# THREE-DIMENSIONAL SIMULATION OF RADIOACTIVE POLLUTANT IN THE ATMOSPHERE FROM NUCLEAR POWER PLANT

**Daniela Buske, danielabuske@gmail.com**
Universidade Federal de Pelotas, DME/IFM, Pelotas / RS / Brazil

**Davidson Martins Morreira, davidson@pq.cnpq.br**
Universidade Federal do Pampa, Unipampa Campus Bagé, Bagé / RS / Brazil

**Marco Túllio de Vilhena, vilhena@pq.cnpq.br**
Universidade Federal do Rio Grande do Sul, PROMEC, Porto Alegre / RS / Brazil

***Abstract.*** *In this work we report numerical simulations using the GILTT (Generalized Integral Laplace Transform Technique) approach to simulate radioactive pollutant dispersion in the Planetary Boundary Layer (PBL). To study the dispersion and the possible scenarios arising from accidental emissions, the results obtained with the GILTT method are compared with experimental data obtained at the Nuclear Power Plant of Angra dos Reis under neutral/moderately unstable conditions. Furthermore, to a better description of the wind profile for the irregular ground level terrain, we consider the wind profile as solution of the MM5 mesoscale model. The statistical indices point out a reasonable good agreement is obtained between experimental data and GILTT model.*

***Keywords****: GILTT, Laplace Transform, Advection-Diffusion Equation, Radioactive Pollutant Emission*

## 1. INTRODUCTION

The safety has been an important consideration from the very beginning of the development of nuclear reactors. Although construction and operation of nuclear power plants are closely monitored and regulated by the Research Council (CNEN – Brazilian National Commission of Nuclear Energy), an accident is possible. The potential danger from an accident at a nuclear power plant is exposure to radiation. This exposure could come from the release of radioactive material from the plant into the environment, usually characterized by a plume (cloud-like) formation. The area the radioactive release may affect is determined by the amount released from the plant, wind direction and speed and weather conditions which would swiftly drive the radioactive material to the ground, hence causing increased in the deposition of radionuclides.

With mathematical models it is possible to do forecasts or to simulate concentration fields of contaminants in accidents in accordance with action plans to give safety to the population. The mathematical models of pollutant dispersion in the atmosphere represent an important technical instrument, as for the knowledge of the state of the atmosphere, united to monitoring network, as for the environmental management. Both our scientific understanding and technical developments have been greatly increased by the use of empirical, analytical and numerical models to predict the air pollution concentration in atmosphere. For this purpose, the advection-diffusion equation has been largely applied in operational atmospheric dispersion models. In principle, from this equation it is possible to obtain the dispersion from a source given appropriate boundary and initial conditions plus knowledge of the mean wind velocity and concentration turbulent fluxes.

In a recent work Moreira et al. (2005a) proposed an analytical solution for radioactive pollutant dispersion in atmosphere, solving the time-dependent two-dimensional advection-diffusion equation by the ADMM (Advection Diffusion Multilayer Model) method. The main idea of the ADMM method is based on a discretization of the PBL in $N$ sub-layers, where in each sub-layer the advection-diffusion equation is solved by the Laplace transform technique, considering an average value for eddy diffusivity and the wind speed. The experiment used in the simulations consisted of the controlled releases of radioactive tritiated water vapor from the meteorological tower close to the power plant at Itaorna Beach (Biaggio et al., 1985). The wind profile was determined using experimental meteorological data and the micrometeorological parameters were calculated from empirical equations obtained in the literature.

Following the above idea, Buske et al. (2006) solved the same equation by the GILTT approach in order to avoid the stepwise discretization of eddy-diffusivity coefficient appearing in the ADMM method. In fact, the main idea of the GILTT approach relies on the expansion of the concentration in series of eigenfunctions obtained from an auxiliary problem, replacing this equation in the advection-diffusion equation and taking moments we come out with a matrix ordinary differential equation that is solved analytically by the Laplace transform technique. In the simulations was used the same experiment, wind profile and micrometeorological parameters described above.

In this work we step forward reporting numerical simulations using the GILTT approach to simulate radioactive pollutant dispersion in the PBL, using the experimental data obtained at the Nuclear Power Plant of Angra dos Reis under neutral/moderately unstable conditions (Biaggio et al., 1985). Furthermore, to a better description of the wind profile for the irregular ground level terrain, we consider the wind profile as solution of the MM5 Mesoscale model.

To reach our objective we organize this paper as follows: in section 2 we report the advection-diffusion equation solution by the GILTT method. In section 3 the experimental data and turbulent parameterizations are presented. The comparison with experimental data are in section 4, and finally in section 5, the conclusions.

## 2. SOLUTION OF THE ADVECTION-DIFFUSION EQUATION BY THE GILTT METHOD

Atmospheric pollution turbulent fluxes can be assumed to be proportional to the mean concentration gradient. This assumption, along with the equation of continuity, leads to the advection-diffusion equation. The cross-wind integration of the advection-diffusion equation leads to:

$$\frac{\partial c(x,z,t)}{\partial t} + u\frac{\partial c(x,z,t)}{\partial x} + w\frac{\partial c(x,z,t)}{\partial z} = \frac{\partial}{\partial z}\left(K_z \frac{\partial c(x,z,t)}{\partial z}\right) - \lambda c(x,z,t) \ , \tag{1}$$

for $0 < z < h$, $t > 0$ and $x > 0$, where $h$ is the height of the PBL, $u$ and $w$ are, respectively, the longitudinal and vertical mean wind, $K_z$ is the vertical eddy diffusivity, $c$ is the crosswind integrated concentration and $\lambda$ is the decay constant. Moreover, we introduce the usual boundary conditions of zero flux at the ground and PBL top, and a source with emission rate $Q$ at height $H_s$:

$$K_z \frac{\partial c(x,z,t)}{\partial z} = 0 \qquad \text{at } z = 0, h \tag{1a}$$

$$uc(0,z,t) = Q\delta(z - H_s) \qquad \text{at } x = 0 \tag{1b}$$

$$c(x,z,0) = 0 \qquad \text{at } t = 0 \tag{1c}$$

in which $\delta$ is the Dirac delta function.

Using the Laplace Transform technique, transforming $t$ into $r$ and $c$ into $C$, applying the initial condition (1c), the Eq. (1) becomes:

$$u\frac{\partial C(x,z,r)}{\partial x} + w\frac{\partial C(x,z,r)}{\partial z} = K_z \frac{\partial^2 C(x,z,r)}{\partial z^2} + K_z' \frac{\partial C(x,z,r)}{\partial z} - \lambda^* C(x,z,r) \ , \tag{2}$$

where $\lambda^* = \lambda + r$. Taking advantage of the well known solution of the stationary problem with advection in the $x$ direction by the GILTT method (Wortmann et al., 2005; Moreira et al., 2005, 2006b), we pose that the solution of problem (1) has the form:

$$C(x,z,r) = \sum_{i=0}^{N} \bar{c}_i(x,r)\Psi_i(z) \ , \tag{3}$$

where $\Psi_i(z)$ are the eigenfunctions of an associated Sturm-Liouville problem ($\Psi_i(z) = \cos(\lambda_i z)$ for $i = 0, 1, 2, 3, ...$ and $\lambda_i = i\pi/h$) and $\bar{c}_i(x,r)$ is the solution of the transformed problem which is given below.

Proceeding in a similar manner of the work of Wortmann et al. (2005), replacing the expansion of the pollutant concentration in a series in terms of the given eigenfunctions (Eq. (3)) in Eq. (1) and taking moments, we obtain the transformed problem that in matrix notation becomes:

$$Y'(x) + FY(x) = 0 \ , \tag{4}$$

where $Y(x)$ and $Y'(x)$ are, respectively, the column vector whose components are $\bar{c}_i(x)$ and $\bar{c}_i'(x)$ and $F = B^{-1}.E$. The entries of the matrices $B$ and $E$ are written as:

$$b_{i,j} = -\int_0^h u\Psi_i\Psi_j dz \ ; \ e_{i,j} = \int_0^h K_z'\Psi_i'\Psi_j dz - \lambda_i^2\int_0^h K_z\Psi_i\Psi_j dz - \int_0^h w\Psi_i'\Psi_j dz - \lambda^*\int_0^h \Psi_i\Psi_j dz$$

The transformed problem represented by the Eq. (4) can be solved analytically by the Laplace transform technique and diagonalization. For more details about the solution of the transformed problem see the work (Wortmann et al., 2005).

Finally, the time dependent concentration is obtained by inverting numerically the transformed concentration $C(x,z,r)$ by a Gaussian Quadrature scheme

$$c(x,z,t)=\sum_{k=1}^{m}\frac{P_k}{t}A_k\sum_{i=0}^{N}\bar{c}_i(x,\frac{P_k}{t})\Psi_i(z) \quad , \tag{5}$$

where $A_k$ and $P_k$ are the weights and roots of the Gaussian Quadrature integration (Stroud and Secrest, 1996). It is important to observe that Eq. (5) accepts any wind and eddy diffusivity vertical profiles.

In this work, we need to calculate three-dimensional concentration. This way, we assumed a Gaussian distribution in the lateral direction and we have taken into account the dispersion parameter $\sigma_y$. Therefore, the final equation to calculate the ground-level centerline concentration is:

$$C(x,0,0)=\frac{c(x,0)}{\sqrt{2\pi}\sigma_y} \quad , \tag{6}$$

where the ground-level crosswind-integrated concentration in Eq. (6) is calculated employing Eq. (5).

## 3. EXPERIMENTAL DATA AND TURBULENT PARAMETERIZATION

The results obtained with the GILTT method (Eq. (6)) are compared with experimental data obtained at the Nuclear Power Plant of Angra dos Reis under neutral/moderately unstable conditions and described in Biagio et al. (1985). The experiment consisted in the controlled releases of radioactive tritiated water vapor from the meteorological tower, 100m height, close to the power plant in Itaorna Beach (Angra dos Reis), during five days (28 November to 4 December 1984). During the whole experiment, four meteorological towers collected the relevant meteorological data. Wind speed and direction are measured at three levels (10, 60 and 100 m), along with the temperature gradient between 10 and 100 m. Some additional data of relative humidity are available in some of the sampling sites, and are used to calculate the concentration of radioactive tritiated water in the air (after measuring the radioactivity of the collected samples). All relevant details, as well as the synoptic meteorological conditions during the dispersion campaign are also described in (Biaggio et al., 1985). The data from experiment 3 was used to obtain the numerical results and are presented in Table 1.

Table 1. Micrometeorological parameters and emission rate for experiment 3.

| Period | $u(m/s)$ | $h(m)$ | $u_*(m/s)$ | $L(m)$ | $w_*(m/s)$ | $Q$(MBq/s) |
|---|---|---|---|---|---|---|
| 1 | 2.2 | 1153 | 0.4 | -967 | 0.6 | 20.5 |
| 2 | 2.0 | 1027 | 0.3 | -861 | 0.5 | 20.5 |
| 3 | 2.6 | 1367 | 0.5 | -1147 | 0.7 | 20.5 |

The micrometeorological parameters shown in the Table 1 are calculated from equations obtained in the literature. The roughness length utilized was $z_0 = 1$m (in according with (Martano, 1992); the Monin-Obukhov length can be written as (Zanetti, 1990):

$$L=-\frac{h}{k}\left(\frac{u_*}{w_*}\right)^3 \quad , \tag{7}$$

in which $k$ is the von Karman constant ($k$ = 0.4), $w_*$ is the convective velocity scale ( $w_* \cong 0.25u$ in neutral conditions (Briggs, 1992), $u_*$ is the friction velocity and $h$ is the height of the boundary layer. The friction velocity is obtained in neutral turbulence conditions by the expression:

$$u_*=\frac{k\cdot u}{\ln(z_r/z_0)} \quad , \tag{8}$$

where $z_r = 10$ m (reference height) and $u$ is the wind speed. The height of the PBL $h$ is obtained by:

$$h = \frac{0.3u_*}{f_c} \quad , \tag{9}$$

where $f_c = 10^{-4}$ represents the Coriolis coefficient.

In the atmospheric diffusion problems, the choice of a turbulent parameterization represents a fundamental aspect for the contaminants dispersion modeling. From a physical point of view a turbulence parameterization is an approximation of nature in the sense that we are putting into mathematical models an approximated relation that in principle can be used as a surrogate for the natural true unknown term. The reliability of each model strongly depends on the way turbulent parameters are calculated and related to the current understanding of the PBL (Moreira et al., 2005a).

For lateral dispersion parameter $\sigma_y$ we followed Nieuwstadt and Van Dop (1982) where the following form is presented:

$$\frac{\sigma_y}{h} = \left(0.26X/(1+0.91X)\right)^{1/2} \quad , \tag{10}$$

where $X$ is a nondimensional distance ( $X = xw_*/uh$ ).

For small variations of the wind direction in relation to the position of the receptors was utilized the expression (10) as lateral dispersion and the follow expression for large variations (Nieuwstadt and Van Dop, 1982):

$$\frac{\sigma_y}{x} = \sigma_\theta \left(1 + 0.031x^{0.46}\right)^{-1} \quad , \tag{11}$$

where $\sigma_\theta = 15$ (Blackadar, 1997) for unstable/neutral conditions.

In terms of the convective scaling parameters the vertical eddy diffusivity derived from Batchelor (1949) can be formulated as (Degrazia et al., 1997):

$$K_z = 0.22w_*h\left(\frac{z}{h}\right)^{1/3}\left(1-\frac{z}{h}\right)^{1/3}\left[1-\exp\left(-\frac{4z}{h}\right)-0.0003\exp\left(\frac{8z}{h}\right)\right] . \tag{12}$$

Thus, in this study we introduce the lateral dispersion parameter (Eqs. (10) or (11)) and the vertical eddy diffusivity (Eq. (12)) in the GILTT model (Eq. (6)) to calculate the ground-level concentration of emissions released from an elevated continuous source point in an unstable/neutral PBL.

## 4. NUMERICAL SIMULATIONS

Bearing in mind that in this work our aim is to show the feasibility of the proposed model to simulate pollutant dispersion in the atmosphere for more realistic problems, we are now in position to specifically apply this methodology on a problem with wind speeds evaluated by the MM5 mesoscale model. In order to apply the GILTT method using the discrete data supplied by MM5, we perform a polynomial interpolation of the wind field ($u$(x,z) and $w$(x,z)) in $z$ variable and take the average in $x$ variable. The grid used was 25x25 km with 1km of resolution and 38 vertical levels. The topography and vegetation used was USGS, 900m.

To simulate a dispersion experiment for radioactive contaminant described in Biaggio et al. (1985), we used the wind field from the model MM5 (MM5V3 Modeling System Version 3). The mesoscale model is a limited-area, non-hydrostatic or hydrostatic (Version 2 only), terrain-following sigma-coordinate model designed to simulate or predict mesoscale and regional-scale atmospheric circulation. It has been developed at the Penn State and NCAR as a community mesoscale model and is continuously being improved by contributions from users at several universities and government laboratories.

Table 2 presents some statistical performances (Hanna, 1989) using in the simulations the Eq. (6) for two cases: wind field from MM5V3 and $u$ constant from micrometeorological tower. The statistical index FB shows if the predicted quantity underestimates or overestimates the observed ones. The statistical index NMSE represents the quadratic error of the predicted quantities related to the observed ones. The best results are expected to have values close to zero for the indices NMSE, FB and FS, and close to 1 in the indices COR, FA2 and FA5 (simulations in complex terrain is usually FA5). Figure 1 shows the observed and predicted scatter diagram of ground-level concentrations. Promptly, we observed from Table 1 and Figure 1 that the model satisfactorily reproduces the concentrations, particularly, with wind field from the MM5 mesoscale model.

Table 2. Statistical evaluation using the experiment 3 of Angra dos Reis.

| | NMSE | COR | FA2 | FA5 | FB | FS |
|---|---|---|---|---|---|---|
| GILTT with MM5 | 0.78 | 0.68 | 0.30 | 0.82 | -0.49 | 0.69 |
| GILTT with u cte | 2.43 | 0.58 | 0.12 | 0.48 | -1.17 | -0.06 |

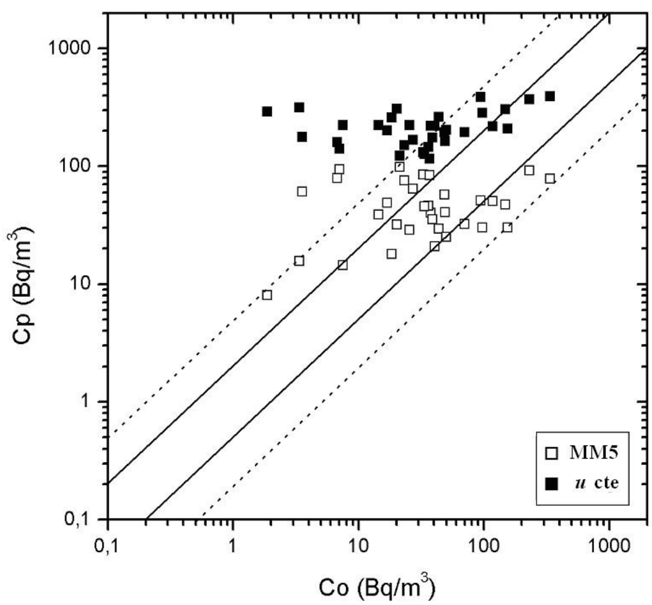


Figure 1. Observed and predicted scatter diagram of ground-level concentrations. Solid lines indicate a FA2, dotted lines a FA5.

## 5. CONCLUSIONS

Air pollution models are an important technical instrument for identifying and managing accidental release from nuclear plants. In this work we compared computed concentration against data used for simulating accidental release from the Nuclear Power Plant of Angra dos Reis, Brazil. The experimental simulation was made releasing tritiated water under neutral/moderately unstable conditions (Biaggio et al., 1985).

We presented a general solution (i.e. for any wind and eddy diffusivity vertical profiles) of the three-dimensional transient advection diffusion equation using the GILTT method. The statistical indices point out that a reasonable good agreement is obtained between experimental data and GILTT model. The computer code for this new solution runs much faster than computer code for a numerical model and can be used for a fast screening of concentration distribution from a given source and as an auxiliary tool in the control of critical events related to air quality.

Remembering that this is a preliminary study, our next step is considering the complex terrain of Angra dos Reis in the model.

## 6. ACKNOWLEDGEMENTS

The authors thank to CNPq for the partial financial support of this work.

## 7. REFERENCES

Moreira, D.M., Vilhena, M.T., Tirabassi, T., Carvalho, J.C., 2005a, “A semi-analytical model for the trítium dispersion simulation in the PBL from the ANGRA I nuclear power plant”, Ecological Modelling, Vol. 189, No 3-4, pp. 413-424.

Biaggio, R., Godoy, G., Nicoli, I., Nicoli, D., Thomas, P., 1985, “First atmospheric diffusion experiment campaign at the Angra site – KfK 3936”, Karlsruhe and CNEN 1201, Rio de Janeiro.

Buske, D.; Vilhena, M.T., Moreira, D.M., Brauer, C.R., 2006, "The GILTT solution for the tritium dispersion simulation in the PBL from the Angra I nuclear power plant", Proceedings of the International Youth Nuclear Conference 2006, Stockholmo, Sweden.

Moreira, D.M., Vilhena, M. T., Tirabassi, T., Costa, C., Bodmann, B., 2006a, "Simulation of pollutant dispersion in the atmosphere by the Laplace transform: the ADMM approach", Water, Air and Soil Pollution, Vol. 177, pp. 411-439.

Heydarian, M., Mullineaus, N., 1989, "Solution of parabolic partial differential equations", Applied Mathematical Modelling, Vol. 5, pp. 448-449.

Stroud, A.H. and Secrest, D., 1996, "Gaussian Quadrature Formulas", Englewood Cliffs, N.J., Prentice Hall Inc.

Moreira, D.M., Vilhena, M.T., Tirabassi, T., Buske, D., Cotta, R.M., 2005b, "Near souce atmospheric pollutant dispersion using the new GILTT method", Atmospheric Environment, Vol. 39, pp. 6289-6294.

Moreira, D.M., Vilhena, M.T., Buske, D., Tirabassi, T., 2006b, "The GILTT solution of the advection-diffusion equation for an inhomogeneous and nonstationary PBL", Atmospheric Environment, Vol. 40, pp. 3186-3194.

Wortmann, S., Vilhena, M.T., Moreira, D.M., Buske, D., 2005, "A new analytical approach to simulate the pollutant dispersion in the PBL", Atmospheric Environment, Vol. 39, pp. 2171-2178.

Martano, 1992, "Dinâmica de fluxo turbulento sobre relevo e aplicação à difusão em pequena escala na camada limite atmosférica", Tese de doutorado, Departamento de Física, PUC-RJ, 117pp.

Zanetti, P., 1990, "Air Pollution Modeling", Computational Mechanics Publications, Southampton (UK).

Briggs, G.A., 1992, "Plume dispersion in the convective boundary layer. Part II: analyses of CONDORS field experimental data", Journal of Applied Meteorology, Vol. 32, pp. 1388-1425.

Nieuwstadt and Van Dop, 1982, "Atmospheric Turbulence and Air Pollution Modelling", D. Reidel Publishing Company.

Blackadar, A.K., 1997, "Turbulence and diffusion in the atmosphere: lectures in Environmental Sciences", Springer-Verlag, 185pp.

Batchelor, G.K., 1949, "Diffusion in a field of homogeneous turbulence, Eulerian analysis", Australian Journal of Science Research, Vol 2, pp. 437-450.

Degrazia, G.A., Campos Velho, H.F., Carvalho, J.C., 1997, "Nonlocal exchange coefficients for the convective boundary layer derived from spectral properties", Contributions to Atmospheric Physics, pp. 57-64.

Hanna, S.R., 1989, "Confidence limit for air quality models as estimated by bootstrap and jacknife resampling methods", Atmospheric Environment, Vol. 23, pp. 1385-1395.

## 8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.