A CFD ANALYSIS OF HEAT TRANSFER IN A 3D BACKWARD-FACING STEP LAMINAR

Rosiane Cristina de Lima, rosianeclima@gmail.com
Cláudia Regina de Andrade, claudia@ita.br
Edson Luiz Zaparoli, zaparoli@ita.br
Grupo de Simulação de Escoamento e Transferência de Calor-GSET, Divisão de Engenharia Mecânica, Instituto Tecnológico de Aeronáutica, CEP 12228-900, São José dos Campos, SP, Brazil

Abstract. *Fluid flows over a backward-facing step with or without heat transfer are often used as benchmark problem for computational fluid dynamic (CFD) codes. Some features of this flow, such as: recirculation zone extensions, spanwise swirling vortex and Nusselt number distributions on the heated stepped wall present several challenges to CFD solutions. In this work, 3D heat and fluid flow in a backward-facing step with aspect ratio of 8 and expansion ratio of 2 are numerically solved by finite volume technique. A segregated coupling of pressure-velocity is employed and a mesh sensitivity study is performed. The results presented good agreement compared with numerical and experimental data from literature.*

Keywords*: CFD, Heat transfer, Laminar convection, Backward-facing step, Three-dimensional flow*

1. INTRODUCTION

Convective heat transfer in separated-reattached flow due to a sudden expansion such as backward-facing step play an important role in the design of a wide variety of engineering applications, such as cooling systems for electronic and power generating equipment electronic, high performance heat exchangers, cooling passages in turbine blades, and dump combustors. In the separated flow region appears a fluid mixing with low and high energy, which affects the heat transfer performance of these devices.

The heat transfer and fluid flow problem in channels with a backward-facing step has been heavily studied, considering forced and/or mixed laminar convection inside vertical, inclined and horizontal retangular ducts. Buoyancy effects also has been analysed. Blackwell (1993) presented a set of eleven papers, showing a large number of interesting experimental and numerical results concerning to steady-state two-dimensional mixed convection flow characteristics of a laminar fluid in vertical backward-facing steps. Reattachment length and Nusselt number results on heated stepped wall were analyzed considering buoyancy effects. In all studies was imposed a difference of constant temperature between the stepped and opposite wall while the other walls were treated as adiabatic surfaces. Abu-Mulawek (2003) also reported data two-dimensionals for laminar mixed convection flow, however flowing over vertical, horizontal and inclined backward- and forward-facing steps.

Iwai et al (2000) provided three-dimensional numerical simulations for flow over a backward-facing step with constant properties, analyzing four aspect ratios (AR=4, 8, 16, 24). They presented results varying the Reynolds number between 125 and 375, showing the effects of the duct aspect ratio on the distributions of Nusselt number and the skin friction coefficient at the bottom wall. It was reported that the maximum Nusselt number did not appear on the centerline but near the two side walls in every duct aspect ratio analyzed.

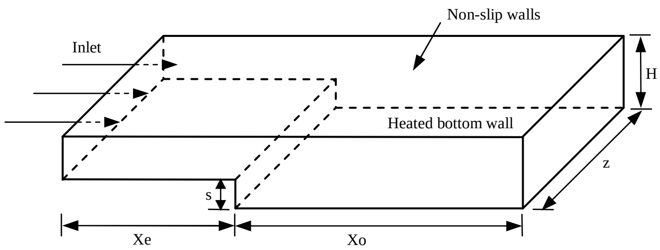
The three-dimensional case of the incompressible laminar forced convection flow over a horizontal backward-facing step was also studied by Nie and Armaly (2002, 2003, 2004). In these three works, the authors used a rectangular duct with expasion ratio ER=2 and aspect ratio AR=8. In the first two studies, a constant heat flux on bottom stepped wall and adiabatic boundary conditions on the remaining walls were imposed; in the last cited work, a constant heat flux on walls downstream from the step was imposed and the upstream walls were treated as adiabatic. A hydrodynamically steady and fully developed flow with a distribution for the streamwise velocity component equal to the one described by Shah and London (1978) for rectangular ducts was taken account in all studies. Analyzes of geometric feature influences as aspect and expansion ratio and downstream duct length on reattachment length and on Nusselt number for different Reynolds numbers are shown. The reattachment length results were compared with experimental data of Li (2001) for Reynolds number 343.

Chen et al (2006) used an inclined three-dimensional backward-facing step in a rectangular duct to carry out numerical studies on laminar forced convection with constant properties and to examine effects of step inclination on flow and heat transfer distributions. They also used the streamwise velocity component equal to the one described by Shah and London (1978). Results showed that the location of the maximum Nusselt number is closely associated with the location where the negative transverse velocity component is maximum and the friction coefficient inside the primary recirculation region increases with the increase of the step inclination angle.

In the present work, the laminar forced convection flow in a three-dimensional horizontal channel with a backward-facing step is numerically solved. The objective of this paper is twofold. First, to analyze the influence of the upstream channel length on recirculation zone extension downstream of the step, already that the majority of the studies cited above used a approximation of the fully developed velocity profile described by Shah and London (1978) for rectangular ducts; and second goal is to analyze the recirculation zone development and Nusselt number distribution as Reynolds number increases. Thus, in the next sections are presented: problem description, mathematical modeling, results and final comments.

2. PROBLEM DESCRIPTION

Three-dimensional laminar convection flow over a backward-facing step is numerically simulated. The configuration of the computational domain and boundary conditions are schematized in Fig. 1. The geometry has a step height (s) of 0.01 m and a width (z) of 0.08 m. The duct heigths upstream (h) and downstream (H) of the step are kept at 0.01 m and 0.02 m, respectively. This provides an expansion ratio (ER=H/h=1+s/h) of two and an aspect ratio (AR=z/h) of eight. Due to symmetry hypothesis of the flow and temperature fields in the spanwise direction, the width of the computation domain was taken equal to half of the actual duct width (L=0.04 m). The computational domain length is 0.5 m downstream (Xo) and upstream (Xe) of the step rating between 0.02 m and 0.5 m.



Height of the inlet channel	h	0.01m
Height of the sudden expansion	H	0.02m
Spanwise width	z L=z/2	0.08m 0.04m
Step height	s	0.01m
Inlet channel streamwise length	Xe	0.02; 0.1; 0.2; 0.5m
Outlet channel streamwise length	Xo	0.5m

Figure 1. Three-dimensional sketch of the backward-facing step with heated bottom wall

The longer calculation domain as upstream as downstream from the step are necessary to reach a fully developed laminar flow conditions and thus ensure no influence of the outflow boundary on the recirculation zones. The origin of the coordinate system is centered at the sharp corner of the step; where x, y and z coordinates denote the streamwise, transverse and spanwise directions, respectively.

The Reynolds number for all calculations is given by the following expression:

$$\mathrm{Re} = \frac{\overline{U}D}{\nu} \tag{1}$$

where $\overline{U}$ is the average velocity at the upstream duct, D is the hydraulic diameter of the inlet channel which is equivalent to twice its height, D=2h, and ν is the fluid kinematic viscosity.

3. MATHEMATICAL MODELING

The laminar steady flow three-dimensional Navier–Stokes and energy equations under constant properties are solved numerically in conjunction with the continuity equation using the finite volume method.

Continuity equation:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial x}(\rho w)=0 \tag{2}$$

Momentum equations:

$$\frac{\partial}{\partial x}\left(\rho u^2\right)+\frac{\partial}{\partial y}(\rho uv)+\frac{\partial}{\partial z}(\rho uw)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}\left(\rho\ v^2\right)+\frac{\partial}{\partial z}(\rho vw)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{4}$$

$$\frac{\partial}{\partial x}(\rho uw)+\frac{\partial}{\partial y}(\rho\ vw)+\frac{\partial}{\partial z}\left(\rho w^2\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{5}$$

Energy equation:

$$\frac{\partial}{\partial x}\left(\rho C_p uT\right)+\frac{\partial}{\partial y}\left(\rho C_p vT\right)+\frac{\partial}{\partial z}\left(\rho C_p wT\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{6}$$

where T is temperature, p represents the pressure, u, v and w are the velocity components in coordinate directions of x, y, and z, respectively. The physical properties are evaluated for air at the inlet temperature (T_0) of 20°C, that is, density (ρ) is 1.205 kg/m^3, specific heat (C_p) is 1005 J/(kg°C), dynamic viscosity (μ) is 1.81x10^{-5} kg/(ms), and thermal conductivity (k) is 0.0259 W/(m°C). Flow at the inlet section upstream of the step (x/s=-2, -10, -20, -50; 0≤y/s≤1;

$0 \le z/s \le 4$) is considered to be isothermal (T_0=20°C) and uniform velocity (u) is imposed. The others two velocity components, v and w, are set to be equal to zero at that inlet section. Non-slip boundary conditions (zero velocities) and thermally adiabatics are applied to all wall surfaces with exception of the bottom wall downstream from the step ($0 \le x/s \le 50$; y/s=-1; $0 \le z/s \le 4$), in which a uniform heat flux, q_w=50W/m^2, is imposed. The boundary conditions above described are mathematically represented in Tab. 1. The Nusselt number for all calculations is given by the following expression:

$$Nu = \frac{q_w s}{(T_w - T_0)k} \tag{7}$$

where T_w is the temperature along the heated bottom wall.

Table 1. Surfaces, coordinates and boundary conditions.

Surfaces	**Boundary conditions**
Inlet	u=uniform, v=w=0; T_0=20°C
Outlet	$p = 0;\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0$
Symmetry	$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial z} = 0$
Heated bottom wall	q_w=50W/m^2
Non-Slip Walls	$\frac{\partial p}{\partial n} = 0$; u=v=w=0; q_w=0

4. RESULTS

The numerical simulations have been performed using a CFD commercial code based on finite volume method (FVM), Fluent (2007). Mathematical modeling for laminar flow over a heated three-dimensional backward-facing step was solved using segregated approach with SIMPLE algorithm as a strategy for the velocity-pressure coupling. Pressure field and advective terms were discretized using second order approaches.

In this work, two problems were studied: 1) the influence of inlet duct length on flow development inside upstream channel and recirculation zone extensions; and 2) the influence of Reynolds number on recirculation zone extensions and on Nusselt number distributions.

Firstly, it was performed a mesh refinement study. A successive hexahedral grid refinement procedure was achieved using structured meshes with uniform element spacing for each different generated mesh. The presented results are for longest inlet channel Xe/s=50. Tab. 2 shows the meshes used in this way, the coarser and finer meshes used in the simulations amount 600,000 and 1,791,567 cells, respectively. The third, fourth and fifth columns show the hexahedral element x, y, z-dimensions. All simulations were carried out until the normalized maximum residuals of the continuity, momentum and energy reach a value lower than 10^{-7}.

Table 2. Mesh characteristics used in the refinement study for Xe/s=50.

Meshes	Cells	x	y	z
1	600,000	0.0009961	0.0010036	0.0009922
2	1,218,750	0.0007975	0.000783387	0.00079524
3	1,791,567	0.00070128	0.0007142	0.000701754

The presented reattachment lines on stepped wall were validated with experimental results of Li (2001) for Reynolds number 343. Thus, the no influence of the mesh refinement using the cell meshes presented in Tab. 2 was shown in Fig. 2. In this figure, a comparison of the reattachment line on stepped wall along the spanwise direction is illustrated. The results for last two meshes (2 and 3 in Tab. 2) demonstrated no influence of grid size. So, the element spacing of the mesh 2 was chosen for remaining simulations.

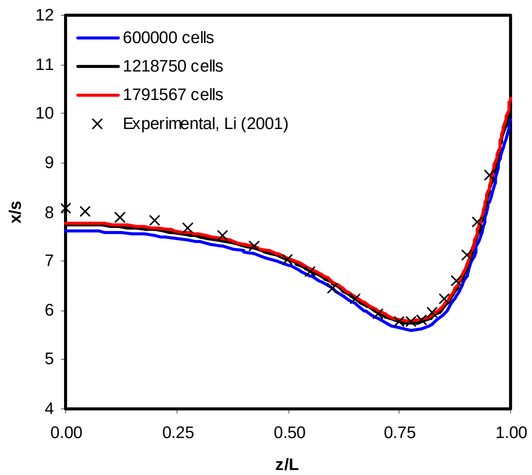


Figure 2. Reattachment line on stepped wall along the spanwise direction for Xe/s=50 upstream channel

Following the report, the influence of inlet duct length (Xe) on both flow development inside upstream channel and on recirculation zone extensions downstream of the step will be presented. Table 3 shows the meshes used in this way. Note that the last line (mesh 7) corresponds to the mesh 2 (Xe/s=50 in Tab. 2) because its results will be tested against others Xe/s. In addition, it was verified that the increase of mesh amount is only due to higher Xe/s values. Constant hexahedral element x,y,z-dimensions are used based on Lima et al (2008).

Table 3. Mesh characteristics used in the inlet channel length study.

Meshes	Xe/s	Cells	x	y	z
4	2	797,500	0.0007975	0.000783387	0.00079524
5	10	862,500			
6	20	975,000			
7	50	1,218,750			

Figure 3 shows the dimensionless centerline streamwise velocity profile (u/uo) along the inlet channel upstream of the step. It is observed that only for Xe/s=50 the flow is fully developed just before the step.

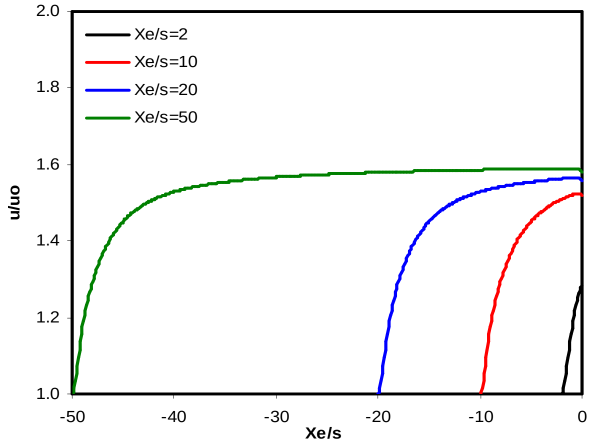


Figure 3. Streamwise velocities along the centerline upstream channel for Re=343

The influence of upstream channel length on recirculation zone size is shown in Fig. 4. The lines of recovering flows were evaluated by the x-wall shear stress sign change on the stepped bottom wall. The obtained numerical solutions with shorter channel (Xe/s=2) present a recirculation zone extension approximately constant near the midplane, showing a very poor agreement with experimental values of Li (2001). The increase of inlet channel length improves the agreement with these experimental values. The better results were obtained for Xe/s=50 corroborating with the information presented in Fig. 2-3.

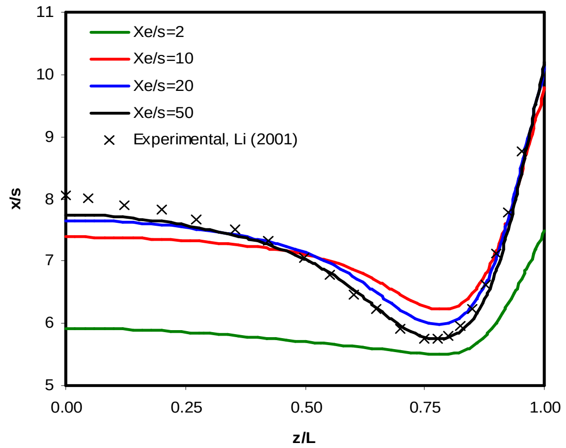


Figure 4. Effects of inlet duct length on reattachment length on stepped wall for Re=343

Reattachment length over entire stepped span wall increases with Reynolds number as expected, Fig. 5. At low Reynolds number, the recirculation zone extensions are approximately constant close to midplane while for higher Reynolds number the flow reattachment occurs more away from the step near the centerline.

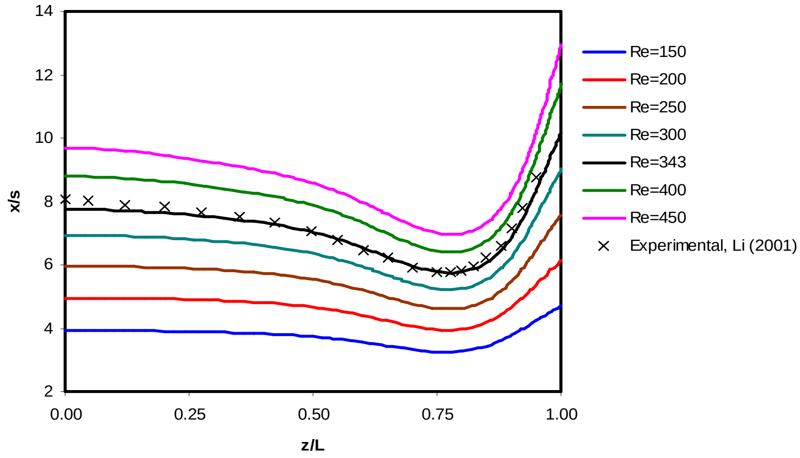


Figure 5. Reattachment length extension on stepped wall varying Reynolds number

A comparison between present work and numerical Nu (maximum and minimum) results of Nie and Armaly (2002) is presented in Tab. 4. These values were obtained over heated stepped wall. The maximum Nusselt number value is in good agreement with the literature one (1.3%), but its location is 12% shifted away. A good agreement (3.4%) is also obtained for minimum Nusselt number. Its location is not comparison due to lack of information.

Table 4. Maximum and minimum Nu number for Re=343.

	Present results		**Nie and Armaly (2002)**	
		x/s		x/s
Nu_{max}	1.521	8.72	1.514	7.612
Nu_{min}	0.705	0.08	0.73	

Figure 6 illustrates the Nusselt number results at different Reynolds numbers as a function of the step distance (x/s). The centerline Nusselt number maximum value ever increases and shifts away as Re elevates. After this peak, the Nu values are greater for higher Re values reversing the behaviour observed near the step.

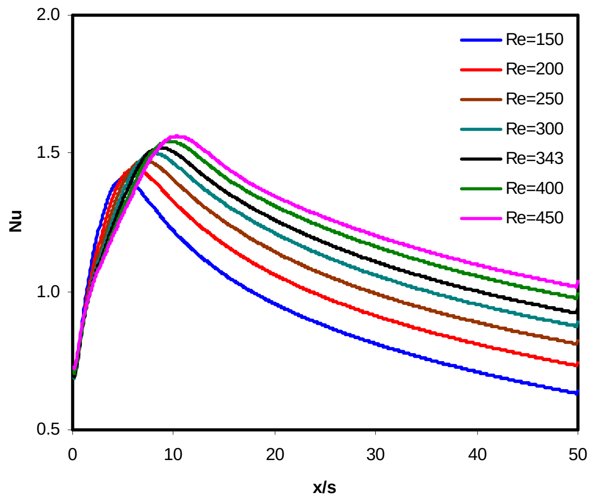


Figure 6. Nusselt number distributions along the centerline for different Reynolds number

For each Reynolds number value, the Nusselt number profile exhibits an asymptotic decay with x/s increase after the maximum point is reached.

6. FINAL COMMENTS

At the present work, a three-dimensional laminar separated flow adjacent to backward-facing step problem with heated bottom stepped wall was numerically solved. The influence of both upstream channel length and Reynolds number on the recirculation zone extension and Nusselt number were evaluated. Results showed that the inlet duct length must be sufficient to ensure fully developed laminar flow conditions just before the step to obtain a good prediction of recirculation zone extension and Nusselt number over heated stepped wall. Authors also intend to investigate the effect of a $Xe/s > 50$ in future works.

7. ACKNOWLEDGEMENTS

The authors would like to acknowledge the CNPq by the scholarship of first author and GSET (Group of Simulation of Fluid Flow and Heat Transfer) by the computational support.

8. REFERENCES

Abu-Mulaweh, H.I., (2003), “A review of research on laminar mixed convection flow over backward- and forward-facing steps”, International Journal of Thermal Sciences, Vol. 42, pp. 897–909.

Blackwell, B.F. and Armaly, B.F., 1993, “Computational aspect of heat transfer benchmark problems”, in: ASME Winter Annual Meeting, in: HTD, Vol. 258.

Chen, Y.T., Nie, J.H., Hsieh, H.T. and Sun L.J., 2006, “Three-dimensional convection flow adjacent to inclined backward-facing step”, International Journal of Heat and Mass Transfer, Vol. 49, pp. 4795–4803.

Fluent Inc. (2007) User’s guide, 6.4.11 version.

Iwai, H., Nakabe, K. and Suzuki, K., 2000, “Flow and heat transfer characteristics of backward-facing step laminar flow in a rectangular duct”, International Journal of Heat and Mass Transfer, Vol. 43, pp. 457-471.

Li, A., 2001, “Experimental and Numerical Study of Three-Dimensional Laminar Separated Flow Adjacent to Backward-Facing Step”, Ph.D. thesis, University of Missouri, Rolla, MO apud Nie, J.H. and Armaly, B.F., 2003, “Reattachment of three-dimensional flow adjacent to backward-facing step”, Transactions of the ASME, Vol. 125, pp. 422-428.

Lima, R.C., Andrade, C.R. and Zaparoli, E.L., 2008, “Numerical study of three recirculation zones in the unilateral sudden expansion flow”, International Communications in Heat and Mass Transfer, doi:10.1016/ j.icheatmasstransfer.2008.07.004.

Nie, J.H. and Armaly, B.F., 2002, “Three-dimensional convective flow adjacent to backward-facing step - effects of step height”, International Journal of Heat and Mass Transfer, Vol. 45, pp. 2431–2438.

Nie, J.H. and Armaly, B.F., 2003, “Reattachment of three-dimensional flow adjacent to backward-facing step”, Transactions of the ASME, Vol. 125, pp. 422-428.

Nie, J.H. and Armaly, B.F., 2004, “Convection in laminar three-dimensional separated flow”, International Journal of Heat and Mass Transfer, Vol. 47, pp. 5407-5416.

Shah, R.K. and London, A.L., 1978, “Laminar Flow Forced Convection in Ducts”, Supl. 1 in: Advances in Heat Transfer (Irvine JR, T.F. and Hartnett, J.P), Academic Press, New York.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.