DISCUSSION

The site chosen for the computer simulation of the photo bioreactor was in Curitiba, Paraná, Brazil. The times of year used in the simulations were the months of January and February. The simulation period was 60 days, during which the air temperature was assumed to be constant at 20^o Celsius. The parameters related to the species of algae were arbitrarily set as the average of a group of algae reported in the literature. The model, however, can give accurate results for a given species if specific parameters for those algae were used.

Three different arrangements of transparent tubes, H x W (in number of tubes) were selected. The first arrangement was 10x10, the second arrangement was 20x10 and the third arrangement was 10x20.

The variation of algal concentration versus time can be seen in Fig. 9. It can be seen that the 10x10 arrangement reached a concentration slightly larger than the 10x20 arrangement. The 20x10 arrangement reached the lowest algae concentration. It is likely due to the fact that arrangements which are sparser receive more light from the sun. In all cases, steady state was reached in approximately 40 days.

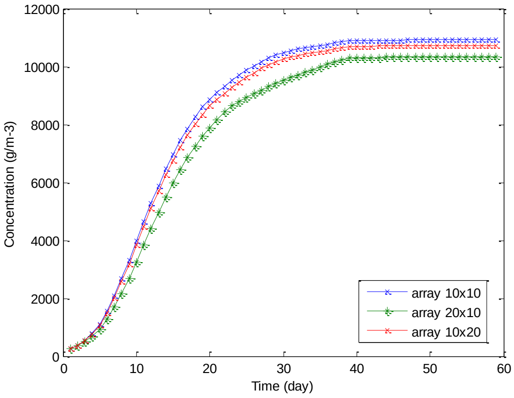

Figure 9. Evolution of the concentration of algae in relation to time for various arrangements

The number of greatest interest to the study the photobioreactors is the *production rate*. The production rate is defined as the amount of biomass produced (kg) over time (days). This number is obtained in a post-processing of simulation results and is the derivative with respect to the time of the concentration curve. The production rate for the three different arrangements studied is shown in Fig. 10. All three arrangements reached a point of maximum production rate during the ninth or tenth day of operation. The 10x20 arrangement reached the highest production rate, followed by the 20x10 arrangement. Again, the 10x20 arrangement makes better use of sunlight and has superior performance. The 10x10 arrangement had the worst performance. This is most likely due to the decreased volume of the culture medium which is half of the volume of the other arrangements.

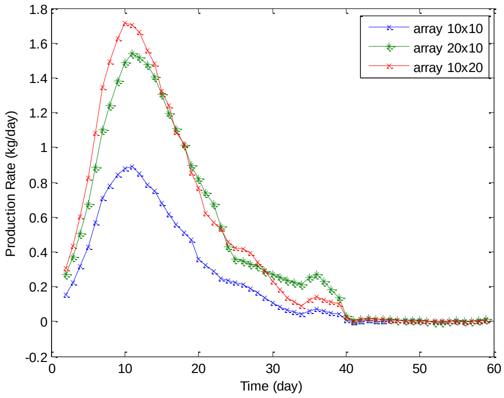

Figure 10. Production rate of algae biomass over time for various arrangements

The area of land occupied by a photobioreactor is another point of great interest. Thus, a relation between the production rate and the surface area occupied was attempted. This relation is shown in Fig. 11 for geometries previously studied. The highest specific production rate (kg/(m^2.day)) among the evaluated arrangements the 20x10 arrangement. This configuration achieved a production 0.27 kg of biomass per square meter per day. The other arrangements amounted to only half of that number.

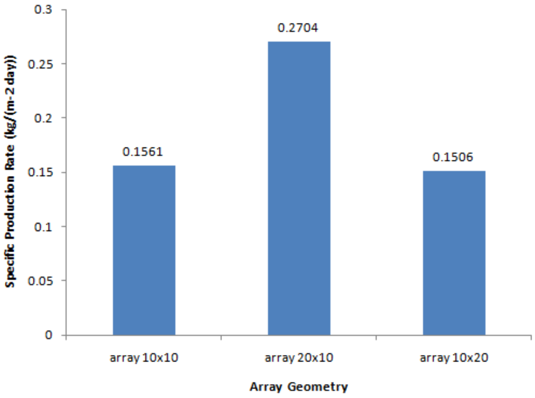

Figure 11. Specific production rate for various arrangements

4. CONCLUSIONS

The cultivation of microalgae is presented as a viable alternative for biofuel production in place of petroleum products. Many technical challenges must be overcome so that this goal is reached. The evaluation of the biomass of microalgae in photobioreactors according to the geometry through mathematical modeling as demonstrated in this article presents a fundamental tool in developing such equipment. The main advantage of tubular photobioreactors is the exploitation of the land area by compacting the tubes and extending vertically. Future studies should focus on optimizing the placement of tubes and maximizing the size of the system. The compaction and vertical extension of the PBR should be performed to ensure the sustainable production of microalgae minimal use of land.

3. ACKNOWLEDGEMENTS

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