CONVECTION IN MICROCHANNELS WITH ELECTROOSMOTIC FLOW: INTEGRAL TRANSFORMS IN PSEUDO-TRANSIENT FORMULATION

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Resumo. *Electroosmotic flow in microchannels has revealed of great applications in many areas, such as biomedicine, pharmacy, environmental monitoring and defense, among others. Drug devilery systems and the analysis and sequencing of DNA/RNA are some examples of these applications. Aiming a better understanding of this process and also providing data to an experiment that is being constructed at the Laboratory of Transmission and Technology of Heat (LTTC - COPPE/UFRJ), this work presents a study about the electroosmotic flow and the transient forced convection in a parallel plate microchannel, with 10* μm *between the plates The Generalized Integral Transform Technique (GITT), with partial transforms, is used to solve the convection problem. The eigenvalue problem that comes from the application of the GITT in the convection problem is also solved by GITT. Using the fact that this kind of problem has small transients, the pseudo—transient formulation of the method is also studied.*

Keywords*: microscale, electroosmotic flow, transient convection, integral transforms*

1. INTRODUCTION

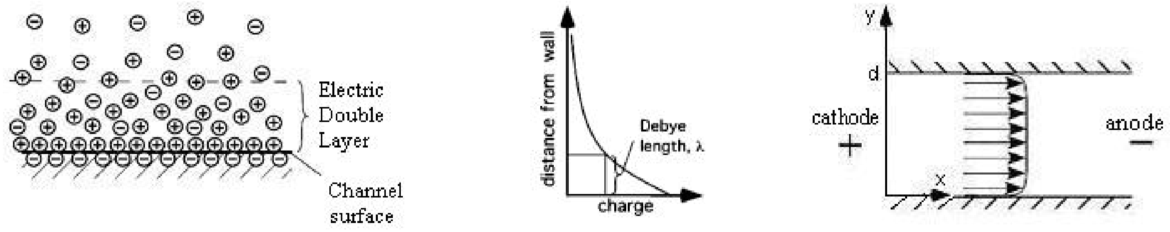
Electroosmotic flow in microchannels has been used with great results in many areas, such as biomedicine, pharmacy, environmental monitoring and defense, among others. Drug delivery, DNA analysis and sequencing and chips with sensors to detect chemical and biological agents are some usual applications of this technology. In all these examples, the reliability and adequacy of all components are essencial for the whole project success and for the correct operation of the entire system. In particular, microvalves and micropumps with moving parts are difficult to project and manufacture, susceptible to mechanical failures and defects on manufacturing, and don´t allow the precise control required by the majority of applications. In this context, electroosmotic flow appears as an excellent alternative, allowing pump and flow control without moving parts, through manipulation of electrical fields. Furthermore, the plug-flow profile, typical of electroosmotic flow, allows fluid transportation without dispersion problems that would be present in a pressure driven flow.

Aiming a theoretical study of this matter, this work presents an analysis of forced convection in a parallel plate microchannel, with purely electroosmotic flow. It has two major goals. The first one is to perform a physical analysis of the problem, providing information and reference data for a lab experiment that is being constructed at LTTC. The second one is to exploit the partial transform in pseudo-transient formulation of the Generalized Integral Transform Technique (GITT), trying to find a new methodology to reach the steady state result in a more efficient way, considering that there is no interest in the transient result in the analysis that is being performed.

2. PROBLEM FORMULATION

Electroosmosis is a process in which an ionized liquid moves in relation to an electrically charged stationary surface, when subjected to an electric field externally applied. Most solid surfaces will acquire a relative electric charge when in contact with an aqueous electrolytic solution, which in turn influences the charge distribution in the solution. If the surface acquires negative charges, as shown in Fig. 1a, ions of opposite charge are attracted towards the surface and ions of the same charge are repelled from the surface (Maynes and Webb, 2004), keeping the bulk of the liquid, far away from the wall, electrically neutral. The ions of opposite charge cluster immediatelly near the wall, forming the *Stern layer*, a layer of typical thickness of one ionic diameter. The ions within the Stern layer are attracted to the wall with very strong electroostatic forces; hence, they are immobilized near the charged surface. Immediatelly after the Stern layer there forms the *diffuse layer*, where the ion density variation obeys the Boltzmann distribution, and goes from its maximum value near the wall to a zero charge in the fluid core (Karniadakis *et al.*, 2005).

Hence, Stern and diffuse layers form the electric double layer (EDL), characterized by Debye length, λ , which is the wall-normal distance over which the net charge has decreased from the charge magnitude near the wall surface to about 37% of the surface charge, as shown in Fig. 1b (Maynes and Webb, 2004).



(a) (b) (c)

Figure 1. Schematic illustration of the electroosmotic flow. (a) the EDL; (b) Debye length; (c) fluid flow.

The electroosmotic flow occurs when an electric field is applied parallel to the surface, in the presence of a stabilized EDL. In Figure 1c it can be seen that, in the presence of an externally applied electric field, positive ions will be attracted by the cathode and repelled by the anode, causing the movement of the whole fluid in the direction of the electric field, as a consequence of the presence of viscous forces (Tabeling, 2005).

In this work we consider a rectangular microchannel with 10 μm x 200 μm x 2 cm (heigth x width x length), with one surface made of glass and the other made of silicon, that has been made on the *Laboratorie de Microfluidique, MEMS et Nanostructures – MMN*, ESPCI, by professor Patrick Tabeling. As the width is 20 times greater than the height, it will be modeled as a parallel plate microchannel (Fig. 2).

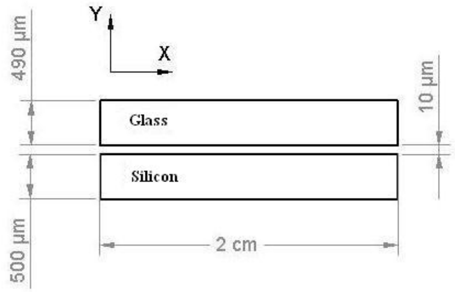


Figure 2. Schematic illustration of the microchannel

2.1. Electroosmotic flow

The following assumptions are considered for the hydrodynamics: a) Newtonian fluid, with constant physical properties; b) incompressible, laminar and fully developed flow; c) steady state regime; d) purely electroosmotic flow (which means that there is no pressure gradient); e) electric field externally applied on the fluid, in the x direction.

With this assumptions, mass conservation equation resumes to $u = u(y)$. After some manipulations, the Navier-Stokes equations take the following dimensionless form, considering boundary condition of no-slip on both surfaces,

$$\begin{cases} \dfrac{d^2U(Y)}{dY^2} - G\,E_x^* \dfrac{d^2\overline{\psi}(Y)}{dY^2} = 0, & 0 < Y < 1 \\ U(Y) = 0 & Y = 0 \\ U(Y) = 0 & Y = 1 \end{cases} \tag{1}$$

with $G = \varepsilon\,\varepsilon_o k_b T\,\varsigma_m / L\,z\,q\,\mu\,u_m$, where E_x^* is the dimensionless electric field externally applied, $\overline{\psi}(y)$ is the dimensionless electrical potential, ε the dielectric constant of the medium, ε_0 the permittivity of vacuum, k_b the Boltzmann constant, T is the absolute temperature, ζ_m a reference zeta potential, L the length of the microchannel, z the ion charge, q the elemental charge, μ the fluid viscosity and u_m a reference velocity.

The dimensionless electric potential is obtained by a combination of Poisson and Boltzmann distribution equations (Yang *et al.*, 2001), and is given by

$$\begin{cases} \dfrac{d^2\overline{\psi}(Y)}{dY^2} = Z^2\overline{\psi}(Y) & 0 < Y < 1 \\ \psi(Y) = \bar{\varsigma}_1 & Y = 0 \\ \overline{\psi}(Y) = \bar{\varsigma}_2 & Y = 1 \end{cases} \tag{2}$$

where $Z = \kappa d = d/\lambda$ is a parameter that relates the length of the channel and the EDL length, λ, that is computed as the inverse of the Debye-Hückel parameter, written as $\kappa^2 = 2n_0 z^2 q^2/\varepsilon\varepsilon_0 k_b T$, where n_0 is bulk concentration of ions, and $\bar{\varsigma}_1$ and $\bar{\varsigma}_2$ are dimensionless zeta potentials (the zeta potential is the value of the EDL field at the top of Stern layer).

Both systems (Eqs. (1) and (2)) are easily solved by integrating and substituting boundary conditions, obtaining

$$\overline{\psi}(Y) = \frac{e^{-YZ}\left[\bar{\varsigma}_1\left(e^{2Z} - e^{2YZ}\right) + \bar{\varsigma}_2\left(e^{Z+2YZ} - e^{Z}\right)\right]}{e^{2Z} - 1} \tag{3}$$

$$\begin{aligned} U(Y) = {} & \frac{e^{-YZ}E_x^* G\left[\bar{\varsigma}_1\left(e^{2Z} + e^{YZ} - e^{2YZ} - e^{2Z+YZ} - Ye^{YZ} + Ye^{2Z+YZ}\right)\right]}{e^{2Z} - 1} + \\ & \frac{e^{-YZ}E_x^* G\left[\bar{\varsigma}_2\left(e^{Z+2YZ} - e^{Z} + Ye^{YZ} - Ye^{2Z+YZ}\right)\right]}{e^{2Z} - 1} \end{aligned} \tag{4}$$

The dimensionless parameters considered were,

$$X = \frac{x}{d}\frac{1}{Pe} \qquad Y = \frac{y}{d} \qquad H = \frac{L}{d\,Pe} \qquad U(Y) = \frac{u(y)}{u_m} \tag{5}$$

$$\overline{\psi}(Y) = \frac{zq}{k_b T}\psi(y) \qquad E_x^* = \frac{E_x L}{\varsigma_m} \qquad Pe = \frac{u_m\, d}{\alpha} \qquad \varsigma_m = \frac{\varsigma_1 + \varsigma_2}{2} \tag{6}$$

2.2. Forced convection

To analyze the thermal problem, we consider that the microchannel is subjected to uniform and constant flux in the silicon surface, and to natural convection with ambient air in the glass surface. The additional following assumptions are considered: a) transient condition; b) axial diffusion; c) all heat that achieves the silicon surface is transfered to the fluid; d) thermal resistance of glass surface. For this problem, energy conservation equation can be written, in the dimensionless form, as

$$\begin{cases} \dfrac{\partial\theta(X,Y,\tau)}{\partial\tau} + U(Y)\dfrac{\partial\theta(X,Y,\tau)}{\partial X} = \dfrac{1}{Pe^2}\dfrac{\partial^2\theta(X,Y,\tau)}{\partial X^2} + \dfrac{\partial^2\theta(X,Y,\tau)}{\partial Y^2}, & 0 < X < H,\ 0 < Y < 1,\ \tau > 0 \\ \theta(0,Y,\tau) = 0 & \\ \left.\dfrac{\partial\theta(X,Y,\tau)}{\partial X}\right|_{X=H} = 0 & \\ \left.\dfrac{\partial\theta(X,Y,\tau)}{\partial Y}\right|_{Y=0} = -\Phi_w & \\ \left.\dfrac{\partial\theta(X,Y,\tau)}{\partial Y}\right|_{Y=1} = -Bi\,\theta(X,1,\tau) & \\ \theta(X,Y,0) = 0 & \end{cases} \tag{7}$$

where the additional following parameters are considered

$$\tau = \frac{\alpha}{d^2}t \qquad \theta(X,Y,\tau) = \frac{T(x,y,t) - T_\infty}{\Delta T} \tag{8}$$

$$\Phi_w = \frac{\phi_w\, d}{k_f\, \Delta T} \qquad Bi = \frac{h_{efe} d}{k_f} \qquad h_{efe} = \frac{k_v h_\infty}{h_\infty esp_v + k_v} \tag{9}$$

In Eqs. (8) and (9), T_∞ is the temperature of ambient air, ΔT the maximum difference temperature in the microchannel, ϕ_w the heat flux at the silicon surface, k_f the thermal conductivity of the fluid, h_{efe} an effective heat transfer coefficient, that considers the thermal resistance of the glass surface, k_v the thermal conductivity of glass, h_∞ the convection heat transfer coefficient and esp_v the thickness of the glass surface.

To analyze the results, we define a mean temperature and the Nusselt numbers for both surfaces,

$$\theta_m(X,\tau)=\frac{\int_{Y=0}^{1} U(Y)\theta(X,Y,\tau)\,dY}{\int_{Y=0}^{1} U(Y)\,dY} \tag{10}$$

$$Nu_1=\frac{2\Phi_w}{\theta(X,0,\tau)-\theta_m(X,\tau)} \qquad Nu_2=2\,Bi\frac{\theta(X,1,\tau)}{\theta_m(X,\tau)-\theta(X,1,\tau)} \tag{11}$$

3. SOLUTION OF ENERGY EQUATION

The Generalized Integral Transform Technique (GITT) is used to solve Eq. (7). The first step in the application of this method is to define an auxiliary eigenvalue problem. For this problem, we define an auxiliary problem that includes the velocity profile (Mikhailov and Cotta, 2005),

$$\begin{cases} \dfrac{d^2\Gamma_i(Y)}{dY^2}+\delta_i^2\,U(Y)\Gamma_i(Y)=0 & 0<Y<1 \\ \left.\dfrac{d\Gamma_i(Y)}{dY}\right|_{Y=0}=0 & Y=0 \\ \left.\dfrac{d\Gamma_i(Y)}{dY}\right|_{Y=1}+Bi\,\Gamma_i(1)=0 & Y=1 \end{cases} \tag{12}$$

that is solved by GITT itself (Mikhailov and Cotta, 1994, Oliva Soares *et al.*, 2005). So, we define an auxiliary problem for Eq. (12),

$$\begin{cases} \dfrac{d^2\Omega_m(Y)}{dY^2}+\nu_m^2\,\Omega_m(Y)=0 & 0<Y<1 \\ \left.\dfrac{d\Omega_m(Y)}{dY}\right|_{Y=0}=0 & Y=0 \\ \left.\dfrac{d\Omega_m(Y)}{dY}\right|_{Y=1}+Bi\,\Omega_m(1)=0 & Y=1 \end{cases} \tag{13}$$

and the following inverse-transform pair,

$$\Gamma_i(x)=\sum_{m=1}^{\infty}\tilde{\Omega}_m(x)\overline{\Gamma}_{i,m} \quad \text{(inverse)} \qquad \overline{\Gamma}_{i,m}=\int_{x_0}^{x_1}\tilde{\Omega}_m(x)\Gamma_i(x)\,dx \quad \text{(transform)} \tag{14a,b}$$

where $\tilde{\Omega}_m(x)$ is a normalized eigenfunction, $\tilde{\Omega}_m(x)=\dfrac{\Omega_m(x)}{N_m^{1/2}}$ and $N_m=\int_{x_0}^{x_1}[\Omega_m(x)]^2\,dx$ is the norm of the eingenfunctions. The solution for Eq. (13) is given by Özişik (1993) as

$$\Omega_m(Y)=\cos(\nu_m Y) \tag{15}$$

where ν_m are the positive roots of $\sin(\nu_m)-(Bi/\nu_m)\cos(\nu_m)=0$.

With this, it's possible to obtain the eigenvectors $\overline{\Gamma}_{i,m}$, $m = 1,2,\ldots,S_{aux}$, and to reconstruct $\Gamma_i(x)$ using Eq. (14a), considering S terms in the sum, large enough to obtain the desirable number of significant digits in the result. Now, it's possible to define an inverse-transform pair to the dimensionless temperature,

$$\theta(X,Y,\tau)=\sum_{i=1}^{\infty}\tilde{\Gamma}_i(Y)\bar{\theta}_i(X,\tau)\ \text{(inverse)} \qquad \bar{\theta}_i(X,\tau)=\int_{Y=0}^{1}U(Y)\tilde{\Gamma}_i(Y)\theta(X,Y,\tau)\,dY\ \text{(transform)} \tag{16a,b}$$

The norm, in this case, is given by $N_i = \int_{Y=0}^{1} U(Y)\left[\Gamma_i(Y)\right]^2 dY$. To apply the integral transform, we operate on Eq. (7) with $\int_{Y=0}^{1}(\cdot)\tilde{\Gamma}_i(Y)dY$ (main equation) and $\int_{Y=0}^{1}(\cdot)U(Y)\tilde{\Gamma}_i(Y)dY$ (boundary and initial conditions), and after some manipulations,

$$\begin{cases} \displaystyle\sum_{j=1}^{\infty}A_{ij}\frac{\partial\bar{\theta}_j(X,\tau)}{\partial\tau}+\frac{\partial\bar{\theta}_i(X,\tau)}{\partial X}=\frac{1}{Pe^2}\sum_{j=1}^{\infty}A_{ij}\frac{\partial^2\bar{\theta}_j(X,\tau)}{\partial X^2}-\delta_i^2\bar{\theta}_i(X,\tau)+\bar{g}_i & 0<X<H,\ \ \tau>0 \\ \bar{\theta}_i(0,\tau)=0 & \\ \displaystyle\left.\frac{\partial\bar{\theta}_i(X,\tau)}{\partial X}\right|_{X=H}=0 & \\ \bar{\theta}_i(X,0)=0 & \end{cases} \tag{17}$$

These coupled system of partial differential equations is solved by the numerical Method of Lines, implemented in routine *NDSolve*, in *Mathematica* software, truncating the infinite sum in a number S of terms, sufficient to ensure convergence with the desired number of significant digits. Once the potentials ($\bar{\theta}_i$) are numerically computed, this routine gives the solution as interpolating functions, that allows the direct application of the inverse formula (Eq. (16a)) to obtain the dimensionless temperature field.

4. RESULTS AND DISCUSS

To generate all the results presented here, we consider a diluted electrolytic solution of KCl in water (ε = 80, μ = 0,9x10^{-3}kg/ms, ρ = 998 kg/m^3, k_f = 0,6W/mK, α = 1,44x10^{-7}m^2/s, z = 1), with molar concentration of 10^{-4}M. Using these data, and considering u_m = 2mm/s (Karniadakis *et al.*, 2005), it is possible to estimate Pe = 0,139 (Peclet) and Re = 0,022 (Reynolds). With this Peclet number, we cannot let axial diffusion out of the modeling. To the molar concentration mentioned above, the zeta potential is equal to -100mV at the silicon surface (Mala *et al.*, 1997) and equal to -59mV at the glass surface (Erickson *et al*, 2000). Using this data, at ambient temperature (T = 298K), Debye-Hückel parameter is κ = 3,25625x10^6 m^{-1}; consequently, the length of the EDL is λ = 307,1nm and Z = 32,562. To generate the results showed here, we consider Z = 25, that is still a typical electroosmotic flow. We apply electric fields of 1V/mm.

To simulate forced convection, we consider heat flux of 1000W/m^2 at silicon surface. A maximum difference temperature of 80ºC is used, to avoid fluid evaporation inside the microchannel. A convection heat transfer coefficient of 25 W/m^{2}K is considered, corresponding to natural convection with air at ambient temperature. The thermal conductivity of the glass is 0,8W/mK, and using all these values it is possible to compute an effective heat transfer coefficient (h_{efe}) of 24,62 W/m^{2}K and Bi = 0,00041.

4.1. Validation

It is expected a parabolic velocity profile for Z = 0,5, because it represents a situation where the EDL length is greater than the channel diameter, simulating a constant pressure gradient across channel section. The value of Z =50 represents a typical electroosmotic flow, and then a plug profile is expected when zeta potentials of the surfaces are the same. It was used to validate the results obtained with Eq. (4), and the expected profile has been obtained (Oliva Soares, 2007).

The eigenvalue problem solved by GITT was compared with an analytical result, obtained by Castellões and Cotta (2005), Mikhailov and Özişik (1984) and Mikhailov and Cotta (1997), in terms of confluent hypergeometric functions, for a parabolic velocity profile. The numerical results showed convergence to the analytical value with S_{aux} = 30 in the sum, with 6 significant digits. Table 1 presents this result; S is the number of terms in the main sum (for the computation of the dimensionless temperature), and S_{exc} is used to assure converge, $S_{aux} = S + S_{exc}$.

The transient energy problem was compared with a simpler version, in steady state regime and without axial conduction, using the same solution methodology. To make the comparison, we considered Pe = 1000 in the complete solution, to simulate a situation without axial conduction. We observed that for τ = 12000 (t ~ 8s) the transient solution reaches the steady-state solution (Oliva Soares, 2007).

Table 1 – Comparing the solution obtained with GITT applied to the eigenvalue problem with the analytical result, for a parabolic velocity profile (S = 10), for some eigenvalues

	$S_{exc} = 10$	$S_{exc} = 20$	$S_{exc} = 30$	$S_{exc} = 40$	$S_{exc} = 50$	***Analytical***
μ_2	4,62942	4,62942	4,62942	4,62942	4,62942	**4,62942**
μ_4	11,15961	11,1596	11,1596	11,1596	11,1596	**11,1596**
μ_6	17,69126	17,6912	17,6912	17,6912	17,6912	**17,6912**
μ_8	24,31113	24,22302	24,22302	24,22302	24,22302	**24,22302**
μ_{10}	43,50276	30,75492	30,75492	30,75492	30,75492	**30,75492**

4.2. Convergence

Our validation has shown that GITT can be used to solve eigenvalue problem, with good results, when it is not possible to obtain the analytical solution. We have verified how many terms are necessary in the sum to reach convergence when an electroosmotic velocity profile is used (Z = 25), finding out that S_{aux} = 40 terms are enough to assure convergence with 5 significant digits, as seen in Tab. 2.

Table 2 – Convergence of the first 10 eigenvalues for a typical electroosmotic flow

	$S_{exc} = 0$	$S_{exc} = 10$	$S_{exc} = 20$	$S_{exc} = 30$	$S_{exc} = 40$	$S_{exc} = 50$
μ_2	19,42276	19,42214	19,42211	19,42211	19,42210	19,42210
μ_4	57,65417	57,63324	57,63238	57,63228	57,63225	57,63224
μ_6	95,62972	95,47825	95,47373	95,47318	95,47306	95,47303
μ_8	134,07163	133,15624	133,14161	133,13997	133,13962	133,13952
μ_{10}	183,12336	170,77145	170,73302	170,72925	170,72847	170,72824

It is also necessary to analyze convergence of the method (GITT) applied to advective-diffusive problem. To do this, some truncation orders are considered, and the one who gives good results with a reasonable computational cost is chosen (time between parenthesis in the first line of Tabs. 3 and 4 is computational time, in a Pentium D, 2,80 GHz and 1 Gb RAM). To give an example, Tab. 3 shows the results for mean temperature, and Tab. 4 shows it for the Nusselt number on silicon surface. A simple filter has been used to verify the possibility of accelerate the convergence. We could observe that, for a fixed number of terms in the sum, it is possible to obtain one or two more significant digits using this technique, mainly for Nusselt numbers (Oliva Soares, 2007).

Table 3 – Dimensionless mean temperature convergence

	X	$S = 5$ *(0,2969 s)*	$S = 10$ *(2,4689 s)*	$S = 15$ *(16,3439 s)*	$S = 20$ *(435,625 s)*	$S = 25$ *(17757,0779 s)*
$\tau = 2000$ $(t = 1{,}39\ s)$	500	0,29471	0,29466	0,29465	0,29464	0,29463
	2000	0,42815	0,42811	0,42811	0,42810	0,42810
$\tau = 6000$ $(t = 4{,}17\ s)$	500	0,44623	0,44619	0,44618	0,44617	0,44617
	2000	0,69696	0,69692	0,69691	0,69691	0,69691
$\tau = 12000$ $(t = 8{,}34\ s)$	500	0,46976	0,46975	0,46974	0,46974	0,46975
	2000	0,75286	0,75284	0,75284	0,75283	0,75285

Table 4 – Dimensionless Nusselt convergence on silicon surface

	X	$S = 5$ *(0,2969 s)*	$S = 10$ *(2,4689 s)*	$S = 15$ *(16,3439 s)*	$S = 20$ *(435,625 s)*	$S = 25$ *(17757,0779 s)*
$\tau = 2000$ $(t = 1{,}39\ s)$	500	8,12735	7,53738	7,34218	7,24004	7,18259
	2000	5,91131	5,48878	5,34873	5,27543	5,23417
$\tau = 6000$ $(t = 4{,}17\ s)$	500	7,29467	6,81334	6,65217	6,56789	6,52032
	2000	5,14985	4,82409	4,71450	4,65719	4,62480
$\tau = 12000$ $(t = 8{,}34\ s)$	500	7,18066	6,71327	6,55654	6,47459	6,42827
	2000	5,01558	4,70560	4,60105	4,54639	4,51545

4.3. Physical results

We briefly present some physical results for the EDL field, velocity profile, mean temperature and Nusselt numbers.

Figure 5 shows variation of EDL with *Z* parameter, for a 10^{-4}M solution, and Fig. 6 shows the same kind of result for the velocity profile. Figure 7 and Fig. 8 presents the dimensionless mean temperature varying with heat flux and electric field. This information is used to see which values can be used in the laboratory in order to no have vaporization of the fluid inside the channel.

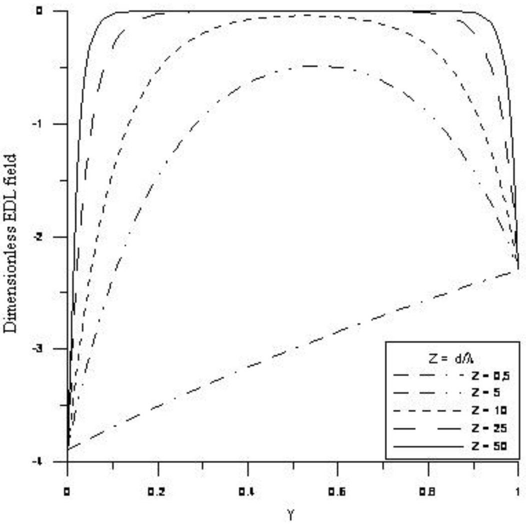


Figure 5 – EDL field variation with *Z* parameter

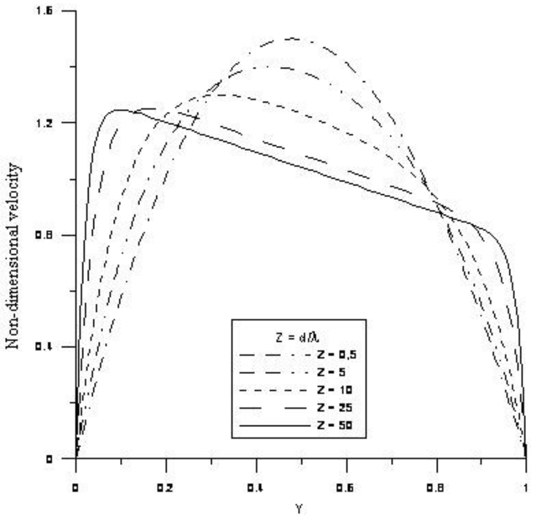


Figure 6 – Velocity profile variation with *Z* parameter

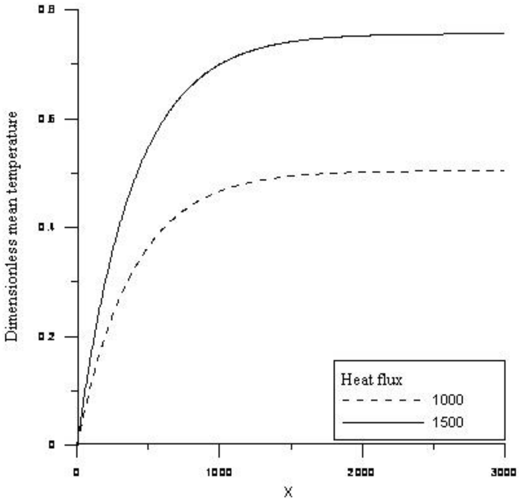


Figure 7 – Temperature variation with heat flux (1V/mm)

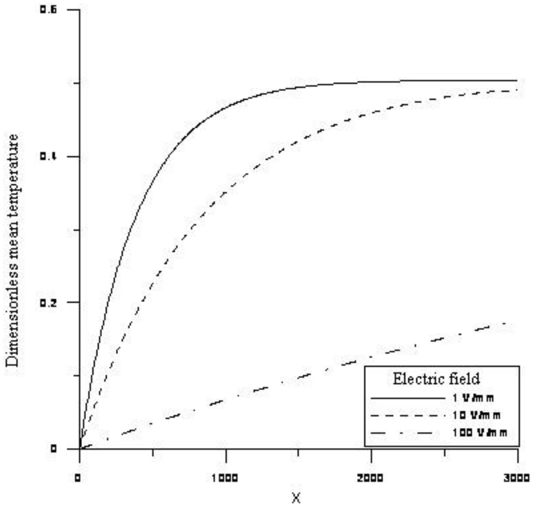


Figure 8 – Temperature variation with electric field ($1000W/m^2$)

Nusselt number is a measure of the relationship between convection and conduction in the fluid. Figures 9 and 10 shows Nusselt numbers on both surfaces (silicon and glass), varying with the velocity profile, represented by a variation in electric field (a greater electric field represents a greater velocity).

4.4. Pseudo-transient formulation

In a practical situation in microscale, our interest is on steady-state solutions (for the case considered at this work, a physical time of about 7,6s is enough to reach steady-state regime). So one could ask why we considered transient regime in energy equations. The transient term is included here to make it possible to obtain the steady-state solution without computational problems. When it is used, the integral transform of energy equation results in an initial value problem, solved directly by *NDSolve* routine. If it is not used, the result of integral transformation of energy equation is a boundary value problem, non-treatable by *NDSolve*, until version 5.2 of *Mathematica*. Routine DBVPFD, of IMSL, could be used in this situation, but it’s not trivial, and it wasn’t our intention to use another programming language.

Considering this, we thought about working with a pseudo-transient problem to reach the steady-state result in a more efficient way, saving computational time. So, have we considered a lowest-order solution (Cotta, 1993), rewriting Eq. (17) substituting A_{ij} in the first sum by A_{ii}. Doing it, we have considered only the diagonal of the matrix, leaving this term with less precision. We have obtained a significant less computational time. *NDSolve* took 5 times less to be executed, comparing with the case when A_{ij} is considered in the sum, without lost of precision (a difference in the final result has been observed only for initial times, in the 6 significant digit).

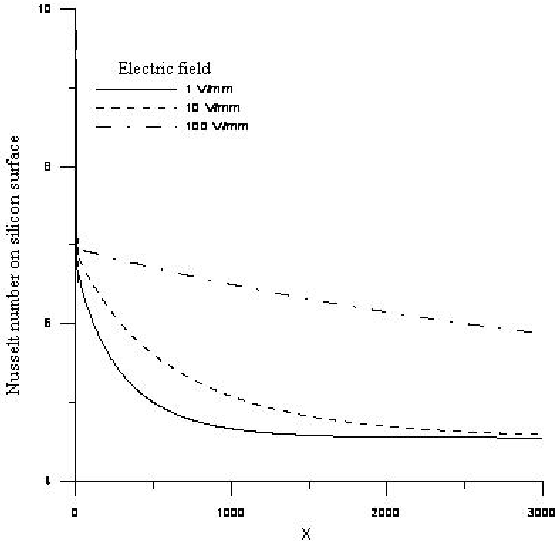


Figure 9 – Nusselt number on silicon surface

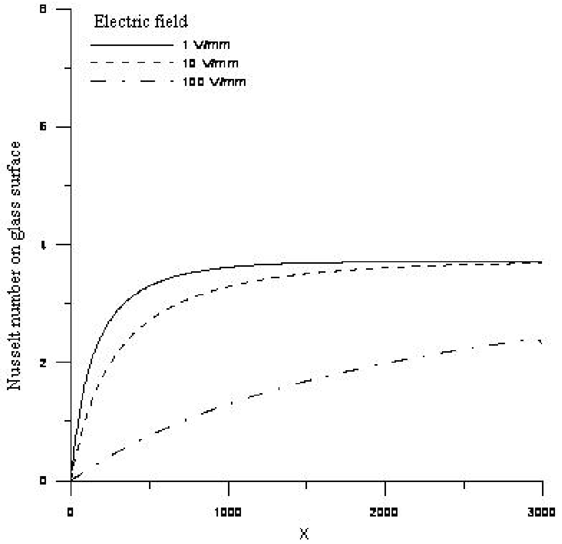


Figure 10 – Nusselt number on glass surface

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