

COOLING OF FRUITS WITH ELLIPSOIDAL SHAPE BY GALERKIN-BASED INTEGRAL METHOD: AN ANALYTICAL STUDY

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Abstract. *Fruits and horticultural products post-harvested present many lost due to inadequate transport and storage. The storage and conservation of fruits and vegetables allow planning to offer the product to the consuming market and to assist the demand for long time after the crop. Several methods are used to food conservation such as: cooling, freezing, drying thermal treatment, modified atmosphere packing; the choice of these methods depends on food properties and economical viability. In this sense, the aim of this work is to simulate cooling of ellipsoidal solids with particular reference to banana and orange. Analytical solution of the heat conduction equation using GBI method considering boundary condition of the first kind and constant thermo-physical properties is presented Results of cooling kinetics and temperature distribution inside the fruits are shown and analyzed. It was verified that the banana have cooled more fast than orange fruit presenting temperature distribution that change strongly in radial and angular directions.*

Keywords: *heat, conservation, foods, fruit, analytical*

1. INTRODUCTION

Fruits and vegetables are highly perishable and need immediate attention after harvest to prolong the shelf life. Cooling and freezing are well known preservation methods widely used in the food industry to maintain the sensorial attributes, nutritious properties and to reduce loss of visual appearance (shiny surface) due to water loss, loss texture and premature ripening of freshly harvested produce mainly fruits and vegetables. Fresh produce starts to deteriorate immediately following harvest. Respiration due to enzymatic oxidation in the growing produce continues after harvest. This process results in the consumption of sugars, starches and moisture without replenishment by the plant. Carbon dioxide and other gases along with heat are generated in the process. If the heat is not removed, the process is accelerated. Growth of molds and the loss of moisture from the produce are also accelerated by heat. Bruising of the produce further accelerates these processes, resulting in the loss of texture, firmness, colour, flavour and appearance. In addition, some nutritional value may also be lost. When these losses occur, the produce is generally considered to have lost its freshness and quality. Rapid lowering of produce temperature and then maintaining it at a constant low temperature minimizes the enzymatic and other processes that cause these losses. Pre-cooling as quickly as practical is therefore a very important requirement for maintaining optimum produce quality, especially for those types with naturally high respiration rates.

Management practices can also affect postharvest quality. Produce that has been stressed by too much or too little water, high rates of nitrogen, or mechanical injury (scrapes, bruises, abrasions) is particularly susceptible to postharvest diseases.

Temperature is the single most important factor in maintaining quality after harvest. Refrigerated storage retards the following elements of deterioration in perishable crops; aging due to ripening, softening, and textural and color changes; undesirable metabolic changes and respiratory heat production; moisture loss and the wilting that results; spoilage due to invasion by bacteria, fungi, and yeasts; undesirable growth, such as sprouting of potatoes.

In these cases, the effect of the temperature along the shelf-life of the product is quite significant.

To preserve quality and prolong the storage life of fruits and vegetables, it is essential to rapidly cool produce to optimum storage temperature. Studies have shown that precooling produce greatly increases storage life. Without precooling, many common fruits and vegetables would not be available in quantity and quality.

Thus, the products should be quickly cooled and/or frozen, reducing its respiratory rate and, consequently, increasing its shelf-life. The cooling and freezing processes combine the favorable effect of low temperatures with the conversion of water into ice. At temperatures lower than $-10\text{ }^{\circ}\text{C}$ the growth of microorganisms will be minimum, chemical reactions are also delayed (Delgado and Sun, 2001).

Other benefits resulting from pre-cooling fruits and vegetables include the following: minimized production losses; improved economics of harvest operations; minimized losses during marketing; improved utilization by consumer; expanded market opportunities;

Several cooling methods, such as, room cooling, forced air-cooling, hydro-cooling, vacuum cooling, evaporative cooling and ice cooling are used (Dincer and Genceli, 1994; Mederos, 2000; Teruel et al., 2004; Jain, 2007).

The amount of heat in produce is governed by the temperature around it. The temperature difference between newly harvested produce and its optimum storage temperature is an indicator of field-heat. Rapidly lowering the temperature of harvested produce to near storage temperature is known as precooling, or removal of field-heat.

Particularly, the problem that consists in to predict the cooling time, involves aspects related with the heat transfer, where the three basic mechanisms are usually involved: conduction, convection and radiation. In cooling systems that use forced air, the heat transfer process occurs fundamentally by convection. In this case, the air flows with a relative high velocity around of the product surface, decreasing fast the temperature of the solid. Some mathematical models to evaluate processes involving fruits refrigeration can be found in the literature. These equations are solved by several numerical methods (finite-differences, finite-volumes, etc.) or analytical methods (separation of variables, Laplace transform, GBI method, etc.), with different restrictions and boundary conditions, for cylindrical, spherical or plane geometries.

The correct mathematical formulation of the physical phenomenon is very important for the reliability of the results. The numerical and analytical solutions of the heat transfer equation, together with the experimental data of cooling kinetics of biological materials, makes it possible to analyze with a higher precision what happens with the temperature, inside the material along the process (through the analysis of the temperature distribution). The knowledge of effect of parameters such as size and initial moisture content in the cooling kinetics is indispensable, because it allows optimizing industrial cooling/freezing systems. The main variables demanded by the customers are related to the product size and to its initial moisture content. These parameters influence directly in the process time and in the final quality of the product (Montague et al., 2003).

For project efficient cooling systems, engineers and researchers should analyze the mass and heat transfer phenomena's during the cooling and/or freezing of biological products; furthermore they must have know-how of all the variables that interfere in the process, and influence the cooling- and evaporation rate. These variables are: density, thermal conductivity, specific heat, heat transfer coefficient, convective mass coefficient, dimensions and form of the sample, as well as the cooling conditions. Know-how off this information is essential to calculate the thermal load of the cooling unit and to optimize these systems, projected to minimize the damages of these materials and longer shelf-life. Less energy use, higher confidence and safety of the quality also has to be considered. In the literature there are several reports of theoretical and experimental studies to estimate some important parameters, such as cooling time, thermal diffusivity and heat transfer coefficient (McDonald and Sun, 2000; Dincer and Ginceli, 1994; Ansari et al., 1984; Ansari and Afaq, 1986; Lin et al., 1996; Marin et al., 1985; Chau and Gaffney, 1990; LeBlanc et al., 1990; Mederos, 2000; Chuntranuluck et al., 1998a-c).

In this sense, the objective of this study is to model and simulate the two-dimensional heat transfer phenomenon during the cooling of solids with ellipsoidal shape. As an application, the model was used to verify the effect of the temperature inside the banana and orange fruits.

2. MATHEMATICAL MODELLING

The transient heat conduction equation without source term is given by:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) \quad (1)$$

According to Payne et al. (1986), the analytical solution of the equation (1) is:

$$T = \sum_{n=1}^N C_n \psi_n e^{-\gamma_n t} + T_e \quad (2)$$

where γ_n is the n^{th} eigenvalue (for all positions in the solid) and C_n is a constant to be determined. For convenience it is assumed that the solid has finite dimensions.

Putting equation (2) into equation (1), by considering γ_n , C_n , ρ , c_p , T_e and k constants and ψ_n independent of the time, for each n value, after the substitution and a series of algebraic operations, we obtain:

$$\rho c_p \gamma_n \psi_n + \nabla \cdot (k \nabla \psi_n) = 0 \quad (3)$$

where

$$\psi_n = \sum_{j=1}^N d_{nj} f_j \quad (4)$$

where f_i is the one element of the set of base functions, and d_{nj} are constants to be determined. Putting equation (4) into equation (3) and using the Galerkin procedure, which consists of multiplying both sides of resultant equation by $f_i dV$ and integrating the resulting function into the volume of the solid, the following is obtained:

$$(\bar{A} + \gamma_n \bar{B}) \bar{d}_n = 0 \quad (5)$$

where \bar{A} e \bar{B} are square matrix of $N \times N$ elements. The elements of the matrix \bar{A} e \bar{B} are given by:

$$a_{ij} = \int_V f_i \nabla \cdot (k \nabla f_j) dV \quad (6a)$$

$$b_{ij} = \int_V \rho c_p f_i f_j dV \quad (6b)$$

matrix \bar{A} is symmetric if the first term on the right side of Eq. (6a) is symmetric. So, it is possible to demonstrate that

$$\int_V f_i \nabla \cdot (k \nabla f_j) dV = \int_S k f_i \left(\frac{\partial f_j}{\partial n} \right) dS - \int_V k \nabla f_i \cdot \nabla f_j dV \quad (6c)$$

When dealing with homogeneous boundary conditions of the first kind (prescribed temperature $f_j=0$) or the second kind (prescribed heat flux $\partial f_j / \partial n = 0$), the first term on the right side of the Eq. (6c) is zero while the second term is always symmetric. For homogeneous boundary conditions of the third kind (convective, $-k \partial f_j / \partial n = h f_j$), the first term on the right side of Eq. (6c) becomes (Beck et al., 1992)

$$\int_S k f_i \left(\frac{\partial f_j}{\partial n} \right) dS = \int_S h f_i f_j dS \quad (6d)$$

which is also symmetric when i and j are switched. Inasmuch as the boundary conditions for f_i and f_j are always homogeneous, matrix \bar{A} is always symmetric.

To obtain coefficients C_n in equation (2) the initial condition is used. Then, when $t = 0$, from Eq. (2) we get:

$$T_o = \sum_{n=1}^N C_n \psi_n + T_e \quad (7)$$

By multiplying both sides of equation (7) by $f_i dV$ and by integrating over the volume of the solid, we obtain

$$\int_V f_i (T_o - T_e) dV = \int_V f_i \left(\sum_{n=1}^N C_n \psi_n \right) dV \quad (8)$$

The results of Eq. (8) will be a set of N linear algebraic equations that allows determination of C_1, C_2, \dots, C_n . This completes the solution of the problem.

The average value of the variable T is given by (Whitaker, 1980):

$$\bar{T} = \frac{1}{V} \int_V T dV \quad (9)$$

where V is the volume of the solid being studied.

In this work the GBI method was used to solve the problem of heat conduction inside spheroidal solids (prolate spheroid, oblate spheroid and sphere) (Figure 1). The contour of the solid is defined by:

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad (10)$$

Since $x^2+y^2=r^2$ (solid of revolution), we have $z = b\sqrt{1-\left(\frac{r}{a}\right)^2}$ e $dV = r dr d\theta dz$. Then, the total volume of solid (half of ellipsoid) (Fig. 1), will be:

$$V = \int_0^{2\pi} \int_0^a \int_0^{b\sqrt{1-\left(\frac{r}{a}\right)^2}} r dr d\theta dz = \frac{2}{3} \pi a^2 b \quad (11)$$

and a_{ij} , b_{ij} and C_n coefficients are given by (boundary condition of the first kind):

$$a_{ij} = - \int_0^{2\pi} \int_0^a \int_0^{b\sqrt{1-\left(\frac{r}{a}\right)^2}} k \nabla f_i \cdot \nabla f_j r dz dr d\theta \quad (12a)$$

$$b_{ij} = \int_0^{2\pi} \int_0^a \int_0^{b\sqrt{1-\left(\frac{r}{a}\right)^2}} \rho c_p f_i f_j r dz dr d\theta \quad (12b)$$

$$\int_0^{2\pi} \int_0^a \int_0^{b\sqrt{1-\left(\frac{r}{a}\right)^2}} f_i (T_o - T_e) r dz dr d\theta = \int_0^{2\pi} \int_0^a \int_0^{b\sqrt{1-\left(\frac{r}{a}\right)^2}} f_i \sum_{n=1}^N C_n \Psi_n r dz dr d\theta \quad (12c)$$

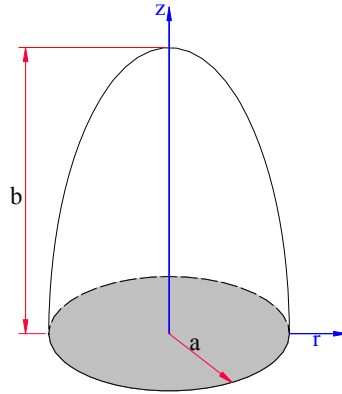


Figure 1. Ellipsoid of revolution and characteristics.

The base functions, f_j , are given by

$$f_j(r, z) = \left(1 - \frac{r^2}{a^2} - \frac{z^2}{b^2}\right) r^{p-q} z^q \quad (13)$$

where $p = 0, 2, 4, \dots, NP$ and $q = 0, 2, 4, \dots, p$. In this work ten base functions, which correspond to $NP = 6$, were used. These base functions are not orthogonal; however, according to Payne et al. (1986), functions Ψ_n are orthogonal.

Thus, the following initial and boundary conditions can be given as follows:

$$T(r, z, t = 0) = T_o; \quad \frac{\partial T}{\partial z}(r, z = 0, t) = 0; \quad \frac{\partial T}{\partial r}(r = 0, z, t) = 0; \quad T(r, z = b\sqrt{1-\left(\frac{r}{a}\right)^2}, t) = T_e \quad (14a-d)$$

As an application the study was used to predict cooling of banana (prata variety) and orange (Valencia variety) fruits. The themophysical properties of the fruits are shown in Table 1.

Table 1. Thermophysical properties used in this work.

Thermophysical properties	Banana (prata variety)	Orange	Source
Thermal conductivity (k) (W/m°C)	0.5	0.5	Mederos (2000)
Specific heat (c _p) (J/kg°C)	3660	3770	
Density (ρ) (kg/m ³)	980	948	
Minor axis of the fruit (a) (cm)	2.04	3.78	
Major axis of the fruit (b) (cm)	7.81	3.89	
Initial temperature of the fruit (T ₀) (°C)	27.6	27.6	
Cooling air temperature (T _e) (°C)	7.0	7.0	

3. RESULTS AND DISCUSSIONS

Using the model presented in this work, the behaviour of the dimensionless average and center temperature for the fruits can be traced as a function of Fourier number (Fig. 2) Analysing this figure, it can be observed that the dimensionless average temperature decreases much faster in few time processes and depends on the shape of fruits. Therefore, it can be observed that the aspect ratio of the solid has a influence direct on the cooling process. This is directly related to area/volume ratio (S/V). In a detailed analysis, it can be affirmed that the larger the area/volume ratio of a solid, the faster it will cooling when maintained under the same experimental conditions.

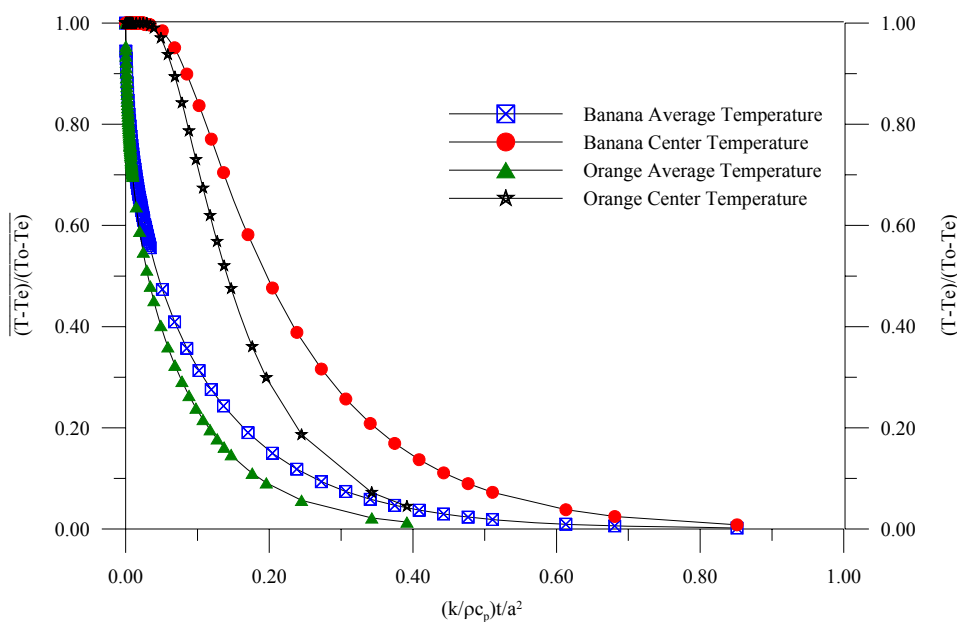


Figure 2. Dimensionless average and center temperature of banana and orange as a function of Fourier number.

It can be verified however that at the center of the spheroid, the behaviour of the curves is smoother than the behaviour of the curves of the dimensionless average temperature. It can be concluded that the center of the fruit is the place where the smallest temperature gradients are seen during the cooling process for the same aspect ratio and the same Fourier number.

Figures 3 and 4 illustrate the dimensionless temperature distribution inside banana and orange as a function of cylindrical coordinates (r, z) for many Fourier numbers. Analysing Fig. 3, it can be observed that the dimensionless temperature distribution has high temperature gradients, mainly on the z axis and in proximity to the surface of the solid. The iso-concentration lines are shown in the form of elliptical lines with the shape of a prolate spheroid. A phenomenon that occurs on the extremity of the spheroid in proximity to the coordinate z=L₂ is observed; cooling is quick in that area, generating high temperature gradients. The areas where high temperature gradients are found are the areas with the largest loss of heat. Thus, this area is more susceptible to thermo-mechanical effects, such as loss of sensorial attributes and nutritious properties. These effects jeopardise the quality of the product after cooling. Several authors also reported this type of effect, for instance Lima (1999), Carmo (2001), Oliveira (2001), Nascimento (2002),

Oliveira and Lima (2002) and Carmo (2004). It can be verified that larger temperature gradients occur for lower Fourier numbers, tending to zero at the end of the process, when the solid reaches its dimensionless equilibrium temperature.

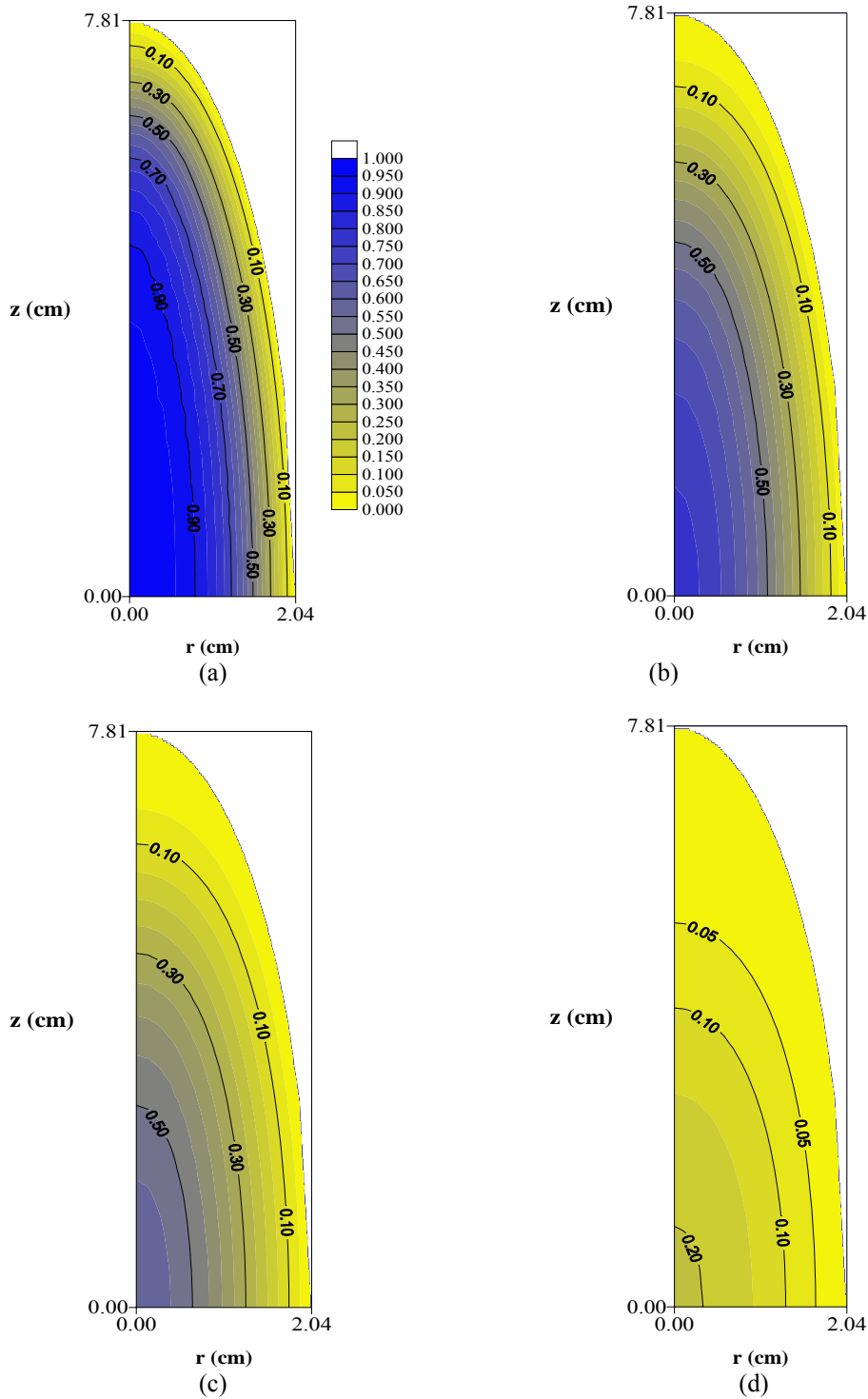


Figure 3: Distribution of the dimensionless temperature inside the banana for a) $t=150\text{ s}$ ($Fo=0.0511$), (b) $t=350\text{ s}$ ($Fo=0.1192$), (c) $t=500\text{ s}$ ($Fo=0.1703$) e (d) $t=1000\text{ s}$ ($Fo=0.3405$).

Analysing Fig. 4, it can be noted that high moisture gradients are found in the spheroid, i.e., in the area close to the surface, the spheroid is practically cool, while at the center it is very heat, as could already be seen in the Fig. 3. The iso-concentration lines are circular in the shape of the spheroid according to boundary conditions used in this work. In Fig. 4 smaller temperature gradients than in the previous case ($t=500\text{ s}$) are seen. This indicates that the heat moves from the center of the solid to the surface of the same.

By comparing Figs. 3 and 4, it can be seen, that the area/volume ratio directly influences the cooling kinetics. The higher the area/volume ratio, the faster the solid will cool. In this work area/volume values 0.786213 and 1.18729 were found for orange and banana, respectively, according to Teruel et al. (2001).

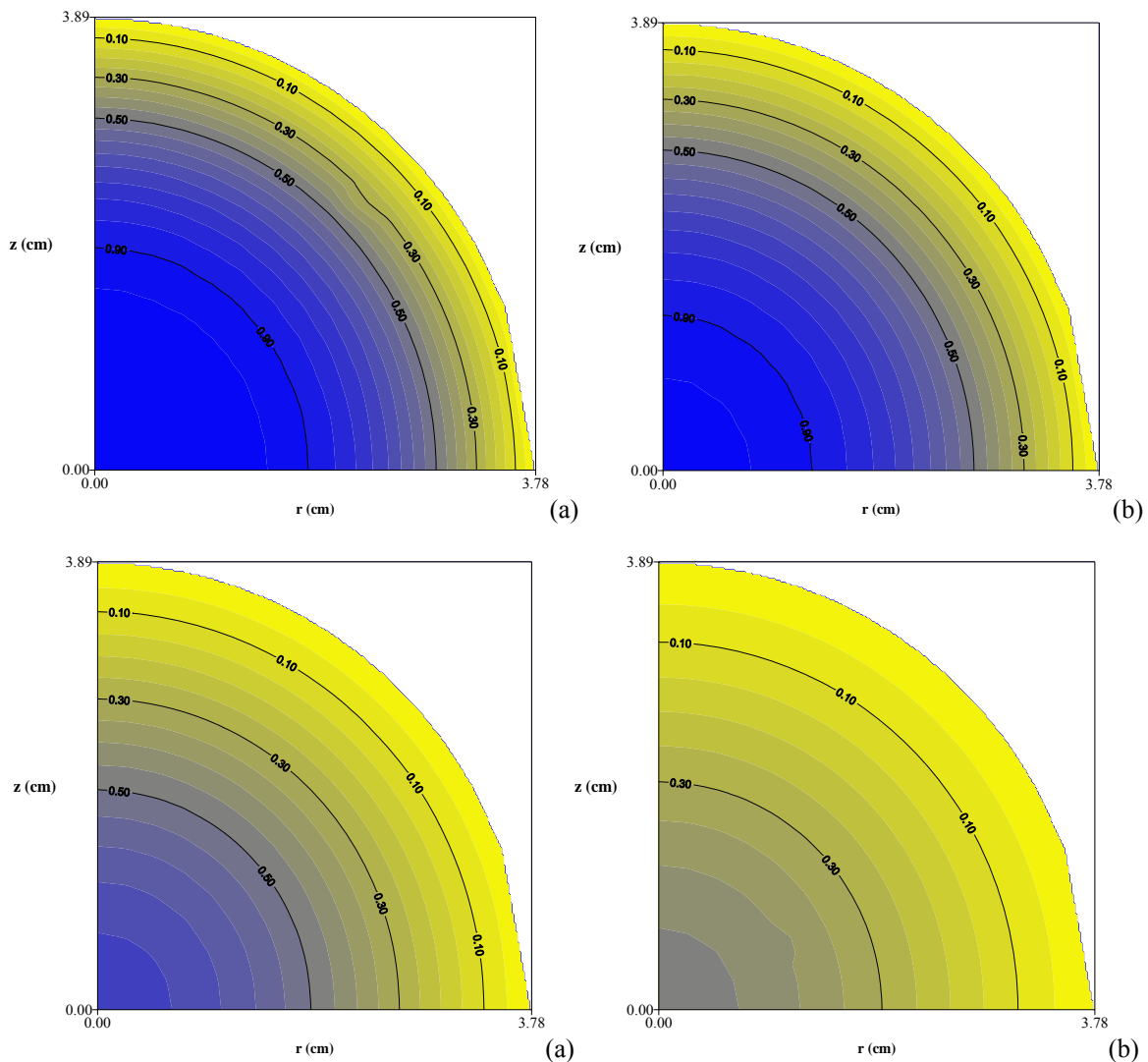


Figure 4: Distribution of the dimensionless temperature inside the orange for (a) $t = 350s$ ($Fo=0.0343$), (b) $t=500s$ ($Fo=0.0490$), (c) $t = 1000s$ ($Fo=0.0979$) e (d) $t=1500s$ ($Fo=0.1469$).

4. CONCLUSIONS

According to data obtained by simulation of the cooling process in bodies with an ellipsoidal shape, it can be concluded that the mathematical modelling and the integral method based on Galerkin used to solve the problem of heat conduction in solids that vary in shape from a circular disk to an infinite cylinder, including a sphere, was appropriate. The solution can also be used to describe other unsteady phenomena such as wetting, drying and/or heating.

It was verified that the dimensionless average temperature in a spheroid as well as the dimensionless temperature at the center and at any point inside the same, it decreases with the increase in Fourier number for any aspect ratio of the fruit. The form and aspect ratio of a fruit directly influence the cooling process, and this is directly related to the area/volume ratio, i.e., as the lower the area/volume ratio, the faster the cooling for a fixed value of Fourier number.

During the cooling process the smallest temperature gradients are found close to the center and the highest are found close to the surface, mainly for short times for any aspect ratio. The dimensionless temperature depends on radial and longitudinal coordinates. The closer to the surface of the banana and orange fruits, the lower the dimensionless temperature will be. The iso-concentration lines in the fruit tend to have the same shape as the surface of the solid. This is due to the boundary condition used in this work. Banana shown a phenomenon called the tip effect, and the area with high temperature gradients is more significant in higher aspect ratio of the spheroid.

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