POLAR REPRESENTATION OF CENTRIFUGAL PUMP HOMOLOGOUS CURVES

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Abstract. *Essential for any mathematical model designed to simulate flow transient events caused by pump operations is the pump performance data. The performance of a centrifugal pump is characterized by four basic parameters: the rotational speed, the volumetric flow rate, the dynamic head, and the hydraulic torque. Any one of these quantities can be expressed as a function of any two others. The curves showing the relationships between these four variables are called the pump characteristic curves, also referred to as four-quadrant curves. The characteristic curves are empirically developed by the pump manufacturer and uniquely describe head and torque as functions of volumetric flow rate and rotation speed. Because of comprising a large amount of points, the four-quadrant configuration is not suitable for computational purposes. However, it can be converted to a simpler form by the development of the homologous curves, in which dynamic head and hydraulic torque ratios are expressed as functions of volumetric flow and rotation speed ratios. The numerical use of the complete set of homologous curves requires specification of sixteen partial curves, being eight for the dynamic head and eight for the hydraulic torque. As a consequence, the handling of homologous curves is still somewhat complicated. In solving flow transient problems that require the pump characteristic data for all the operation zones, the polar form appears as the simplest way to represent the homologous curves. In the polar method, the complete characteristics of a pump can be described by only two closed curves, one for the dynamic head and other for the hydraulic torque, both in function of a single angular coordinate defined adequately in terms of the quotient between volumetric flow ratio and rotation speed ratio. The usefulness and advantages of this alternative method are demonstrated through a practical example in which the homologous curves for a pump of the type used in the main coolant loops of a pressurized water reactor (PWR) are transformed to the polar form.*

Keywords*: centrifugal pump, pump homologous curves, pump failure, flow loop analysis, loss of flow accident*

1. INTRODUCTION

Centrifugal pumps comprise an ample variety of pumps in which pumping of fluid or generation of pressure is affected by the rotary motion of one or several impellers. Every centrifugal pump consists of two principal parts: an impeller which forces the fluid into a rotary motion by impelling action; and the pump casing, which directs the fluid to the impeller and leads it away under a higher pressure. The impeller is mounted on a shaft which is supported by bearings and driven through a flexible or rigid coupling by a driver. The pump casing includes suction and discharge nozzles, supports the bearings, and houses the rotor assembly. Readers are referred to Stepanoff (1957) for theory, design, and application of centrifugal pumps.

Flow loop transient events are strongly influenced by pump transients and non-rarely initiated by them. Pump failures, due to loss of power, shaft break, rotor locking, or inadvertent operator action, are events of relatively common occurrence in all power plants.

Flow transients caused by pump operations are usually severe, and the pipelines should be designed to withstand positive and negative pressures caused by these operations. In nuclear reactor systems, loss-of-flow accidents (LOFAs) caused by failure of one or more pumps in the primary coolant loops may be injurious to the fuel elements as a result of insufficient flow to the reactor core to prevent the degradation of the heat transfer mechanism due to film boiling occurrence. The film boiling regime is characterized by the formation of a continuous vapor film over the heated surface which may cause the rupture of the fuel rod in view of a sharp increase in the fuel cladding surface temperature. On the other hand, if a large quantity of cold coolant from the inactive loop were suddenly inserted into the reactor core having a negative temperature coefficient, there would be an undesired increase in the reactor power output. Therefore, the accurate prediction of loop flow as a function of time during the flow transient is essential for optimizing the core power for maximum safe operation.

Detailed flow transient analysis can be found in Boyd et al. (1961), Fuls (1968), Wylie and Streeter (1978), Chaudhry (1979), Grover and Koranne (1981), Todreas and Kazimi (1990), Araya, Murao and Iwamura (1995), Tong and Weisman (1996). RETRAN-02 (McFadden et al., 1981) and RELAP5 (Ransom et al., 1985) are examples of existing computer codes for transient thermal-hydraulic analysis of complex fluid flow systems. The PANTERA-2 code (Veloso, 2003) has been designed to carry out rod bundle subchannel analysis in conjunction with multi-loop simulation.

Fundamental to any mathematical model designed to simulate flow transient events caused by multi-pump operation is the pump performance data. The basic parameters that characterize the pump performance are the rotational speed (ω), the volumetric flow rate or discharge (Q), the dynamic head (H), and the shaft torque (τ). Two of these quantities may be considered independent, that is, any one of the four quantities ω, Q, H, and τ can be expressed as a function of any two others. The curves describing the relationships between these four variables are called the pump characteristic curves. The forms in which the pump characteristics have been handled for computational purposes are discussed in the remaining sections.

2. DYNAMICS OF A CENTRIFUGAL PUMP

The angular velocity of the rotating parts of a motor-pump assembly is related to the torques acting on the assembly by the angular momentum equation. Treating the rotating assembly as a rigid body, the equation of motion may be expressed as

$$I\frac{d\omega}{dt}=\tau(t), \tag{1}$$

where I is the moment of inertia (kg m^2) of the rotating parts about the rotation axis, ω is the angular velocity (s^{-1}), and $\tau(t)$ denotes the net torque (Nm) exerted on the rotating parts as a function of time t in seconds.

The net torque is given by

$$\tau(t)=\tau_m(t)-\tau_h(t)-\tau_f(t), \tag{2}$$

where $\tau_m(t)$ represents the motor torque, $\tau_h(t)$ is the hydraulic torque exerted on the fluid by the pump impellers, and $\tau_f(t)$ expresses the frictional and viscous torque acting on the pump shaft, commonly referred to as bearing friction and windage torque. Under steady-state operating conditions, the motor torque just balances the hydraulic torque and the friction and windage torque, resulting in constant rotational speed.

The hydraulic torque is calculated as a function of volumetric flow rate Q(t) and angular velocity $\omega(t)$ from the pump performance curves. The motor torque versus rotational speed is normally available from the motor manufacturer. The frictional and viscous torque is usually related to the rotational speed by an empirical relationship. The value of this torque is dependent on the sign of the pump speed.

In the analysis of flow loop transient events caused by operations of pumps, Eq. (1) must be solved together with the fluid conservation equations. During any transient, the motor provides torque for (1) overcoming the hydraulic resistances in the loops, (2) overcoming frictional losses in the pumping rotating parts, (3) accelerating the fluid in the loops, and (4) accelerating the pump rotating parts. At steady-state, no acceleration is involved, and only the first two items need be considered.

3. PUMP PERFORMANCE CURVES

Figure 1 illustrates the various modes of pump operation. During a flow loop transient caused by a pump failure, also called coastdown transient, the pump passes from the zone of normal pump (quadrant I) to the zone of energy dissipation (quadrant II), with reverse flow and positive rotation. Then, unless the rotor shaft is provided with an antireverse ratchet, the pump may pass to the zones of normal turbine (quadrant III) and reverse pump (quadrant IV), with negative and positive flows, respectively.

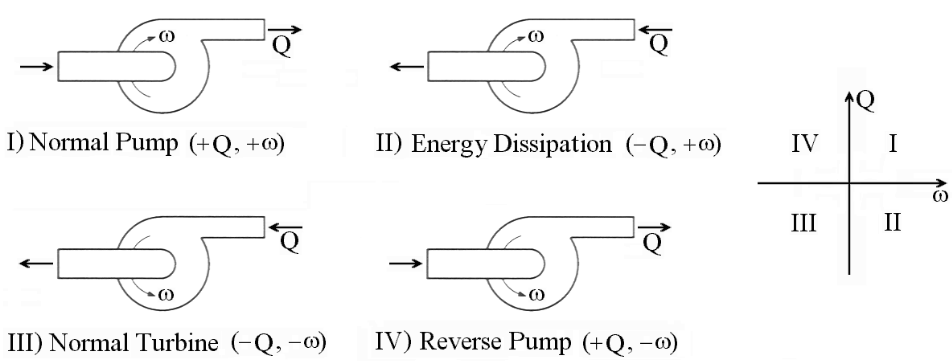

Figure 1. Operation modes of a centrifugal pump.

Interations between the pump and the fluid can be described by the pump characteristic curves which normally relate the pump head (H) and torque (τ) to the volumetric flow rate (Q) and pump angular velocity (ω). These curves, frequently referred to as four-quadrant curves, are generally available from pump manufacturers. A typical set of four-quadrant curves as reported by Stepanoff (1957) is shown in Fig. 2. Both positive and negative values for each of the four quantities are represented. In order to make the data applicable to all similar pumps, irrespective of pump size and speed, the head, volumetric flow rate, torque, and rotation speed are expressed in percentage of those values at the best efficiency point.

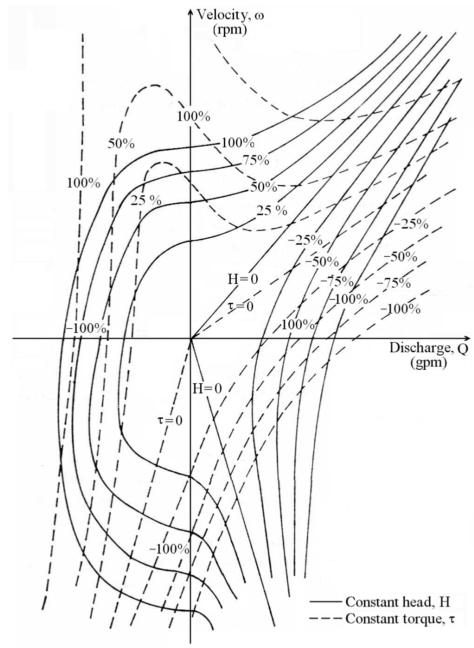

Figure 2. Typical pump characteristic four-quadrant curves.

If the complete characteristic data are not available, the characteristics of a pump having about the same specific speed may be used as an approximation. The specific speed of a centrifugal pump is defined as

$$n_s = \frac{n_r \sqrt{Q_r}}{H_r^{3/4}} . \tag{3}$$

In metric units, n_r is in revolutions per minute (rpm), Q_r is in m^3/s, and H_r is in m; in American units, n_r is in rpm, Q_r is in gallons per minute (gpm), and H_r is in ft. The subscript r denotes the rated quantities, that is, the values of variables at the point of best efficiency.

The disadvantage in using the four-quadrant curves for numerical purposes are the need for two-dimensional interpolation, the large amount of points needed to define the entire data domain, and the fact that the domain is infinite in extent. These difficulties can be overcome by development of homologous curves by using a homologous transformation based on the centrifugal pump similarity relationships. Such a transformation collapses the four-quadrant characteristic curves for head (or torque) onto a single, bounded, dimensionless curve covering eight octants. Thus there is a separate set of eight partial curves for head and torque. Conversion of four-quadrant characteristic curve to homologous curves is a simple although somewhat tedious procedure.

The homologous curves express head and torque ratios as a function of volumetric flow and speed ratios. The functional relationships may be written as

$$\frac{h}{\alpha^2} = f\left(\frac{v}{\alpha}\right) \quad or \quad \frac{h}{v^2} = f\left(\frac{\alpha}{v}\right)$$

and

$$\frac{\beta}{\alpha^2} = f\left(\frac{v}{\alpha}\right) \quad or \quad \frac{\beta}{v^2} = f\left(\frac{\alpha}{v}\right).$$

The dimensionless-homologous quantities v, α, h, and β are defined as follows:

$$v = \frac{Q}{Q_r} \quad \textit{(volumetric flow ratio)}$$

$$\alpha = \frac{\omega}{\omega_r} = \frac{n}{n_r} \quad \textit{(speed ratio)}$$

$$h = \frac{H}{H_r} \quad \textit{(dynamic head ratio)}$$

$$\beta = \frac{\tau}{\tau_r} \quad \textit{(hydraulic torque ratio)}$$

where n = ω/2π is the pump speed in revolutions per second. As before, subscript r represents the rated quantities, i.e., the values of Q, ω, n, h, and τ at the point of maximum efficiency.

For the purpose of constructing the homologous representation, each of the eight octants are identified by three capital letters, according to the following convention:

1st letter

H – curve for dynamic head
B – curve for hydraulic torque

2nd letter

A – abscissa is v/α and ordinate is h/α^2 or β/α^2
V – abscissa is α/v and ordinate is h/v^2 or β/v^2

3rd letter

N – normal pump (+Q, +ω)
D – energy dissipation (−Q, +ω)
T – normal turbine (−Q, −ω)
R – reverse pump (+Q, −ω)

Homologous head and torque curves for a centrifugal pump of the type used in primary coolant loops of a pressurized water reactor (PWR) are shown in Fig. 3 and Fig. 4, respectively. These figures were constructed in the light of the homologous data presented in Tab. 1 and Tab. 2. The intermediate points that compose each curve segment in these figures were obtained from the tabular data by using a Hermite cubic spline interpolation scheme.

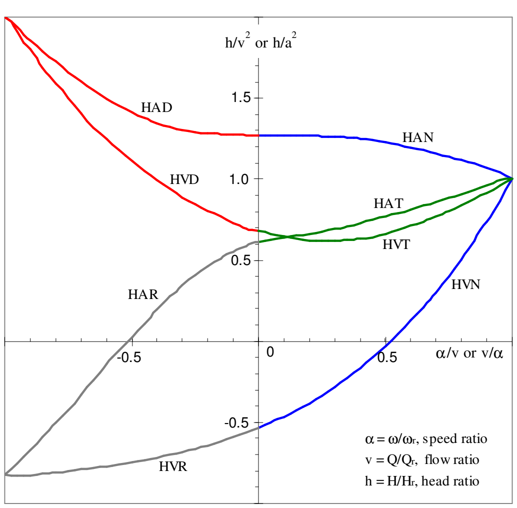

Figure 3. Homologous pump head curves.

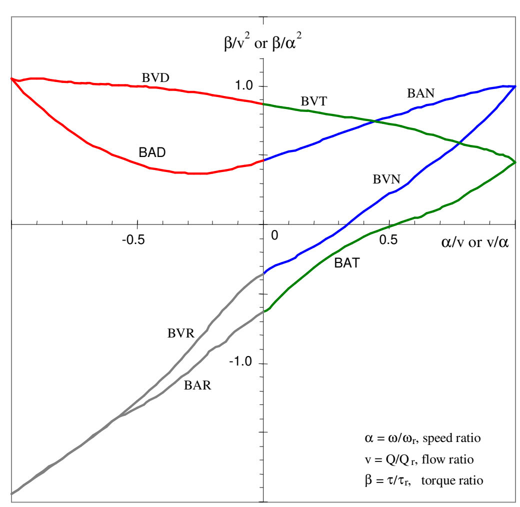

Figure 4. Homologous pump torque curves.

Table 1. Homologous pump head data.

HAN		HVN		HAD		HVD		HAT		HVT		HAR		HVR	
v/α	h/α²	α/v	h/v²	v/α	h/α²	α/v	h/v²	v/α	h/α²	α/v	h/v²	v/α	h/α²	α/v	h/v²
0.0	1.269	0.0	-0.532	-1.0	1.998	-1.0	1.998	0.0	0.611	0.0	0.680	-1.0	-0.824	-1.0	-0.824
0.1	1.268	0.1	-0.466	-0.9	1.854	-0.9	1.799	0.1	0.639	0.1	0.645	-0.9	-0.671	-0.9	-0.826
0.2	1.268	0.2	-0.384	-0.8	1.724	-0.8	1.593	0.2	0.662	0.2	0.623	-0.8	-0.504	-0.8	-0.811
0.3	1.262	0.3	-0.284	-0.7	1.605	-0.7	1.405	0.3	0.692	0.3	0.617	-0.7	-0.330	-0.7	-0.791
0.4	1.251	0.4	-0.165	-0.6	1.497	-0.6	1.249	0.4	0.731	0.4	0.631	-0.6	-0.142	-0.6	-0.772
0.5	1.228	0.5	-0.032	-0.5	1.413	-0.5	1.112	0.5	0.773	0.5	0.663	-0.5	0.024	-0.5	-0.749
0.6	1.194	0.6	0.127	-0.4	1.346	-0.4	0.995	0.6	0.812	0.6	0.715	-0.4	0.196	-0.4	-0.717
0.7	1.157	0.7	0.300	-0.3	1.302	-0.3	0.889	0.7	0.858	0.7	0.770	-0.3	0.344	-0.3	-0.684
0.8	1.117	0.8	0.501	-0.2	1.282	-0.2	0.806	0.8	0.906	0.8	0.839	-0.2	0.462	-0.2	-0.642
0.9	1.073	0.9	0.739	-0.1	1.275	-0.1	0.731	0.9	0.962	0.9	0.914	-0.1	0.554	-0.1	-0.591
1.0	1.000	1.0	1.000	0.0	1.269	0.0	0.680	1.0	1.000	1.0	1.000	0.0	0.611	0.0	-0.532

Table 2. Homologous pump torque data.

BAN		BVN		BAD		BVD		BAT		BVT		BAR		BVR	
v/α	β/α²	α/v	β/v²	v/α	β/α²	α/v	β/v²	v/α	β/α²	α/v	β/v²	v/α	β/α²	α/v	β/v²
0.0	0.462	0.0	-0.354	-1.0	1.056	-1.0	1.056	0.0	-0.632	0.0	0.872	-1.0	-1.941	-1.0	-1.941
0.1	0.524	0.1	-0.258	-0.9	0.867	-0.9	1.054	0.1	-0.461	0.1	0.839	-0.9	-1.815	-0.9	-1.815
0.2	0.587	0.2	-0.158	-0.8	0.718	-0.8	1.031	0.2	-0.312	0.2	0.812	-0.8	-1.689	-0.8	-1.689
0.3	0.653	0.3	-0.045	-0.7	0.595	-0.7	1.018	0.3	-0.187	0.3	0.784	-0.7	-1.559	-0.7	-1.559
0.4	0.721	0.4	0.097	-0.6	0.506	-0.6	1.008	0.4	-0.084	0.4	0.760	-0.6	-1.417	-0.6	-1.418
0.5	0.775	0.5	0.221	-0.5	0.437	-0.5	0.996	0.5	-0.012	0.5	0.729	-0.5	-1.324	-0.5	-1.274
0.6	0.835	0.6	0.342	-0.4	0.392	-0.4	0.982	0.6	0.045	0.6	0.683	-0.4	-1.207	-0.4	-1.113
0.7	0.886	0.7	0.478	-0.3	0.367	-0.3	0.961	0.7	0.115	0.7	0.630	-0.3	-1.065	-0.3	-0.916
0.8	0.935	0.8	0.622	-0.2	0.376	-0.2	0.935	0.8	0.207	0.8	0.579	-0.2	-0.898	-0.2	-0.708
0.9	0.984	0.9	0.793	-0.1	0.413	-0.1	0.903	0.9	0.323	0.9	0.532	-0.1	-0.754	-0.1	-0.503
1.0	1.000	1.0	1.000	0.0	0.465	0.0	0.872	1.0	0.451	1.0	0.451	0.0	-0.632	0.0	-0.354

3. POLAR REPRESENTATION

From the computational viewpoint, the homologous curves are difficult to handle because v, α, h, and β may all change sign and go through zero during a transient. The signs of v and α depend upon the zones of operation. In addition to the need to define a different characteristic curve for each zone of operation, α/v becomes infinite for v = 0.

In numerical simulations of flow transient events involving pump operations, the polar representation reported by Wylie and Streeter (1978) and Chaudhry (1979) appears as the simplest way to represent the homologous curve. In the polar approach, defining a new abscissa θ,

$$\theta = tan^{-1}\frac{\alpha}{v}, \tag{4}$$

and the ordinates

$$F_h = \frac{h}{\alpha^2 + v^2} \tag{5}$$

$$F_\beta = \frac{\beta}{\alpha^2 + v^2}, \tag{6}$$

the complete characteristics of a pump may be represented by only two closed curves, one for the dynamic head ($F_h \times \theta$) and other for hydraulic torque ($F_\beta \times \theta$). By definition, θ is always finite, and its values varies between 0 and 2π for the four zones of operation (see Tab. 3).

From plots or tables for h/α^2 (or h/v^2) as a function of v/α (or α/v) one can always determine F_h, since

$$F_h = \frac{h}{\alpha^2 + v^2} = \frac{h}{\alpha^2} \times \frac{1}{1+(v/\alpha)^2} = \frac{h}{v^2} \times \frac{1}{1+(\alpha/v)^2} \; . \tag{7}$$

The same procedure may be used to determine F_β from plots or tables for β/α^2 (or β/v^2) as a function of v/α (or α/v). The corresponding relation assumes the form

$$F_\beta = \frac{\beta}{\alpha^2 + v^2} = \frac{\beta}{\alpha^2} \times \frac{1}{1+(v/\alpha)^2} = \frac{\beta}{v^2} \times \frac{1}{1+(\alpha/v)^2} \; . \tag{8}$$

Table 3. Zones of pump operation.

Zone of Operation	**v**	**α**	**Range of θ**
Normal pump	+	+	$0 \le \theta \le \pi/2$
Energy dissipation	–	+	$\pi/2 \le \theta \le \pi$
Normal turbine	–	–	$\pi \le \theta \le 3\pi/2$
Reverse pump	+	–	$3\pi/2 \le \theta \le 2\pi$

The homologous head and torque curves in the polar form for the reference pump are shown in Fig. 5. The two data sets used to plot these curves were obtained from Tab. 1 and Tab. 2 by using Eqs. (7) and 8 to evaluate F_h and F_β. Each data set in tabular form comprised 73 points corresponding to equally spaced abscissa (θ) values between 0 and 2π. As before, cubic spline was used to interpolate F_h and F_β at a given value of θ.

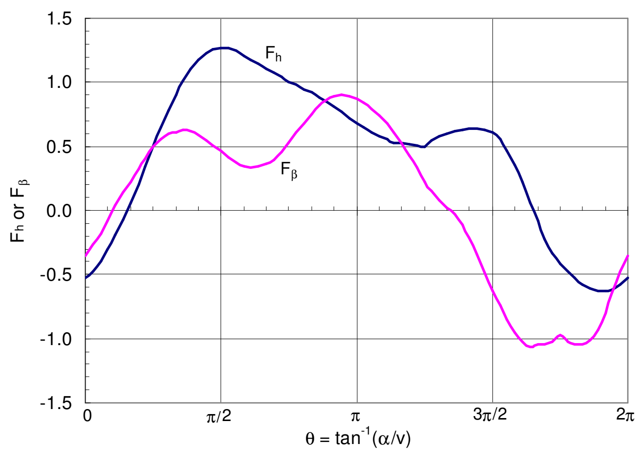

Figure 5. Polar homologous head and torque curves.

For any value of α and v, except when both α and v are simultaneously zero, the value of $\theta = \tan^{-1}(\alpha/v)$ may be determined with the Fortran ATAN2 intrinsic function defined in Tab. 4. However, this function returns the value of θ between 0 and π and between 0 and $-\pi$, whereas the range of interest is between 0 and 2π. This limitation can be circumvented by adding 2π to the computed value of θ if $\theta < 0$.

Table 4. Function $\theta = \mathrm{ATAN2}(\alpha, v)$.

Domain	**Definition**	**Range**
$v < 0, \alpha < 0$	$-\pi + \tan^{-1}(\alpha/v)$	$-\pi < \theta < -\pi/2$
$v = 0, \alpha < 0$	$-\pi/2$	
$v > 0$	$\tan^{-1}(\alpha/v)$	$-\pi/2 < \theta < \pi/2$
$v = 0, \alpha > 0$	$\pi/2$	
$v < 0, \alpha > 0$	$\pi + \tan^{-1}(\alpha/v)$	$\pi/2 < \theta < \pi$
$v = 0, \alpha = 0$	error	

Given a tabular function $F_i = F(\theta_i)$ at n distinct point $\theta_1, \theta_2, \ldots, \theta_n$, where $\theta_1 < \theta_2 < \ldots < \theta_n$, each segment of the curve connecting three consecutive points may be approximated by a quadratic polynomial (Fig. 6). If the table contains a sufficient number of points, then the error introduced by approximating the curve by segmental parabola is negligible. Assuming equally spaced abscissas, it is possible to demonstrate that the interpolation equation may be expressed as

$$F(\theta) = F_i + \frac{1}{2}(F_{i+1} - 2F_i + F_{i-1})\xi^2 + \frac{1}{2}(F_{i+1} - F_{i-1})\xi \ , \tag{9}$$

where $\xi = (\theta - \theta_i)/\Delta\theta$, and $\Delta\theta$ is the abscissa increment. The location of the data point in the table is given by

$$i = min\{ n-1, max[2, int(\theta/\Delta\theta)+1]\} \ . \tag{10}$$

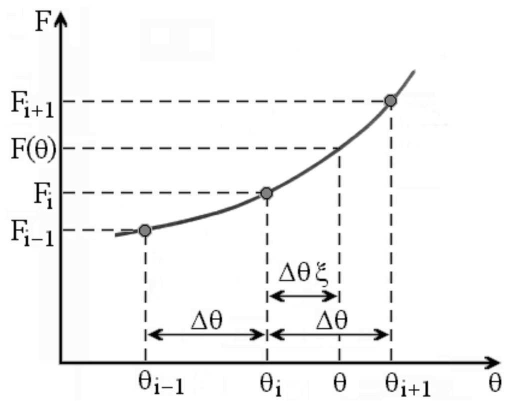

Fig 6. Parabolic interpolation.

If tables of $F_{h,i} = F_h(\theta_i)$ and $F_{\beta,i} = F_\beta(\theta_i)$ are available, Eq. (9) together with Eq. (4) may be used to determine head and torque ratios at given values of α and v. Such tables should contain a sufficient number of entries so that the curves are well defined. An abscissa increment $\Delta\theta$ of $2\pi/72$, yielding 73 values of F_h and 73 values of F_β, has proven to be a reasonable number.

4. CONCLUSIONS

No matter the form in which pump manufacturers present the complete pump characteristic curves, it is always possible to convert them to a polar configuration. Besides preserving the homologous relationships, such a configuration has been the preferred form to represent the pump performance curves for the purpose of analyzing flow loop transient events involving operation of pumps.

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