A SOFTWARE FOR COMPUTER AUTOMATED RADIOACTIVE PARTICLE TRACKING

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Abstract. *TRAÇO-1 is the first software developed in Brazil for optimization and diagnosis of multiphase chemical reactors employing the technique known as "Computer Automated Radioactive Particle Tracking" whose main idea is to follow the movement of a puntual radioactive particle inside a vessel. Considering that this particle has a behavior similar of the phase under investigation, important conclusions can be achieved. As a preliminary TRAÇO_1 evaluation, a simulation was carried out with the aid of a commercial software called MICROSHIELD, version 5.05, to obtain values of photon counting rates at four detector surfaces. These countings were related to the emission of gamma radiation from a radioactive source because they are the main TRAÇO-1 input variables. Although the results that has been found are incipient, the analysis of them suggest that the tracking of a radioactive source using TRAÇO-1 can be well succeed, but a better evaluation of the capabilities of this software will only be achieved after its application in real experiments .*

Keywords: nuclear techniques, CARPT, dynamical systems, multiphase fluids

1. INTRODUCTION

Nowadays, several nuclear techniques for the control and the optimization of processes that occur in the interior of closed units have a worldwide acceptance. This work is concerned with the technique known as Computer Automated Radioactive Particle Tracking (CARPT), whose theoretical beddings are based on the Ergodic Hypothesis of Dynamical systems, a concept useful to describe point's positions in its ambient space as a function of time, like the motion of water in a pipe.

The technique CARPT consists of tracking the movement of one of the phases of a process with a radioactive particle in the form of a small sphere whose behavior is similar of the marked phase in order to estimate the mean velocity profile and the conditions of the process.

Recently, the Washington University had applied CARPT for the characterization of fluids in systems liquid-liquid, gas-solid and liquid-solid inside stirred tank reactors with satisfactory results (Rammohan et al., 2000, Dudukovic et al., 2005, Guha et al., 2007)

The general objective of this research was to develop a software to accomplish the application of the technique CARPT in the characterization of the phases of processes that occur in the interior of chemical reactors so that the quantity of radiation detectors used in the tests is the minimum one.

Conceptually, the part of the code that locates radioactive particle is innovative. Instead of adopting a calibration method for the determination of the counting curves versus detector-particle distance (or the determination of the counting maps through the Monte Carlo Method) and, after that, to use another method to optimize the localization of the particle from that curve, it was created an original combination of a mathematical model, properly adapted from emission tomography, and the Bayes´ iterative method for image reconstruction. Then, the software, called TRAÇO_1, is capable to transform part of the procedures of a CARPT application more simple. Now, it is possible to solve simultaneously, with only one iterative method: 1) The inverse problem of the localization of a radioactive particle; 2) the problem of the uncertainties due to the statistical nature of gamma radiation and to the small number of detectors.

2. METHODOLOGY

2.1 Radiation measurement simulation

In this new CARPT approach, the basic mathematical model applied in emission tomography was adapted so that the well known iterative algorithm of the maximization of the expectation (EM), a Bayesian solution of the inverse problem in tomographic image reconstructions derived from the Entropy method (Lange and Carlson., 1984), could be used in a 3D tracking of a radioactive particle, instead of given the traditional estimative of the source distribution in a plane. The EM algorithm was chosen for this research because it considers the image density distribution related to emission of gamma radiation as a Poisson distribution.

In order to perform the simulation considered in this work, the commercial software called MICROSHIELD 5.05 was used to calculate the counting rates at four points in the air medium from the gamma radiation flux emitted by a gold puntual radioactive source after it had been placed in several positions inside a water cylinder chosen as a mathematical phantom.

The coordinates of each point and the instantaneous position of the radioactive source had been chosen in way that they had the same origin in a XYZ rectangular coordinate system that was coincident with the center of the lower base of the cylinder, as shown in Fig. (1), where PD is the distance travelled by a photon emitted by the source and detected in D. The segment PD was divided into two distinct stretches, PC and CD, corresponding to the distances travelled by the photon in the water and in the air, respectively. The points A, B and D had the same high, equal that of detector´s center. This last information was useful to calculate PC and CD using trigonometric relations.

In addition, the simulated geometry consisted of involving the cross section of the cylinder by four radiation detectors and the reactor volume inscribed in a three-dimensional arrangement with fixed cells (reconstruction matrix) so that the linear dimension of each cell or voxel had the same lenght of the particle´s diameter as it is illustrated in the Fig. (2). Ten of the all source locations are described in Tab.(1) and the other simulated parameters are described in Tab.(2).

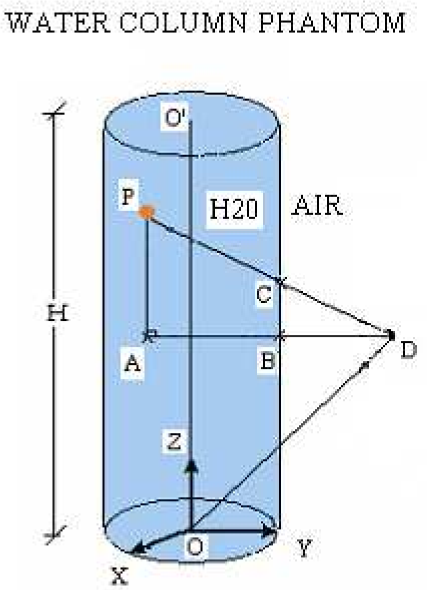

Figure 1. Schematic view of the phantom , one of the detecion points used in the simulations (point D) and the arbitrary position of the ^{198}Au radioactive puntual source inside the phantom (red point - P). H=32.0cm

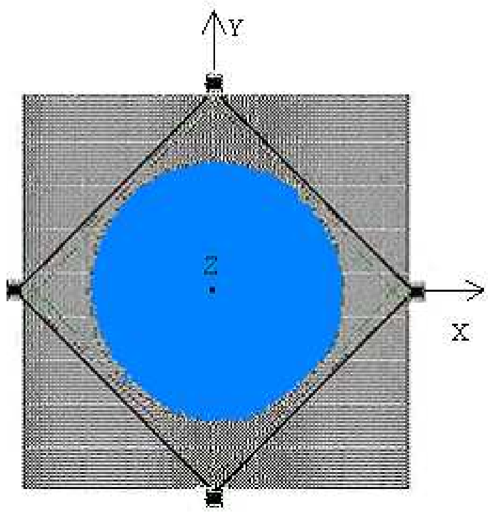

Figure 2. Upper view of the reactor surrounded by four detectors. The 2D black bottom matrix contains $4x10^4$ voxels . Scale: 1:10.

Table 1. Radioactive particle´s simulated positions ($x10^{-2}$ m).

Position / Coordinates	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
X	0.00	-12.00	-10.00	-10.15	-12.00	0.00	12.0	10.00	10.00	12.00
Y	15.00	14.00	5.00	-5.00	-14.00	-15.00	-14.00	-5.00	5.00	14.00
Z	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00

Table 2. The main simulated parameters

1	Mixer high ($x10^{-2}$m)	32.0
2	Cross section diameter ($x10^{-2}$m)	40.0
3	Material in the mixer	water
4	Radioisotope/source geometry/Activity (Bq)	^{198}Au/point/3.1471 $x10^6$
5	Linear attenuation coefficient($x10^2m^{-1}$)	0.1061
6	Photon energy (Mev)	0.4118
7	Distance detector -cross section center ($x10^{-2}$m)	35.0
8	Diameter of the detector´s surface ($x10^{-2}$m)	2.54
9	Lenght of the detector´s collimator ($x10^{-2}$m)	0.0
10	Coordinates of detector´s surface center ($x10^{-2}$m)	(0,-35,30); (35,0,25); (0,35,30); (-35,0,25)

2.2 Application of the EM Algorithm for the localization of the radioactive particle

The following steps lead sumarilly to the Expectation Maximization (EM) algorithm developed by Lange and Carson (1984). Although their achievements had originally been devoted for 2D image reconstruction processes (CT scan or SPECT), we can use a 3D coordinate system of the pixels (voxels) in order to extend those results for 3D radioactive particle tracking like this:

Nj is the probability density function of the photons emitted by voxel j per unit area with expected value v_j;

The product $v_j \times p_{ij}$ is the expected value of the quantity of photons emitted by the j^{th} voxel and detected in the i^{th} detector´s position ;

The product $v_j \times \sum_{i=1}^{m} p_{ij}$, equal to f_j , is the expected value of the quantity of photons emitted by the j^{th} voxel and detected in any of the m^{th} detector´s position;

The product $\sum_{j=1}^{n} v_j \times p_{ij}$ is the expected value of the quantity of photons detected in the i^{th} detector´s position that have been originated in n voxels;

ni´ is the value of counts in the detector´s position i only in the photopeak region so that ni´ Є $\gamma_i = \sum_{j=1}^{n} N_{ij}$ .and the expected value of ni´ is expressed in the Eq.(1)

$$\sum_{j=1}^{n} v_j \times p_{ij} \tag{1}$$

The objective of this algorithm is to find the elements of the vector $v_j' = (v_1\, v_2 \ldots v_n)^T$ from the known values p_{ij} and $n_i´$. Therefore , it can be able to extract the values v_j from the values of f_j given in the Eq. (2):

$$f_j = v_j \times \sum_{l=1}^{m} p_{lj} \tag{2}$$

Consider the matrix W whose elements are defined in the Eq.(3)

$$w_{ij} = \frac{p_{ij}}{\sum_{l=1}^{m} p_{lj}} \tag{3}$$

The expected value of $n_i´$ is expressed in the Eq. (4):

$$E(y_i) = \sum_{j=1}^{n} v_j \times p_{ij} \tag{4}$$

But $n_i´$ can also be expressed, after a simple arrangement among Eq.(2), Eq. (3) and Eq. (4), by Eq. (5):

$$E(\gamma_i) = \sum_{j=1}^{n} \frac{f_j}{\sum_{l=1}^{m} p_{lj}} \times w_{ij} \times \sum_{l=1}^{m} p_{lj} = \sum_{j=1}^{n} f_j \times w_{ij} \tag{5}$$

For the sake of simplicity, Eq. (5) can be described in a more compact form like that by Eq.(6):

$$E(\gamma) = W \times f \tag{6}$$

The elements W_{ij} are probabilities ($0<p_{ij}<1$) and the f_{ij} elements are the mean values of the quantities of photons emitted by the voxels (f_j>0). Then , the estimation of the probability E (γ_i) of the photons emitted by the j^{th} voxel to be detected in the i^{th} detector´s position , as expressed in Eq.(7)

$$E(\gamma_i) = (Wf)_i > 0, \forall i \text{ and } \sum_{i=1}^{n} w_{ij} = 1, \forall j \tag{7}$$

Maximizing the Eq.(5), the biggest probability of occurrence of the events n_i is gotten for specific values of the vector (f_j). So, these values are gotten after the maximization of the “likelihood function” L(f), like this:

The function p(k) is the Poisson probability for a random variable γ related to the values n_i in accordance with Eq.(11) resulting in the Eq. (8).

$$\mathrm{p(k)} = \frac{(E(\gamma_i))k}{k!} \times \exp[-(\mathrm{E}(\gamma_i))] = \frac{(W.f)_{ik}}{k!} \times e\mathrm{xp}(-\mathrm{Wf})_i \tag{8}$$

The likelihood function L(f) for the unknown parameter $(Wf)_i$ is the probability of occurrence of the mean values of n_i, defined by Eq.(9):

$$L(f) = \prod_{i=1}^{m} \mathrm{p}((\mathrm{Wf})\mathrm{i}, \mathrm{n_i}') = \prod_{i=1}^{m} \frac{(W.f)^{n_i'}}{n_i'!} \times e\mathrm{xp}(-\mathrm{Wf})_i \tag{9}$$

The biggest value of L(f) is proportional to the biggest value of log L(f). Then, after some calculations, the wanted value of L(f) is that viewed in the Eq. (10):

$$L(f) = \log L(f) = \sum_{i=1}^{m} [n_i' \times \log(Wf)_i - (Wf)_i] \tag{10}$$

So, the EM algorithm search the vector f that maximizes the function L(f), being an iterative method able to get the Bayes solution after k iterations , as presented in the Eq. (11).

$$f_{k+1} = f_k \times (\mathrm{W^T}) \frac{n'}{Wf} \text{, k= 0,1,....} \tag{11}$$

2.3 Mathematical Model

It was adopted the following nomenclature to charaterize the mathematical model for the direct problem of the emission of photons by the puntual radioactive source located on a voxel and its corresponding counting rates in the detection point on the detector:

W - reconstruction matrix with n elements or voxels (the squares with the smallest area in Fig. (2))
m - quantity of radiation detectors (there are m positions for point D);
p_{ij} - probability of a photon emitted by the j^{th} voxel to be detected in the i^{th} detector´s surface center;
N - vector with m values of counts measured during Δt and multiplied by the detector´s surface area (a circle with 2.54 x10 $^{-2}$m diameter);
f - vector with n-1 null values and only one unity value corresponding to the instantaneous position of the particle on the matrix W (f was the particle´s image vector during the time interval “ t+Δt”),e.g., f=[0 0 0 1 0 ...0] ;
H_{ij} - solid angle subtended by the i^{th} detector´s surface center when viewed from the j^{th} voxel's center;
A_{ij} - attenuation factor for a photon that was emitted by the j^{th} voxel's center and measured in the i^{th} detector´s surface center ;
R_{ij} - radioactive particle´s probability to be on the j^{th} voxel considering the direct relation between W_{ij} and N_i;
$d_{ij'}$ - distance that a photon had travelled along the j^{th} voxel before it was measured in the detector i;

Then, the factors H_{ij} were determined by the Sympson's rule while each Aij was obtained by direct calculation of the exponential function given by Eq. (12), considering that the distances dij were measured on two stretches: one in the water and another in the air , as it had already seen in the section 2.1.

$$A_{ij} = \exp[-\sum \mu_{j'} \times d_{ij'}] \tag{12}$$

while the probabilities p_{ij} were calculated in accordance with the product given by Eq. (13):

$$p_{ij} = H_{ij} \times A_{ij} \tag{13}$$

and the equality of Eq.(14) was an estimate of the total probability of a photon measured in detector i had been originated in the j^{th} voxel:

$$W_{ij} = p_{ij} \times R_{ij} \tag{14}$$

Then, the direct problem resulting from these mathematical model established a relationship between the intensity of the source on the the j^{th} voxel (f_j=0 or f_j=1) and the photon counts on the i^{th} detector´s position, that is, the linear system expressed by Eq. (15):

$$N_i = \sum_{j=1}^{n} W_{ij} \times f_j \qquad (15)$$

2.4 Development of the algorithm

In order to evaluate the f intensities using the iterative solution of Eq.(15), it was written in C++ language the algorithm of the first version of the software, called TRAÇO_1. The main input variables of TRAÇO_1 are the m counts and the time interval between two successive counts whilst its output variables are the calculated source positions and mean velocities of the particle from which the concentration field or velocity profiles of the particle are obtained.

The software MICROSHIELD 5.05 was used to simulate the counting rates at D derived from the ^{198}Au radioactive particle fixed at ten positions. These positions, that were introduced into TRAÇO_1 with a time interval equal 1s, had a circular symmetry in relation to the positions P1 to P4 described in Tab. (1) in order to simulate the simple movement of the particle in a closed track passing around the center of the phantom. Next, the parameters of Tab.(2) had been introduced in the code TRAÇO_1 and it was run using a processor with 3.0 Gb clock frequency, 1Gb RAM and 80Gb HD to provide us, in the sequence bellow:

1. the reconstruction matrix W with n=4x10^6 voxels;
2. twelve f vectors related to the source positions ;
3. the values of each X, Y and Z position coordinates;
4. the three components of the particle's instantaneous velocity between two successive positions.

3. RESULTS

The code ran perfectly and it was last, approximately, 20s to calculate the three components of each particle's instantaneous velocity employing the PC configuration cited in the last section.

The data shown in Tab.(3) were generated by the software TRAÇO_1 and the mean square errors associated to each source position were determined and they are all shown in Fig.(3).

Table3. Values of the coordinates from P1 to P10 evaluated by TRAÇO_1

Position ($\times 10^{-2}$ m)	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
X	0.7	-11.55	-10.15	-10.15	-11.9	0.00	11.9	10.15	10.15	11.9
Y	15.05	12.95	5.25	5.25	-13.3	-15.05	-13.3	-5.6	5.6	13.3
Z	11.55	8.4	10.85	10.85	8.4	11.55	10.85	10.85	8.4	11.55

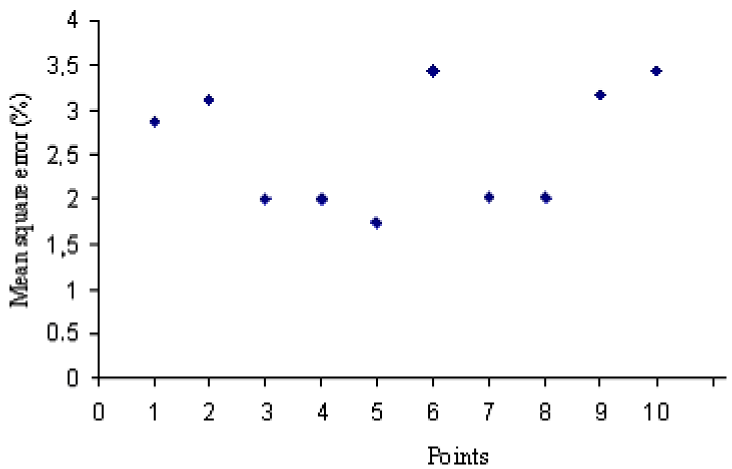

Figure 3. Distribution of the errors associated to the localization of each point of Tab (3).

Figure (4) is the output of the velocity field in the XY plane at Z= 0.10 m calculated by the sofware TRAÇO_1 from the counts simulated by MICROSHIELD5.05 considering the time interval between two instantaneous positions equal to 1.0 s. The total of vectors is 12 and they form a counterclock track around the Z axis.

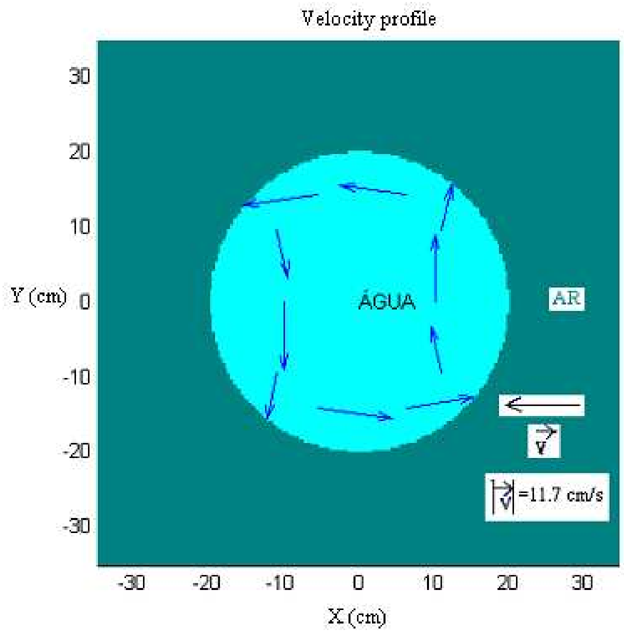

Figure 4.Velocity profile generated from TRAÇO_1 data considering that the ^{198}Au radioactive particle was tracked through the XY plane at Z= 0.10m with a time interval equal 1s.

4.CONCLUSIONS

The authors are hopeful with the main features of TRAÇO_1 although these first simulated results don't mean the final test for a complete evaluation of its potential for troubleshooters. After TRAÇO-1 had been executed to track displacements of the radioactive source in the phantom, it was verified that the maximum value of the mean square error found in the calculus of the instantaneous particle´s position is significantly low. Consequently, the determination of the radioactive particle trajectories and its velocity profiles in the medium by TRAÇO_1 seem feasible and the continuity of this research, that is, the application of the software in a real tracking, is now justified and it will demonstrate the accuracy of TRAÇO_1 outputs more clearly.

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