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Transient Liquid Temperature at Thermometer Surface Caused by its Heating Effect

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Abstract. *The calibration of solid volume using hydrostatic method needs the liquid temperature to be measured with high accuracy - resolution about 0.001 K. The solid has to be weighed in air and liquid to calculate the solid volume from the difference of buoyancy in the air and the liquid. During the calibration it is necessary to maintain the liquid motionless to avoid cause influence on the balance. In this condition it was noted that the semiconductor thermometer shows the raising of temperature, but if the liquid is moved around the thermometer the temperature goes down to the initial temperature. To explain the above observation and predict the temperature error ΔT, a simple mathematical model is developed and programmed by using the new dynamic interactive computing, introduced by Wolfram Research with the release of Mathematica 6. When the model parameters are varied in predefined ranges, the temperature automatically is updated to always reflect the current values of the parameters. The paper demonstrates a new technology that could be applied in any other subject of ENCIT 2008.*

Keywords: *hydrostatic method, transient temperature, mathematical tool*

1. Model development

The calibration of solid volume using hydrostatic method needs the liquid temperature to be measured with high accuracy - resolution about 0.001 ^oC. The solid has to be weighed in air and liquid to calculate the solid volume from the difference of buoyancy in the air and the liquid. During the calibration it is necessary to maintain the liquid motionless to avoid cause influence on the balance. In this condition it was noted that the semiconductor thermometer shows the raising of temperature, but if the liquid is moved around the thermometer the temperature down to initial temperature.

To explain the above observation a mathematical model is developed using the following simplifying assumptions:

1. The water is motionless,
2. The increasing of the sensor temperature is very small,
3. The sensor is considered as a sphere with radius r_0,
4. The thermal energy generated in the sensor, Q, is a known and time-independent quantity,
5. The water temperatue T_0 is a constant equal to the initial temperature of the sensor,
6. The sensor temperature is equal to the fluid temperature at the sensor surface.

At time $t = 0$, electricity current starts generating a thermal energy Q in the sensor. The model has to predict the average temperature of the sensor $T_s(t)$ as a function of time.

Since the water is motionless (assumption 1) the temperature $T(r,t)$ in the region $r_0 \leq r \leq \infty$ and time $t \geq 0$ is described by the heat conduction equation:

$$\frac{\partial T(r,t)}{\partial t} = \alpha \left(\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{2}{r} \frac{\partial T(r,t)}{\partial r} \right), \tag{1}$$

where α is diffusivity of the water.

Because the increase in the sensor temperature is very small (assumption 2), the thermal capacity of the sensor is neglected. Consequently, all generated thermal energy enters in the water through the sensor surface. The sensor is considered as a sphere with radius r_0 (assumption 3) and surface $4\pi r_0^2$. The heat balance at the surface r_0, after taking into account Fourier law, gives

$$Q = -4k\pi r_0^2 \frac{\partial T(r_0,t)}{\partial r}, \tag{2}$$

where Q is a constant (assumption 4), and k is the thermal conductivity of the water.

Because the difference between the sensor and the water temperature is very small (assumption 2), the thermal properties α and k are constants depending on the initial temperature T_0.

The boundary temperature at $r = \infty$ and the initial temperature at $t = 0$, are equal to T_0 (assumption 5):

$$T(\infty, t) = T_0, \tag{3}$$

$$T(r, 0) = T_0. \tag{4}$$

Once the model, represented by Eqs. (1) to (4), is solved, the according sensor temperature follow from the condition (assumption 6):

$$T_s(t) = T(r_0, t) \tag{5}$$

2. Dimensionless model

To obtain the simplest possible dimensionless model the following variables are used:

$$\xi = \frac{r}{r_0}, \qquad \tau = \frac{t\alpha}{r_0^2}, \qquad \theta(\xi, \tau) = (T(r, t) - T_0)\,\frac{4k\pi r_0}{Q}, \qquad \theta_s(\tau) = (T_s(t) - T_0)\,\frac{4k\pi r_0}{Q}, \tag{6}$$

where ξ is the dimensionless radius, τ is the dimensionless time, $\theta(\xi, \tau)$ is the dimensionless fluid temperature, and $\theta_s(\tau)$ is the dimensionless sensor temperature.

The partial differential equation, Eq. (1), after introducing the dimensionless variables shown in Eq. (6), becomes:

$$\frac{\partial \theta(\xi, \tau)}{\partial \tau} = \frac{2}{\xi}\frac{\partial \theta(\xi, \tau)}{\partial \xi} + \frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2}. \tag{7}$$

The dimensionless form of the balance condition, Eq. (2), at $\xi = 1$ is:

$$\frac{\partial \theta(1, \tau)}{\partial \xi} = -1. \tag{8}$$

The boundary condition at infinity, Eq. (3), and the initial condition, Eq. (4), become

$$\theta(\infty, \tau) = 0, \tag{9}$$

$$\theta(\xi, 0) = 0. \tag{10}$$

The dimensionless form of the relation given by Eq. (5) has the form:

$$\theta_s(\tau) = \theta(1, \tau). \tag{11}$$

It is evident that the dimensionless model, Eqs. (7) to Eq. (11), is simpler than the original model, given by Eqs. (1) to (5).

3. Laplace transform solution

To solve the problem defined by Eqs. (7) to Eq. (11) we use the Laplace transform

$$\theta[s][\xi] = \int_0^\infty e^{-s\tau}\theta(\xi, \tau)d\tau. \tag{12}$$

In Laplace domain the Eq. (7), after using Eq. (10), becomes:

$$s\,\theta[s][\xi] = \frac{2}{\xi}\theta[s]'[\xi] + \theta[s]''[\xi]. \tag{13}$$

The Laplace transforms of the conditions, Eqs. (8), (9), and (11) are:

$$\theta[s]'[1] = -\frac{1}{s}, \tag{14}$$

$$\theta[s][\infty] = 0, \tag{15}$$

$$\theta s[s] = \theta[s][1]. \tag{16}$$

The solution of the ordinary differential equation, Eq. (13), is:

$$\theta[s][\xi] = C_1 \frac{e^{-\sqrt{s}\xi}}{\xi} + C_2 \frac{e^{\sqrt{s}\xi}}{2\sqrt{s}\xi}. \tag{17}$$

To satisfy the boundary condition at infinity, Eq. (15), we have to assume $C_2 = 0$. Than the solution given by Eq. (17) becomes:

$$\theta[s][\xi] = C_1 \frac{e^{-\sqrt{s}\xi}}{\xi}. \tag{18}$$

The constant C_1 is determined from the condition given by Eq. (14):

$$C_1 = \frac{e^{\sqrt{s}}}{(1+\sqrt{s})s}. \tag{19}$$

Introducing the constant C_1 from Eq. (19) into Eq. (18) we obtain the solution in Laplace domain:

$$\theta[s][\xi] = \frac{e^{\sqrt{s}(1-\xi)}}{(1+\sqrt{s})s\xi}. \tag{20}$$

Introducing Eq. (20) into the relation given by Eq. (16) we obtain the sensor temperature at Laplace domain:

$$\theta s[s] = \frac{1}{(1+\sqrt{s})s}. \tag{21}$$

The desired dimensionless sensor temperature as function of time τ is given by the inverse Laplace transform:

$$\theta s(\tau) = 1 - e^{\tau} Erfc\left(\sqrt{\tau}\right). \tag{22}$$

The plot of the obtained dimensionless sensor temperature is shown in Fig. 1.

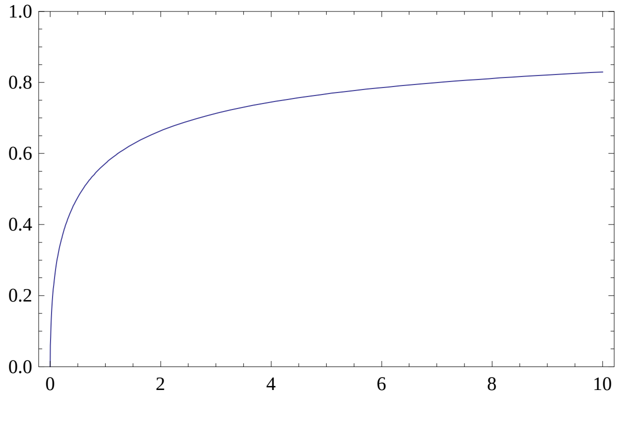

Figure 1: Behaviour of the dimensionless sensor temperature as a function of time τ.

To analyze the influence of the parameter we transform the dimensionless solution, Eq. (22), into one including all parameters of the model

$$\Delta T = \frac{Q}{4k\Pi r_0}\left(1 - e^{t\alpha/r_0^2} Erfc\left[\sqrt{t\alpha/r_0^2}\right]\right), \tag{23}$$

where ΔT is the difference between the sensor temperature $T_s(t)$ and the water temperature T_0.

4. The influence of the parameters

The parameters are assumed to vary in the following intervals: $15 \leq T_0 \leq 30$ in Celsius, $0 \leq Q \leq 0.001$ Watt, $0.001 \leq r_0 \leq 0.01$ meter, $0 \leq t \leq 3600$ second.

The conductivity k and diffusivity α for the water have been determined from $T_0 = 15$ to 30 ^{o}C in steps of 1 ^{o}C by using the Fluid Properties Calculator (see list of References). The data obtained have been used to determine the constant C_0, C_1, and C_2 in the expression $C_0 + CT + C_2T^2$ by using the built-in *Mathematica* function *Fit*.

The developed interactive *Mathematica* tool is shown on Fig. 2. It could be run on *The Mathematica Player*. The latter can be downloaded for free from the site given in the References.

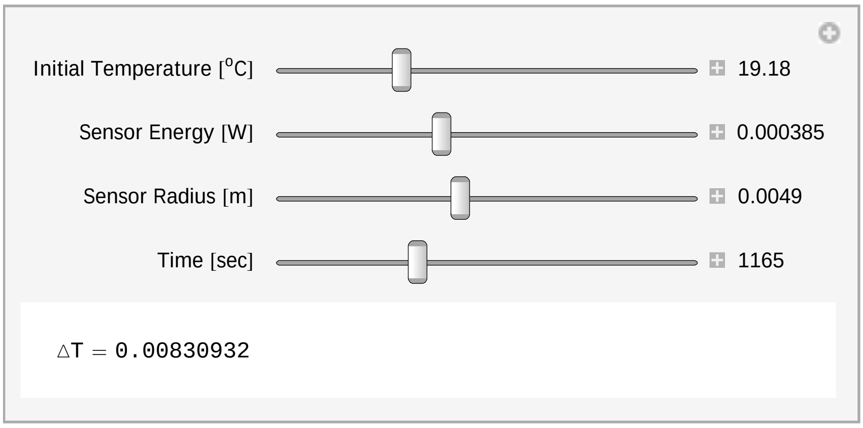

Figure 2: Example of the developed interactive *Mathematica* tool and the slider feature.

The parameters are changed in the above specified interval by using the slyder. The difference between the sensor and water temperatures imediatelly is updated to always reflect the current values of the slyders.

5. Conclusions

A simple mathematical model is developed to predict the difference between the temperatures of a spherical sensor and motionless water ΔT, as a result of the thermal energy generated in the sensor. The model is rewritten in dimensionless form and solved by using the Laplace transform. Finally, an explicit formula is obtained for ΔT as function of the parameters - water temperature, energy generated in the sensor, sensor radius, time, conductivity and diffusivity of the water. This formula is explored to demonstrate the new dynamic interactive computing, introduced by Wolfram Research with the release of Mathematica 6. The input parameter are varied in predefined ranges by using sliders. Empirical correlations between conductivity, diffusivity and temperature of the water are used. The desired temperature difference are automatically updated to always reflect the parameter values corresponding to the current slider's positions.

6. Acknowledgements

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7. References

Fluid Properties Calculator, <http://www.mhtl.uwaterloo.ca/old/onlinetools/airprop/airprop.html>

Mathematica Player, <http://www.wolfram.com/products/player/>