THE RAREFIED GAS DYNAMICS: GAS-SURFACE INTERACTION

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Abstract. *In this work, numerical results are presented for velocity and heat-flow profiles and particle and heat-flow rates of a rarefied gas confined in a plane channel defined by two parallel plates subject to a temperature gradient. The problem is modelled by the BGK kinetic equation and two different types of surface-gas interaction law are analyzed, i. e., the Cercignani-Lampis model that is defined in terms of normal and tangential accommodation coefficients and the usual Maxwell model. Here, the two parallel plates have different chemical compositions, i. e., with different accommodation coefficients. A modern analytical version of the discrete-ordinates method (ADO) is used to solve this problem and a detailed analysis is performed in regard to the influence of the surfaces on the physical quantities.*

Keywords: *thermal-creep problem, rarefied gas dynamics, gas-surface interaction law*

1. INTRODUCTION

Problems involving rarefied gas dynamics may be modeled by Boltzmann equation, which is initially approximated by kinetic equations. In these problems, gas interactions with the surface play an important role inasmuch as the surface temperature, its rugosity and chemical composition influence these interactions directly (gas-surface) leading to different accommodation coefficients [4]. In mathematical models, boundary conditions take into account the type of surface through scattering kernels. Among the different types of scattering kernels expressing different interactions between gas and surface are: the scattering kernel of Maxwell [13], which considers the fraction $(1-\alpha)$ of reflected particles specularly and the remaining ones reflected diffusely, and the scattering kernel of Cercignani-Lampis [3], which features two accommodation coefficients to better represent the physical properties of the surface: the accommodation coefficient of the tangential momentum (α_t) and the kinetic energy accommodation coefficient due to a normal velocity component (α_n). In recent studies, the Cercignani-Lampis kernel associated with the analytical method of discrete-ordinates was used to solve a class of problems based on kinetic equations [6, 7] aiming to find results for the analysis of surface effects, which are important for rarefied gas dynamics phenomena. Therefore, it is extremely important to consider different accommodation coefficients. The purpose of this paper is to use the boundary conditions of Maxwell and Cercignani-Lampis (which describe the gas-surface interaction) associated with the analytical method of discrete-ordinates (ADO) [1] to find the solution to the Creep-Thermal problem, based on the BGK model [2], considering that the plates through which the gas flows have different chemical compositions [5].

2. MATHEMATICAL FORMULATION

Take the written kinetic equation in terms of perturbation $h(y,\mathbf{c})$ for a distribution function of a local Maxwellian as

$$c_y\frac{\partial}{\partial y}h(y,\mathbf{c})+\varepsilon h(y,\mathbf{c})=\varepsilon\pi^{-3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-c'^2}\boldsymbol{F}(\mathbf{c}',\mathbf{c})h(y,\mathbf{c}')\mathrm{d}c_x'\mathrm{d}c_y'\mathrm{d}c_z'+S(\mathbf{c}), \tag{1}$$

where the scattering kernel for the BGK model is given by

$$\boldsymbol{F}(\mathbf{c}',\mathbf{c})=1+2(\mathbf{c}'\cdot\mathbf{c})+(2/3)(c'^2-3/2)(c^2-3/2).$$

In addition, we consider the unidimensional case for the adimensional variable y (measurement in terms of mean-free path l), the components of the velocity vector (c_x,c_y,c_z) expressed in adimensional units,

$$S(\mathbf{c})=-c_x(c_x^2+c_y^2+c_z^2-5/2) \qquad \text{e} \qquad \varepsilon=\sigma_0^2 n_0\pi^{1/2}l,$$

so that σ_0 is the diameter of the gas particles collision in the approximation of the rigid spheres and n_0 is the density of the gas particles balance.

In this work, two specific problems are investigated, both described by Eq. (1) and differentiated by boundary conditions. For one of the problems, the boundary conditions of Maxwell are used and for the other, the Cercignani-Lampis [4, 3] boundary conditions are used,respectively, given by

$$h(\mp a,c_x,\pm c_y,c_z)=\alpha_l\Big[2c_zu_{wl}+\frac{T_w-T_0}{T_0}(c^2-2)\Big]+(1-\alpha_l)h(\mp a,c_x,\mp c_y,c_z)$$

$$+\frac{2\alpha_l}{\pi}\int_{-\infty}^{0} c_x' \mathrm{d}c_x' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-c'^2} h(\mp a, c_x', \mp c_y', c_z')\mathrm{d}c_y'\mathrm{d}c_z', \tag{2}$$

where $l = 1, 2$ represents the walls of the channel, u_{wl} is the plate velocity and T_0 is the constant temperature of reference, and

$$h(\mp a, c_x, \mp c_y, c_z) = \int_{0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\mp a, c_x', \mp c_y', c_z') R_l(c_x', \mp c_y', c_z' : c_x, \pm c_y, c_z)\mathrm{d}c_x'\mathrm{d}c_z'\mathrm{d}c_y', \tag{3}$$

so that

$$R_l(c_x', c_y', c_z' : c_x, c_y, c_z) = \frac{c_y'}{\pi\alpha_{n_l}\alpha_{t_l}(2-\alpha_{t_l})} T_l(c_x' : c_x) S_l(c_y' : c_y)\, T_l(c_z' : c_z), \tag{4}$$

$$T_l(x : y) = \exp\left[-\frac{[(1-\alpha_{t_l})y - x]^2}{\alpha_{t_l}(2-\alpha_{t_l})}\right]$$

and

$$S_l(x : y) = \exp\left[-\frac{[(1-\alpha_{n_l})^{1/2}y - x]^2}{\alpha_{n_l}}\right]\widehat{I}_0\left[\frac{2(1-\alpha_{n_l})^{1/2}|xy|}{\alpha_{n_l}}\right].$$

For computational effects, it is written as

$$\widehat{I}_0(w) = I_0(w)e^{-w},$$

where $I_0(w)$ is Bessel's modified function

$$I_0(w) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{w\cos\phi}\mathrm{d}\phi.$$

Note that in Eq. (4) the kernel of Cercignani-Lampis includes two accommodation coefficients defined as accommodation coefficient of kinetic energy due to the normal component of velocity ($\alpha_n \in [0, 1]$) and the accommodation coefficient of the tangential momentum ($\alpha_t \in [0, 2]$) [3, 4] . Physically, the Cercignani-Lampis kernel admits rear reflection which may occur on rugose surfaces. In the limit case, when $\alpha_t = 2$ and $\alpha_n = 0$, the velocity changes signal after colliding with a surface, thus changing its direction. When $\alpha_t = 1$ and $\alpha_n = 1$ the Cercignani-Lampis kernel (4) coincides with a diffuse reflection. On the other hand, in the limit case $\alpha_t = 0$ and $\alpha_n = 0$ becoming a specular reflection.

To evaluate physical quantities Siewert [10] is followed, and for the velocity profile we get

$$u(y) = \pi^{-3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-c'^2} h(y, c_x, c_y, c_z)c_x\mathrm{d}c_x\mathrm{d}c_y\mathrm{d}c_z \tag{5}$$

and for the heat-flow profile

$$q(y) = \pi^{-3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-c'^2} h(y, c_x, c_y, c_z)(\mathbf{c}^2 - 5/2)c_x\mathrm{d}c_x\mathrm{d}c_y\mathrm{d}c_z. \tag{6}$$

Taking into account the definition of physical quantities in terms of momentums of function h, we multiply Eq. (1) respectively by

$$\phi_1(c_x, c_z) = \frac{1}{\pi}c_x e^{-({c_x}^2+{c_z}^2)}$$

and

$$\phi_2(c_x, c_z) = \frac{1}{\pi\sqrt{2}}c_x({c_x}^2 + {c_z}^2 - 2)e^{-({c_x}^2+{c_z}^2)},$$

we integrate it in relation to c_x and c_z introducing a new notation $\xi = c_y$ we define

$$h_1(y, \xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_1(c_x, c_z)h(y, c_x, \xi, c_z)\mathrm{d}c_x\mathrm{d}c_z$$

and

$$h_2(y, \xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_2(c_x, c_z)h(y, c_x, \xi, c_z)\mathrm{d}c_x\mathrm{d}c_z$$

and we find the equations

$$\xi\frac{\partial}{\partial y}h_1(y,\xi)+h_1(y,\xi)=\pi^{-1/2}\int_{-\infty}^{\infty}e^{-\xi'^2}h_1(y,\xi')\mathrm{d}\xi'-\frac{1}{2} \tag{7}$$

and

$$\xi\frac{\partial}{\partial y}h_2(y,\xi)+h_2(y,\xi)=0. \tag{8}$$

3. A VECTORIAL FORMULATION

Considering $\mathbf{H}(y,\xi)$ a vector with components $h_1(y,\xi)$ and $h_2(y,\xi)$ we may rewrite Eq. (7) and (8) in the vectorial form as

$$\xi\frac{\partial}{\partial y}\mathbf{H}(y,\xi)+\mathbf{H}(y,\xi)=\int_{-\infty}^{\infty}\mathbf{\Psi}(\xi')\mathbf{H}(y,\xi')\mathrm{d}\xi'+\mathbf{S}^*(\xi), \tag{9}$$

where

$$\mathbf{\Psi}(\xi')=\pi^{-1/2}e^{-\xi'^2}\mathbf{Q}, \tag{10}$$

$$\mathbf{Q}=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\qquad \mathbf{H}(y,\xi)=\begin{bmatrix}h_1(y,\xi)\\ h_2(y,\xi)\end{bmatrix}\qquad \text{and}\qquad \mathbf{S}^*(\xi)=\begin{bmatrix}-\frac{1}{2}\xi^2+\frac{1}{4}\\ -\frac{1}{\sqrt{2}}\end{bmatrix}.$$

The methodology used to obtain Eq. (9) is also applied to the boundary conditions. Therefore, for the boundary conditions of Maxwell, Eq. (2) we find the equation in the vectorial form

$$\mathbf{H}(\mp a,\pm\xi)=(1-\alpha_l)\mathbf{H}(\mp a,\mp\xi). \tag{11}$$

For the boundary conditions of Cercignani-Lampis, Eq. (3) we have

$$\mathbf{H}(\mp a,\pm\xi)=\mathbf{A}_l\int_0^{\infty}\mathbf{H}(\mp a,\mp\xi')f_l(\xi',\xi)\mathrm{d}\xi', \tag{12}$$

where

$$\mathbf{A}_l=\begin{bmatrix}1-\alpha_{t_l} & 0\\ 0 & (1-\alpha_{t_l})^3\end{bmatrix} \tag{13}$$

and

$$f_l(\xi',\xi)=\frac{2\xi'}{\alpha_{n_l}}\exp\left[-\frac{[(1-\alpha_{n_l})^{1/2}\xi-\xi']^2}{\alpha_{n_l}}\right]\widehat{I}_0\left[\frac{2(1-\alpha_{n_l})^{1/2}\xi'\xi}{\alpha_{n_l}}\right], \tag{14}$$

with $l=1,2$.

Based on the vectorial notation, we can express physical quantities , Eqs (5) and (6) as

$$u(y)=\pi^{-1/2}\int_{-\infty}^{\infty}e^{-\xi^2}[1\quad 0]\mathbf{H}(y,\xi)\mathrm{d}\xi \tag{15}$$

and

$$q(y)=\pi^{-1/2}\int_{-\infty}^{\infty}e^{-\xi^2}\left[(\xi^2-1/2)\quad\sqrt{2}\right]\mathbf{H}(y,\xi)\mathrm{d}\xi, \tag{16}$$

which respectively represent the velocity profile and the heat-flow profile.

In addition to these physical quantities, we also evaluate the flow rate of particles

$$U=\frac{1}{2a^2}\int_{-a}^{a}u(y)\mathrm{d}y \tag{17}$$

and the heat-flow rate

$$Q=\frac{1}{2a^2}\int_{-a}^{a}q(y)\mathrm{d}y. \tag{18}$$

4. DEVELOPING THE SOLUTION

The problem defined by Eq. (9) is non-homogeneous, therefore its solution is written as follows

$$\mathbf{H}(y,\xi) = \mathbf{H}^h(y,\xi) + \mathbf{H}^p(y,\xi). \tag{19}$$

As for the homogeneous problem, a particular solution is found

$$\mathbf{H}^p(y,\xi) = \begin{bmatrix} -\frac{1}{2}\xi^2 + \frac{1}{4} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

Substituting Eq. (19) in Eqs. (9), (11) and (12), we conclude that the homogeneous solution must satisfy equation

$$\xi\frac{\partial}{\partial y}\mathbf{H}^h(y,\xi) + \mathbf{H}^h(y,\xi) = \int_{-\infty}^{\infty}\mathbf{\Psi}(\xi')\mathbf{H}^h(y,\xi')\mathrm{d}\xi', \tag{20}$$

the boundary conditions of Maxwell

$$\mathbf{H}^h(\mp a,\pm\xi) - (1-\alpha_l)\mathbf{H}^h(\mp a,\mp\xi) = \mathbf{R}_l^*(\xi) \tag{21}$$

and the boundary conditions of Cercignani-Lampis

$$\mathbf{H}^h(\mp a,\pm\xi) - \mathbf{A}_l\int_0^{\infty}\mathbf{H}^h(\mp a,\mp\xi')f_l(\xi',\xi)\mathrm{d}\xi' = \mathbf{R}_l(\xi), \tag{22}$$

where $\mathbf{\Psi}(\xi')$ is given by Eq. (10),

$$\mathbf{R}_l^*(\xi) = (1-\alpha_l)\mathbf{H}^p(\mp a,\mp\xi) - \mathbf{H}^p(\mp a,\pm\xi) \tag{23}$$

and

$$\mathbf{R}_l(\xi) = \mathbf{A}_l\int_0^{\infty}\mathbf{H}^p(\mp a,\mp\xi')f_l(\xi',\xi)\mathrm{d}\xi' - \mathbf{H}^p(\mp a,\pm\xi). \tag{24}$$

Here, $\mathbf{A}_l$ e $f_l(\xi',\xi)$ are given by Eqs. (3.5) and (3.6), respectively.

Note that l in Eqs. (21)-(24) assumes values (quantities?) 1 or 2, representing the channel walls.

4.1 A discrete-ordinates solution

We solve the homogeneous problems, Eqs. (20)-(22), through the analytical method of discrete-ordinates. In this sense, we define the quadrature scheme and rewrite the Eq. (19) in the discrete-ordinates version as

$$\pm\xi_i\frac{d}{dy}\mathbf{H}^h(y,\pm\xi_i) + \mathbf{H}^h(y,\pm\xi_i) = \sum_{k=1}^{N}\omega_k\mathbf{\Psi}(\xi_k)[\mathbf{H}^h(y,\xi_k) + \mathbf{H}^h(y,-\xi_k)]. \tag{25}$$

For Eqs. (25), the following solutions are proposed

$$\mathbf{H}^h(y,\pm\xi) = \mathbf{\Phi}(\nu,\pm\xi)\, e^{-y/\nu}$$

and after substituting in Eq. (25), we find

$$(\nu\mp\xi_i)\mathbf{\Phi}(\nu,\pm\xi_i) = \nu\sum_{k=1}^{N}\omega_k\mathbf{\Psi}(\xi_k)[\mathbf{\Phi}(\nu,\xi_k) + \mathbf{\Phi}(\nu,-\xi_k)].$$

Now we have vectors $\mathbf{\Phi}_+(\nu)$ and $\mathbf{\Phi}_-(\nu)$ of $2N\times 1$ dimension, with N components 2×1, respectively defined by $\mathbf{\Phi}(\nu,\xi_{\mathbf{k}})$ and $\mathbf{\Phi}(\nu,-\xi_{\mathbf{k}})$. Writing

$$\mathbf{U} = \mathbf{\Phi}_+(\nu) + \mathbf{\Phi}_-(\nu)$$

and after some algebraic manipulations we find the problem of self-values, with self-values $1/\nu^2$, thus obtaining the values of the separation constant ν. Therefore, the solution in discrete-ordinates for the homogeneous equation is given by

$$\mathbf{H}_{\pm}^h(y) = A_1\mathbf{\Phi}^1 + B_1\mathbf{\Phi}_{\pm}^2 + \sum_{j=2}^{2N}[A_j\mathbf{\Phi}_{\pm}(\nu_j)e^{-(a+y)/\nu_j} + B_j\mathbf{\Phi}_{\mp}(\nu_j)e^{-(a-y)/\nu_j}], \tag{26}$$

where we introduce the exact solutions $\mathbf{\Phi}^1$ and $\mathbf{\Phi}^2_\pm$ of dimensions $2N\times 1$, respectively defined by N (vectors) components of the form

$$\mathbf{F}^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{e} \qquad \mathbf{F}^2_\pm(y) = \begin{bmatrix} y \mp \xi \\ 0 \end{bmatrix}. \tag{27}$$

Now, aiming to find the physical quantities, taking into account rarefied gases confined between parallel plates with different chemical compositions,we do not use the symmetry condition, $\mathbf{H}(\mathbf{y},\xi) = \mathbf{H}(-\mathbf{y},-\xi)$, here as it was used in Knackfuss and Barichello [6]. This fact is directly reflected in the number of arbitrary constants present in the solution of the homogeneous problems, that is, analytically we obtain a square algebraic system of $4N$ order, differently from the $2N$ system obtained in Knackfuss and Barichello [6].

To determine the arbitrary constants A_j and B_j, $j = 1,\dots,2N$ for the boundary conditions of Maxwell case, we substitute the Eqs. (26) in Eq. (21) and for the conditions of Cercignani- Lampis case, we substitute the Eq. (26) in Eq. (22) obtaining two systems of linear algebraic equations.

Finally, we find the complete solution

$$\mathbf{H}_\pm(y) = \mathbf{H}^p_\pm(y) + A_1\mathbf{\Phi}^1 + B_1\mathbf{\Phi}^2_\pm + \sum_{j=2}^{2N}[A_j\mathbf{\Phi}_\pm(\nu_j)e^{-(a+y)/\nu_j} + B_j\mathbf{\Phi}_\mp(\nu_j)e^{-(a-y)/\nu_j}]. \tag{28}$$

Here, $\mathbf{H}^p_\pm(y) = \mathbf{H}^p(y,\pm\xi_k)$ denotes a vector $2N\times 1$ with N components 2×1.

5. PHYSICAL QUANTITIES OF INTEREST

To obtain physical quantities in the discrete-ordinates version, we substitute Eq. (28) in Eqs. (15)-(18).

The following definitions are used next

$$\mathbf{N}(\nu_j) = [\omega_1\mathbf{\Lambda}(\xi_1)\ \ \omega_2\mathbf{\Lambda}(\xi_2)\ \ \cdots\ \ \omega_N\mathbf{\Lambda}(\xi_N)][\mathbf{\Phi}_+(\nu_j) + \mathbf{\Phi}_-(\nu_j)]$$

with components $N_1(\nu_j)$ and $N_2(\nu_j)$, where

$$\mathbf{\Lambda}(\xi) = \pi^{-1/2}e^{-\xi^2}\begin{bmatrix} 1 & 0 \\ \xi^2 - 1/2 & \sqrt{2} \end{bmatrix}$$

and the expressions

$$\begin{aligned} M(y) &= A_j\ e^{-(a+y)/\nu_j} \\ Q(y) &= B_j\ e^{-(a-y)/\nu_j} \\ O(y) &= A_j\ \nu_j\ (1 - e^{-(2a)/\nu_j}) \\ P(y) &= B_j\ \nu_j\ (1 - e^{-(2a)/\nu_j}) \end{aligned}$$

to write:

velocity profile

$$u(y) = A_1 + B_1 y + \sum_{j=2}^{2N}\Big[M_j(y) + Q_j(y)\Big]N_1(\nu_j);$$

heat-flow profile

$$q(y) = -\frac{5}{4} + \sum_{j=2}^{2N}\Big[M_j(y) + Q_j(y)\Big]N_2(\nu_j);$$

flow rate of particles

$$U = \frac{1}{2a^2}\Bigg[2aA_1 + \sum_{j=2}^{2N}\Big[O_j(y) + P_j(y)\Big]N_1(\nu_j)\Bigg];$$

heat-flow rate

$$Q = \frac{1}{2a^2}\Bigg[-\frac{5a}{2} + \sum_{j=2}^{2N}\Big[O_j(y) + P_j(y)\Big]N_2(\nu_j)\Bigg].$$

6. NUMERICAL RESULTS

To find numerical results, the computational implementation was developed through FORTRAN language programs. Initially, the quadrature scheme is defined in association with the analytical method of discrete-ordinates (ADO) to implement solutions. For a lot of problems in the rarefied gas dynamics, the following procedure has proved to be adequate [5, 7]. Aiming to calculate the integrals in the interval $[0, \infty)$, we use the non-linear transformation

$$u(\xi) = \mathrm{e}^{-\xi}$$

to map $\xi \in [0, \infty)$ under $u \in [0, 1]$, and then the quadrature scheme of Gauss-Legendre linearly mapped in the interval $[0, 1]$ is used. The next step is to determine the self-values (constants of separation) and self-vectors. Lastly, we find the arbitrary constants so as to obtain the physical quantities of interest.

Results are shown in Tables 1 to 4 and in Figures 1 to 3 for different gases obtained with $N = 60$ quadrature points. The numbers in brackets in these tables represent powers of ten. DE and CL notations represent diffuse- specular boundary conditions (Maxwell) and boundary conditions of Cercignani-Lampis, respectively. In the kinetic equation , given by Eq. (1), the parameter ε was considered arbitrary. In the case of the BGK model, when ε is evaluated in terms of viscosity or thermal conductivity, its value is equal to 1, that is, $\varepsilon = \varepsilon_p = \varepsilon_t = 1$.

To obtain the numerical results shown in the tables and graphs coming up next, the following gases are considered: Ne (Neon), Xe (Xenon) and Ar (Argon). The values for the coefficients of tangential accommodation (α_{t_1}) and accommodation coefficient (α_1) for surface 1, are formulated in terms of experimental values given by Lord [8]. For surface 2, the coefficient values (α_{t_2} and α_2) were reproduced from Sharipov [9] who follows the experimental work of Porodnov *et al* [11].

In relation to the normal accommodation coefficient (α_{n_1} e α_{n_2}), as far as we know, experimental results do not exist, therefore numerical values are chosen based on the thermal accomomodation coeffient of the gases listed above, presented in the work of Thomas [12].

Ne: $\alpha_{t_1} = 0,31$, $\alpha_{n_1} = 0,178$, $\alpha_{t_2} = 0,849$ e $\alpha_{n_2} = 0,082$
Xe: $\alpha_{t_1} = 0,95$, $\alpha_{n_1} = 0,77$, $\alpha_{t_2} = 1,014$ e $\alpha_{n_2} = 0,68$
Ar: $\alpha_{t_1} = 0,67$, $\alpha_{n_1} = 0,44$, $\alpha_{t_2} = 0,916$ e $\alpha_{n_2} = 0,222$

Table 1. Thermal-creep flow: velocity profile $u(y)$, BGK model, $2a = 1$

y/a	Ne		Xe		Ar	
	DE	CL	DE	CL	DE	CL
0,0	1,96663(–1)	1,92295(–1)	1,70039(–1)	1,70138(–1)	1,78803(–1)	1,75957(–1)
0,2	1,90684(–1)	1,94052(–1)	1,67501(–1)	1,68025(–1)	1,75256(–1)	1,74686(–1)
0,4	1,81283(–1)	1,92527(–1)	1,60103(–1)	1,61237(–1)	1,67627(–1)	1,69138(–1)
0,6	1,67406(–1)	1,87266(–1)	1,47402(–1)	1,48766(–1)	1,54904(–1)	1,58442(–1)
0,8	1,46528(–1)	1,76846(–1)	1,26036(–1)	1,27908(–1)	1,34502(–1)	1,40219(–1)
1,0	1,05366(–1)	1,53234(–1)	8,12923(–1)	8,39940(–1)	9,25639(–1)	1,01487(–1)

Table 2. Thermal-creep flow: heat-flow profile $q(y)$, BGK model, $2a = 1$

y/a	Ne		Xe		Ar	
	DE	CL	DE	CL	DE	CL
0,0	–9,24828(–1)	–8,49847(–1)	–7,87685(–1)	–7,82379(–1)	–8,49444(–1)	–7,95456(–1)
0,2	–9,06489(–1)	–8,52612(–1)	–7,79281(–1)	–7,75553(–1)	–8,37501(–1)	–7,89891(–1)
0,4	–8,77749(–1)	–8,43552(–1)	–7,56150(–1)	–7,53912(–1)	–8,12911(–1)	–7,69944(–1)
0,6	–8,34752(–1)	–8,20334(–1)	–7,14338(–1)	–7,13625(–1)	–7,71888(–1)	–7,31978(–1)
0,8	–7,68318(–1)	–7,76291(–1)	–6,43387(–1)	–6,44429(–1)	–7,04812(–1)	–6,66169(–1)
1,0	–6,31486(–1)	–6,74377(–1)	–4,87967(–1)	–4,91807(–1)	–5,61409(–1)	–5,20247(–1)

Table 3. Thermal-creep flow: flow rate of particles U, BGK model

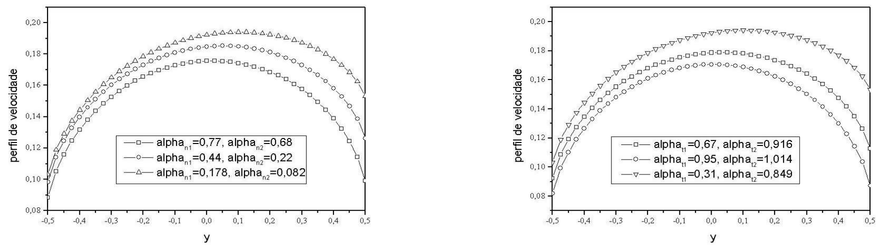
a	Ne		Xe		Ar	
	DE	CL	DE	CL	DE	CL
0,1	8,32005(–1)	7,44537(–1)	5,66656(–1)	5,58912(–1)	6,74913(–1)	6,07628(–1)
1,0	2,26824(–1)	2,26256(–1)	2,04387(–1)	2,06370(–1)	2,08640(–1)	2,10961(–1)
10,0	2,22940(–2)	3,24119(–2)	3,17063(–2)	3,56128(–2)	2,38784(–2)	3,44094(–2)

Table 4. Thermal-creep flow: heat-flow rate Q, BGK model

a	Ne		Xe		Ar	
	DE	CL	DE	CL	DE	CL
0,1	–4,55721	–3,60082	–3,09562	–3,04326	–3,72527	–3,13029
1,0	–1,02991	–9,64696(–1)	–9,05145(–1)	–9,00083(–1)	–9,62584(–1)	–9,13232(–1)
10,0	–1,22575(–1)	–1,21770(–2)	–1,20918(–1)	–1,20849(–1)	–1,21692(–1)	–1,21055(–1)

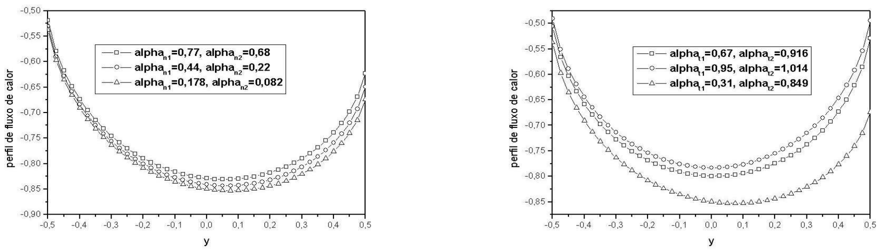
Below are some graphs to illustrate the results obtained, 1 to 3.

In 1 and 2, we observe that the results are not sensitive to the coefficients of tangential and normal accommodation. In 3, we notice that either in 3(a) or in 3(b) there is a similarity between the curves when different scattering kernels are used (Maxwell and Cercignani-Lampis), showing that the results of the physical quantities are not sensitive to the boundary conditions adopted, when $\alpha_t = \alpha$ are considered.



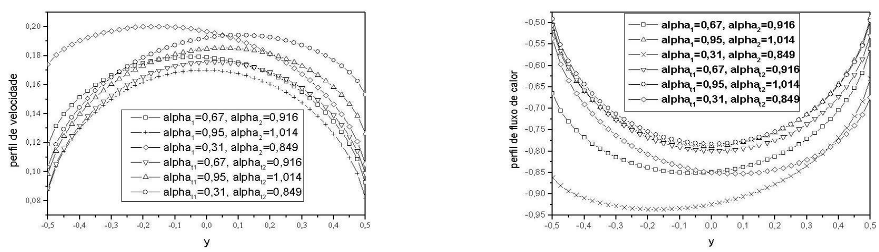
(a) $\alpha_{t1} = 0,31$, $\alpha_{t2} = 0,849$ (b) $\alpha_{n1} = 0,178$, $\alpha_{n2} = 0,082$

Figure 1. Thermal-creep flow - BGK model - Cercignani-Lampis boundary condition - Velocity Profile, $2a = 1$.



(a) $\alpha_{t1} = 0,31$, $\alpha_{t2} = 0,849$ (b) $\alpha_{n1} = 0,178$, $\alpha_{n2} = 0,082$

Figure 2. Thermal-creep flow - BGK model - Cercignani-Lampis boundary condition - Heat-flow profile, $2a = 1$.



(a) Perfil de Velocidade (b) Perfil de Fluxo de Calor

Figure 3. Thermal-creep flow - BGK model - Diffuse specular and Cercignani-Lampis boundary conditions, $2a = 1$, $\alpha_{n1} = 0,178$ e $\alpha_{n2} = 0,082$.

7. FINAL CONSIDERATIONS

The analytical version of the discrete-ordinates method, based on the quadrature scheme of the half-range type, was used to develop the solution to the thermal-creep problem in the rarefied gas dynamics, with the gas-surface interaction through the kernels of Maxwell and Cercignani-Lampis, taking into consideration surfaces with different chemical compositions.

The results based on the BGK model with boundary conditions of Maxwell did not present a significant difference as compared to the results using the boundary conditions of Cercignani-Lampis.

The symmetry condition not used in this work, makes the analysis of the behavior of rarefied gas dynamics more flexible in the sense that one may vary the plates materials through which the gas flows.

8. ACKNOWLEDGEMENTS

The authors would like to thank to FIPE/UFSM for financial support to this and other works.

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