

NUMERICAL INVESTIGATION OF STRATIFIED GAS-LIQUID TWO-PHASE DOWNWARD FLOW

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Abstract. *This work reports numerical solutions of stratified gas-liquid two-phase downward flow in slightly inclined circular pipes. The physical-numerical model proposed by De Sampaio et al (2008) has been applied to predict liquid height (or liquid holdup) and pressure gradient for fully developed turbulent stratified gas-liquid downward flow, with smooth and horizontal interface in inclined circular pipes with inclination angles varying from 0 to -10 degrees. The Reynolds averaged Navier-Stokes equations with the $k-\omega$ turbulence model are solved by using the Newton-Raphson scheme and the finite element method. For a given pair of gas and liquid volumetric flow rates, the Newton-Raphson scheme determines the interface position and the pressure gradient which feed an inner iteration loop that solves the non-linear differential equations by the finite element method. Accurate numerical solutions are obtained by constructing finite element meshes through a mesh generation routine based on a bipolar mapping, and providing a selective refinement close to the pipe wall and at the gas-liquid interface. The numerical results obtained show a good agreement with experimental and numerical data available in the literature.*

Keywords: *gas-liquid two-phase downward flow, smooth stratified downward flow, $k-\omega$ model, finite element method*

1. INTRODUCTION

Two-phase stratified flow in pipes is frequently encountered in practical applications such as the flow of oil and natural gas in pipelines, and the steam generation and refrigeration equipment. In a nuclear power plant it may occur in the reactor core along the process of heat transfer from the fuel rods to the coolant, and in the reactor loops as a flow of steam and water during a hypothetical loss-of-coolant accident (LOCA). In a PWR reactor fuel channel, an accurate prediction of the void fraction is essential for the reactor operation and safety analyses, which has a direct effect on the pressure drop and critical heat flux calculations, and hence the overall mass flow rate circulating through the fuel channel.

The mechanistic model due to Taitel and Dukler (1976) has been widely used, which is a one-dimensional two-fluid model with closure relations for the wall and interfacial shear stresses calculated with single-phase flow correlations. However, the Taitel-Dukler model neglects the detailed velocity profile structure and calculates the wall and interfacial shear stresses via empirical correlations based on the averaged velocities.

Shoham and Taitel (1984) presented one of the earliest two-dimensional numerical solutions of fully developed turbulent-turbulent gas-liquid flow in horizontal and inclined pipes. The gas phase was treated as bulk flow, while the liquid phase momentum equation in the bipolar coordinate system with an algebraic turbulent model was solved by using a finite difference method. Also using the bipolar coordinate system, Issa (1988) modeled stratified flow, with a smooth interface surface, but solved the axial momentum equation in both gas and liquid phases with the standard $k-\epsilon$ model. Wall functions were used in the solid boundaries. The results for flow in a 25.4 mm diameter pipe agree reasonably well with predictions given by the mechanistic model of Taitel and Dukler (1976). More recently, stratified two-phase flow in inclined pipes has also been studied numerically and experimentally, Ottens *et al.* (2001), Ghajar and Tang (2007), Biberg (2007), Berthelsen and Ytrehus (2007).

The influence of the liquid flow field on interfacial structure of long distance two-phase stratified pipe flow was studied experimentally by Lioumbas *et al.* (2005), through local axial velocity measurements in the liquid phase in conjunction with other liquid layer characterization experiments. The results revealed the influence of the liquid flow field development on the interfacial structure, suggesting that the onset of the interfacial waves is strongly affected by the liquid flow structure. Banerjee and Isaac (2006) performed a numerical study to determine the rate of evaporation of gasoline while flowing through an inclined two-dimensional channel. Two-phase vapor-liquid stratified flow was solved by using the volume-of-fluid (VOF) method.

This work reports numerical solutions of stratified gas-liquid two-phase downward flow in slightly inclined circular pipes. The physical-numerical model proposed by De Sampaio *et al.* (2008) has been applied to predict liquid height (or liquid holdup) and pressure gradient for fully developed turbulent stratified gas-liquid downward flow, with smooth and horizontal interface in inclined circular pipes with inclination angles varying from 0 to -10 degrees. The Reynolds

averaged Navier-Stokes equations (RANS) are solved with the $\kappa\omega$ model for a fully developed stratified gas-liquid two-phase flow using the finite element method. Accurate numerical solutions are obtained by constructing finite element meshes through a mesh generation routine based on a bipolar mapping, and providing a selective refinement close to the pipe wall and at the gas-liquid interface. The closure model was developed by Wilcox (2000) and is considered substantially more accurate than $\kappa\epsilon$ model in the near wall layers (Menter *et al*, 2003). The main drawback of the model is that the ω -equation shows a strong sensitivity to the values of ω in the freestream outside the boundary layer, Menter (1992), which has largely prevented the ω -equation from replacing the ϵ -equation as the standard scale-equation in turbulence modeling. However, it is expected that the $\kappa\omega$ model should have a better performance in the prediction of gas-liquid two-phase stratified flow, as no freestream boundary condition of ω is needed in the modeling.

Following Issa (1988) and Newton and Behnia (2000), a smooth and horizontal interface surface is assumed without considering the interfacial waves. The continuity of the shear stress across the interface is enforced with the continuity of the velocity being automatically satisfied by the variational formulation. The mathematical model and the variational formulation are presented in the next section. The numerical techniques are then described. Next the numerical results are presented in comparison with the experimental data of Lioumbas *et al*. (2005) for the interface position at various downward flow conditions, and with available numerical data (Taitel and Dukler, 1976; Shoham and Taitel, 1984; Issa, 1988).

2. MATHEMATICAL MODEL

Let us consider a fully developed stratified gas-liquid two-phase downward flow in a slightly inclined circular pipe. In view of the symmetry of the flow with respect to the vertical plane, only a half-pipe cross-section is considered in the present model. Figure 1 shows schematically the open bounded domains occupied by the gas and liquid phases, which are denoted by Ω_g and Ω_f , respectively. We consider that the volumetric flow rates of the phases, Q_g and Q_f , are given.

The interface between the phases is assumed to be a stable smooth horizontal plane without onset of the waves. However, the interface position y_{int} is unknown. In fact, it will be determined as a function of the given flow rates, pipe diameter d and the physical properties of the gas-liquid phases. Referring to Figure 1, the gas-liquid interface is represented by Γ_{int} , the symmetry boundary is denoted by Γ_s and the pipe wall is Γ_c . We also define the overall open bounded domain $\Omega = \Omega_f \cup \Omega_g \cup \Gamma_{int}$.

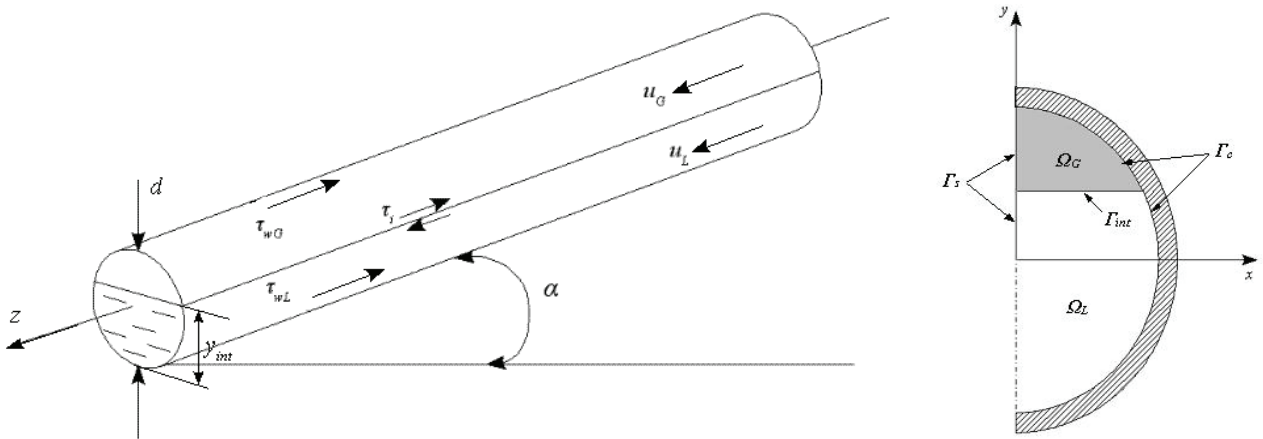


Figure 1. Idealized gas-liquid flow in a slightly inclined pipe with the open bounded domains.

The Reynolds Averaged Navier-Stokes (RANS) approach is adopted to describe the turbulent flow in both phases. For developed turbulent flow, the two-phase flow model is described by the following equations, defined within each open bounded domain Ω_i ($i=1$ meaning phase f and $i=2$ phase g).

$$-\nabla \cdot (A_i \nabla u) + \frac{dp}{dz} = \rho_i \bar{g} \cdot s \quad (1)$$

$$\nabla \cdot (B_i \nabla \kappa) - \beta_2 \rho_i \kappa \omega + S_i = 0 \quad (2)$$

$$\nabla \cdot (C_i \nabla \omega) - \beta_i \rho_i \omega^2 + \frac{\alpha_1 \omega}{\kappa} S_i = 0 \quad (3)$$

Establishing that the gravitational force vector is according to the Fig. 2 then, in terms of the unit vectors e_y and s , we have

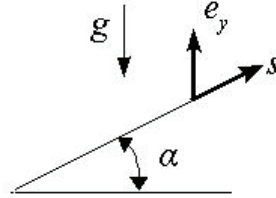


Figure 2. Schematic of the gravitational force acting over an inclined pipe flow.

$$\vec{g} = g e_y \quad (4)$$

$$\vec{g} \cdot s = -g s \sin \alpha \quad (5)$$

Thus the Eq. (1) assumes the form

$$\nabla \cdot (A_i \nabla u) - \frac{dp}{dz} - \rho_i g s \sin \alpha = 0 \quad (6)$$

In the above equations the flow, with velocity u , is aligned to co-ordinate z . The kinetic energy of turbulence is represented by κ and the dissipation per unit turbulence kinetic energy is denoted by ω . Because the flow is assumed to be fully developed, the same pressure gradient dp/dz is considered for both phases. Note though, that like the interface position y_{int} , dp/dz is an unknown variable that will be determined as a function of the given volumetric flow rates. Other terms appearing in Eqs. (1) - (3) are $A_i = \mu_i + \mu_{ti}$, $B_i = \mu_i + \sigma_2 \mu_{ti}$, $C_i = \mu_i + \sigma_1 \mu_{ti}$ and $S_i = A_i \nabla u \cdot \nabla u$. The eddy viscosity for phase i is $\mu_{ti} = \alpha_2 \rho_i \kappa / \omega$. The κ - ω model parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_1$ and σ_2 are non-dimensional quantities. The symbol ∇ denotes the gradient operator in the cross-section analyzed. Thus, in terms of the canonical base given by the Cartesian unit vectors e_x and e_y , we have $\nabla = \frac{\partial}{\partial x} e_x + \frac{\partial}{\partial y} e_y$. The model is completed by

introducing boundary and interfacial conditions. The conditions on the symmetry boundary Γ_s are $\nabla u \cdot n = 0$, $\nabla \kappa \cdot n = 0$ and $\nabla \omega \cdot n = 0$ by the fact that the total momentum exchange of each phase through the symmetry vertical plane vanishes, where n is the outward pointing unit vector on Γ_s . The pipe boundary Γ_c is split into Γ_{cf} and Γ_{cg} , according to the phase which is in contact with the wall. Thus, the boundary conditions on Γ_c are $u = 0$, $\kappa = 0$ and $\omega = \overline{\omega}_{ci}$ on Γ_{ci} , meaning that the prescribed value depends on the properties of the phase which is in contact with the pipe surface.

It is well known that ω goes to infinity on smooth pipe walls. In order to avoid this singular behavior, we employ the same wall boundary condition implemented in the DEFT incompressible flow solver (Segal, 2006), which is given by

$$\overline{\omega}_{ci} = \frac{2 \mu_i}{\beta_o \rho_i Y_p^2} \quad (7)$$

where $\beta_o = 0.072$ is a model constant and Y_p is the distance of the closest grid point to the wall. At the interface Γ_{int} , we impose continuity of the shear stress and consider that the interface is smooth. Thus the turbulence kinetic energy at the interface is set as $\kappa_{int} = 0$. The value imposed for ω at the interface is denoted by $\overline{\omega}_{int}$ and is also computed using Eq. (7). However, Eq. (7) yields different values when computed from the different sides of the interface. Here we

choose $\bar{\omega}_{int}$ to be the higher of these values in order to ensure that the interface is approximated as smooth when viewed from either side. The conditions are completed setting $\sum_{i=f,g} A_i \nabla u \cdot n_i = 0$.

In the next step the Eqs. (2), (3), (6) are written in a dimensionless form. Defining the new non-dimensional variables as

$$\begin{aligned} x^* &= \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad z^* = \frac{z}{d} \\ u^* &= \frac{u}{u_o} \\ p^* &= \frac{p}{\rho_o u_o^2} \\ \rho^* &= \frac{\rho}{\rho_o} \\ \kappa^* &= \frac{\kappa}{u_o^2}, \quad \omega^* = \frac{\omega}{u_o} \end{aligned} \quad (8)$$

where u_o e ρ_o are arbitrary reference quantities. Thus the Eqs. (2), (3), (6) in a dimensionless form turn on

$$\nabla^* \cdot (A_i^* \nabla^* u^*) - \frac{dp^*}{dz^*} - \frac{1}{Fr^2} \rho^* \sin \alpha = 0 \quad (9)$$

$$\nabla^* \cdot (B_i^* \nabla^* \kappa^*) - \beta_2 \rho_i^* \kappa^* \omega^* + S_i^* = 0 \quad (10)$$

$$\nabla^* \cdot (C_i^* \nabla^* \omega^*) - \beta_1 \rho_i^* \omega^{*2} + \frac{\alpha_l \omega^*}{\kappa^*} S_i^* = 0 \quad (11)$$

where $A_i^* = A_i / \rho_o u_o d$, $B_i^* = B_i / \rho_o u_o d$, $C_i^* = C_i / \rho_o u_o d$ and $S_i^* = A_i \nabla^* u^* \cdot \nabla^* u^*$. The boundary and interfacial conditions are rewritten as: $\nabla^* u^* \cdot n = 0$, $\nabla^* \kappa^* \cdot n = 0$ and $\nabla^* \omega^* \cdot n = 0$ on Γ_s^* ; $u^* = 0$, $\kappa^* = 0$ on Γ_c^* and $\omega^* = \bar{\omega}_{ci}^*$ on Γ_{ci}^* ; $\kappa_{int}^* = 0$, $\omega^* = \bar{\omega}_{int}^*$ and $\sum_{i=f,g} A_i^* \nabla^* u^* \cdot n_i = 0$ on Γ_{int}^* .

The problem described above can be recast in variational form as follows: find $u^* \in V_u$, $\kappa^* \in V_\kappa$ and $\omega^* \in V_\omega$, for any $\phi \in V_\phi$ and any $\varphi \in V_\varphi$, such that

$$\sum_{i=f,g} \int_{\Omega_i^*} A_i^* \nabla^* \phi \cdot \nabla^* u^* d\Omega_i^* = - \sum_{i=f,g} \int_{\Omega_i^*} \phi \frac{dp^*}{dz^*} d\Omega_i^* - \sum_{i=f,g} \int_{\Omega_i^*} \phi \frac{\rho_i^* \sin \alpha}{Fr^2} d\Omega_i^* \quad (12)$$

$$\sum_{i=f,g} \int_{\Omega_i^*} B_i^* \nabla^* \phi \cdot \nabla^* \kappa^* d\Omega_i^* = \sum_{i=f,g} \int_{\Omega_i^*} \phi A_i^* \nabla^* u^* \cdot \nabla^* u^* d\Omega_i^* - \sum_{i=f,g} \int_{\Omega_i^*} \phi \beta_2 \rho_i^* \omega^* \kappa^* d\Omega_i^* \quad (13)$$

$$\sum_{i=f,g} \int_{\Omega_i^*} C_i^* \nabla^* \phi \cdot \nabla^* \omega^* d\Omega_i^* = \sum_{i=f,g} \int_{\Omega_i^*} \phi \alpha_l \frac{\omega^*}{\kappa^*} A_i^* \nabla^* u^* \cdot \nabla^* u^* d\Omega_i^* - \sum_{i=f,g} \int_{\Omega_i^*} \phi \beta_1 \rho_i^* \omega^{*2} d\Omega_i^* \quad (14)$$

where,

$$V_u = \{u^* \in H_1(\Omega^*), u^* = 0 \text{ on } \Gamma_c^*\} \quad (15)$$

$$V_\kappa = \{\kappa^* \in H_1(\Omega_f^* \cup \Omega_g^*), \kappa^* = 0 \text{ on } \Gamma_c^*, \kappa^* = 0 \text{ on } \Gamma_{int}^*\} \quad (16)$$

$$V_\omega = \left\{ \omega^* \in H_1(\Omega_f^* \cup \Omega_g^*), \omega^* = \overline{\omega_{ci}^*} \text{ on } \Gamma_{ci}^*, \omega^* = \overline{\omega_{int}^*} \text{ on } \Gamma_{int}^* \right\} \quad (17)$$

$$V_\varphi = \left\{ \varphi \in H_1(\Omega^*), \varphi = 0 \text{ on } \Gamma_c^* \right\} \quad (18)$$

$$V_\varphi = \left\{ \varphi \in H_1(\Omega_f^* \cup \Omega_g^*), \varphi = 0 \text{ on } \Gamma_c^*, \varphi = 0 \text{ on } \Gamma_{int}^* \right\} \quad (19)$$

Note that the problem described above is not closed: the pressure gradient dp^*/dz^* and the interface position y_{int}/d , which ultimately defines the domains Ω_f^* and Ω_g^* , are unknown. The equations that close the model come from the requirement to meet the imposed flow rates Q_f^* and Q_g^* , i.e.,

$$Q_f^* = 2 \int_{\Omega_f^*} u^* d\Omega^* \quad (20)$$

$$Q_g^* = 2 \int_{\Omega_g^*} u^* d\Omega^* \quad (21)$$

The solution of the problem is obtained by using an iterative process that combines two numerical techniques. The first is an external Newton-Raphson method aimed to adjust y_{int}/d and dp^*/dz^* , in order to satisfy Eqs. (13), (14). The second, which we call the flow solver, runs internally and involves the finite element solution of the non-linear problem given by Eqs. (12) – (14), for given values of y_{int}/d and dp^*/dz^* . For a given pair of y_{int}/d and dp^*/dz^* , we have a numerical method to approximate and solve Eqs. (13), (14), which is described in detail by De Sampaio *et al.* (2008). Then we can compute the mismatch of the flow rates obtained for a given pair of y_{int}/d and dp^*/dz^* and the flow rates Q_f^* and Q_g^* imposed as problem data. Thus given the interface position and the pressure gradient, the finite element method is used to obtain an approximate numerical solution of Eqs. (12) – (14). A finite element mesh is constructed using a simple mesh generation routine based on the bipolar mapping described in literature by Shoham and Taitel (1984), Issa (1988), Newton and Behnia (2000), Biberg and Halvorsen (2000). The mesh generator employed provides a selective refinement of mesh close to the pipe wall and at the gas-liquid interface. Here the bipolar mapping is used only to generate a suitable mesh. The problem is solved using the standard Cartesian co-ordinate system (not the bipolar co-ordinate system). More details can be found in De Sampaio *et al.* (2008).

3. NUMERICAL RESULTS

In this section we present results for the interface position at various flow conditions, comparing our numerical data with the experimental data of Lioumbas *et al.* (2005).

As shown in Fig. 3, the agreement between the present numerical data and Lioumbas *et al.* (2005)'s data is good for $Re_{Ls} < 1000$ and is reasonable for $Re_{Ls} > 1000$ range and an inclination angle of -1 degree. As it can be noticed in De Sampaio *et al.* (2008) when the numerical model was tested in the single-phase simulation, it over-estimates the friction factor at transition. This explains the discrepancy observed between the present numerical simulation and Lioumbas *et al.* (2005) experimental results in that Reynolds number range. In addition, the $Re_{Ls} > 1000$ range approaches to the transition from smooth to wavy flow.

Figure 4 shows that the present numerical data are in excellent agreement with the sets of experimental data of the Lioumbas *et al.* (2005) for an inclination angle of -8 degrees, in spite of data have the majority of the liquid phase undergoing transition. There is only a small discrepancy between the numerical and experimental data on the $Re_{Ls} < 500$ range. As pointed out by Lioumbas *et al.* (2005) the liquid phase takes a main role in the transition from smooth to wavy stratified two-phase downward flow which may occurs between $2000 < Re_{Ls} < 2300$.

Table 1 presents a comparison between the present numerical data and those computed by Taitel and Dukler (1976), Shoham and Taitel (1984) and Issa (1988), for the values of y_{int}/d and the gas multiplier ϕ_G for the cases where air and water are flowing at superficial velocities of 1.0 m/s and 0.1 m/s, respectively, in a pipe of 25.4 mm inside diameter with two inclinations: horizontal (0°) and downward (-10°). The definition of ϕ_G is

$$\phi_G = \sqrt{\frac{(dp/dz)_{2P}}{(dp/dz)_{Gs}}} \quad (22)$$

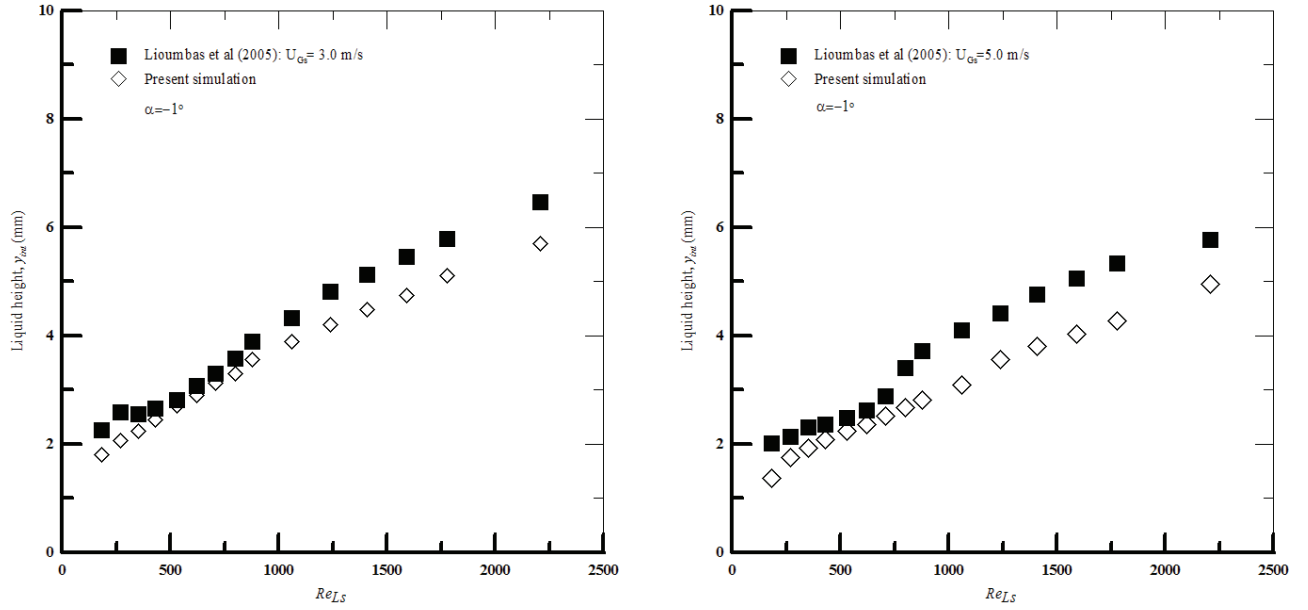


Figure 3. Numerical liquid height, y_{int} , as a function of Reynolds number, Re_{LS} , comparison with experimental data of Lioumbas *et al.* (2005).

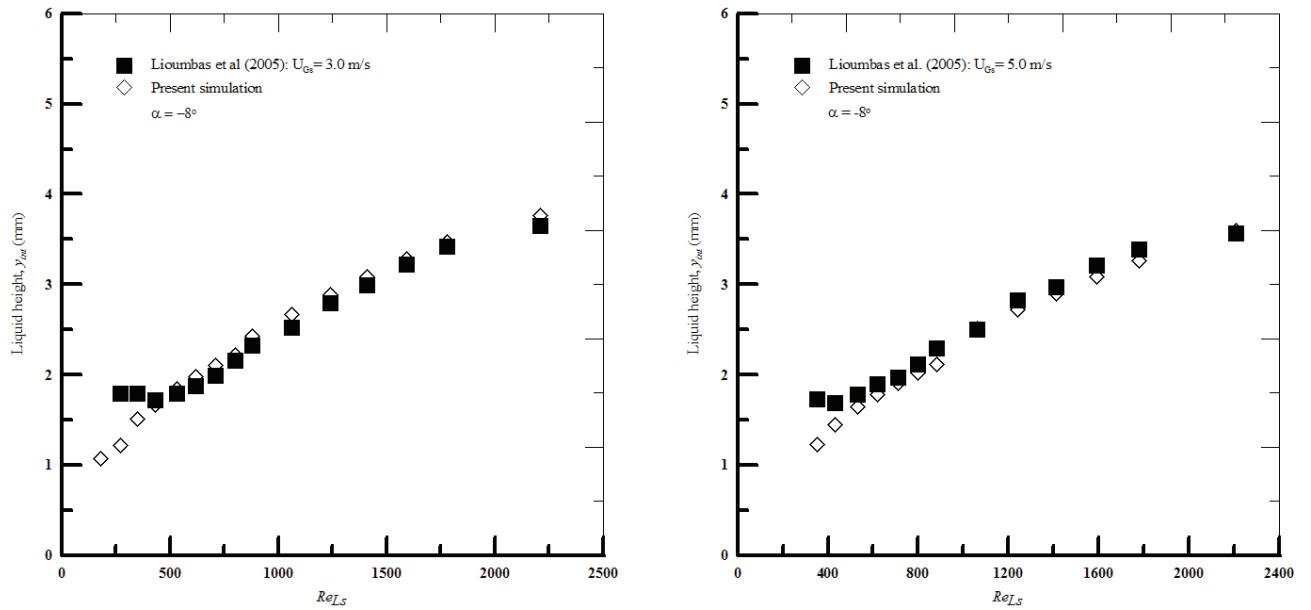


Figure 4. Numerical liquid height, y_{int} , as a function of Reynolds number, Re_{LS} , comparison with experimental data of Lioumbas *et al.* (2005).

Table 1. Comparison of the present numerical data with the numerical data of various authors.

	Present numerical data		Taitel and Dukler (1976)		Shoham and Taitel (1984)		Issa (1988)	
	y_{int}/d	ϕ_G	y_{int}/d	ϕ_G	y_{int}/d	ϕ_G	y_{int}/d	ϕ_G
$\alpha = 0^\circ$	0.53	2.8	0.59	3.7	0.66	5.5	0.59	3.7
$\alpha = -10^\circ$	0.19	0.81	0.17	1.86	0.18	1.85	0.16	0.73

where the subscript $2P$ means two-phase flow and G_s is for the gas single-phase flow case. The pressure gradient calculations for the gas flowing alone in that pipe consist of assigning the same gas properties and flow rates for both phases. We have also relaxed the interface conditions on κ and ω . This permits mimicking a gas single-phase computation using our two-phase computer code. Here, the agreement is close, in terms of the dimensionless liquid height y_{int}/d , with the all cited works. The parameter ϕ_G calculated by the present method is observed to be close with the results of Taitel and Dukler (1976) and Issa (1988) for the horizontal (0°) case while the downward (-10°) case shows a better agreement with the work of Issa (1988).

4. CONCLUSIONS

It was reported numerical solutions of stratified gas-liquid two-phase downward flow in slightly inclined circular pipes, applying the physical-numerical model proposed by De Sampaio *et al.* (2008) to predict liquid height (or liquid holdup) and pressure gradient for fully developed turbulent stratified gas-liquid downward flow, with smooth and horizontal interface in inclined circular pipes with inclination angles varying from 0° to -10° . The simulation data were compared with Lioumbas *et al.* (2005) experimental data and with those models of three reputed authors. The results have showed an excellent agreement with experimental data for an angle of inclination of -8 degrees, and a reasonable agreement for an angle of inclination of -1 degrees. The comparison with other numerical models indicates that the present model is close them in terms of the liquid height, while in terms of the parameter ϕ_G the agreement is close with two authors for 0 degrees and with one author for -10 degrees.

Finally, a concluding remark is that the effects of pipe inclination associated with the transition from smooth to wavy flow requires a better understanding on how to impose interfacial values for κ and ω if we are to expect a closer agreement with the stratified smooth-wavy two-phase downward flow.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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