APPLICATION OF INVERSE PROBLEM IN THE ANALYSIS OF MASS TRANSFER - DETERMINATION OF EFFECTIVE DIFFUSIVITY OF MASS

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Abstract. *Due to the relevant applications in the food industries, there is a growing demand for the formulation and solution of inverse mass transfer problems. In this work an inverse mass diffusion problem is solved using two optimization methods, Differential Evolution and Levenberg-Marquardt, to estimate effective diffusivity of mass of mushroom of the species Agaricus blazei at different drying temperatures. The solution of the mass diffusional equation is made with a separation of variables method to obtain the desired solution for the moisture content distribution. Before the solution of the inverse problem of parameters estimation was made a sensitivity analysis to the parameters of the model, where three case studies are investigated, changing two operating conditions: temperature and speed of air drying. The analysis shows no significant differences between reported and estimated effective diffusivity of mass, by Differential Evolution and Levenberg-Marquardt methods.*

Keywords: *Inverse Problem, Levenberg-Marquardt, Differential Evolution, Mass Transfer, Effective Diffusivity of Mass.*

1. INTRODUCTION

Simulation models of the drying processes are used for designing new or improving existing drying systems or even for the control of the drying process. All parameters (transfer coefficients, effective moisture diffusivity, etc) used by simulation models are directly related to the drying conditions, i.e. temperature and velocity of the drying medium inside the mechanical dryer. Several researches have investigated the drying kinetics of different agricultural products in order to determine the effective diffusivity of mass, namely El-Aouar et al. (2003), Azzous et al. (2002), Martins et al. (2004). The parameter estimation can be made through the Direct Methods or Inverse Methods.

The use of inverse analysis techniques represents a new research paradigm. The results obtained from numerical simulations and from experiments are not simply compared a posteriori, but a close synergism exists between experimental and theoretical researches during the course of study, in order to obtain the maximum information regarding the physical problem under consideration (Beck, 1999). Inverse approach to parameter estimation in the last few decades has become widely in various scientific disciplines, Simpson e Cortés (2004), Mendonça et al. (2005), Vasconcellos et al. (2002), Zueco et al., (2003), Anderson et al., (2006) e Mariani et al. (2007) using the inverse method to estimate the thermophysical properties of foods, and Colaço et al. (2004) e Huang et al. (2008). Many methods have been proposed to solve inverse problems, including the deterministic and stochastic methods.

Some deterministic methods are: Conjugate Gradient Method, the Newton Method, Steepest Descent Method, Gauss Method and Levenberg-Marquardt Method that are based on gradient information, to minimize the objective function. The Levenberg-Marquardt Method has been successfully implemented in several areas (Santos et al., 2002; Yang e Gao, 2007; Kanevce et al., 2005; Mendonça et al., 2005; Silva et al., 2006).

Among the stochastic methods are Simulated Annealing Method, Differential Evolution and the Particle Swarm Method (Colaço et al., 2004). The Differential Evolution algorithm was first introduced by Storn and Price (1995), and was successfully applied in the optimization of some well-known non-linear, non-differentiable and non-convex functions by Storn (1997). This method has been applied successfully in various fields of science and may be cited the work of Kanevce et al. (2003), Arantes et al. (2006), Purcina e Saramago (2007) e Mariani et al. (2007).

Deterministic methods are in general computationally faster than stochastic methods, although they can converge to a local minima or maxima, instead of the global one. On the other hand, stochastic algorithms they are ideally converge to a global maxima or minima, although they are computationally slower than the deterministic ones (Colaço et al., 2004). This paper presents a procedure to estimate effective diffusivity of mass of mushrooms in drying process, using Differential Evolution and Levenberg-Marquardt Methods as technique for obtain parameters of piecewise function through of inverse method.

2. DIRECT PROBLEM

In the direct problem of mass transfer, the application of Fick's second law allows the development of a mathematical model that reproduces the behavior assumed by some products when submitted to air drying process. In order to model the mass transfer, the following assumptions were made:

- The process occurs in transient regime and there is not mass generation inside the product;
- The resistance to moisture flow is uniformly distributed throughout the interior of the material during the process; and the volume shrinkage is negligible;
- The mass transfer is predominantly one dimensional;
- The effective diffusivity of mass is constant.

The general form, in rectangular coordinates, of the mass diffusional equation, is:

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z}\left(D_{eff} \frac{\partial X}{\partial z} \right) \quad (1)$$

where X: moisture content (g H_2O/g dry matter)
D_{eff}: effective diffusivity of mass (m^2/s)
z: space variable (m)
t: time (s)

Using the following initial and boundary conditions:

$$\frac{\partial X(0,t)}{\partial z} = 0, \text{ in } z = 0 \text{ e } \quad t > 0 \quad (2)$$

$$X(L,t) = X_{eq}, \text{ in } z = L \text{ e } \quad t = \infty \quad (3)$$

$$X(z,0) = X_0, \text{ in } 0 < z < L \quad \text{e} \quad t = 0 \quad (4)$$

In this work, the method of separation of variables was utilized to obtain the desired solution for the moisture content distribution. Therefore, the solution is written in the following way:

$$X = X_{eq} + \left(X_0 - X_{eq}\right) \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{-(2n+1)^2 \pi^2 Def \frac{t}{4L^2}} \quad (5)$$

where: X: moisture content at instant t (g H_2O/g dry matter)
X_0: initial moisture content (g H_2O /g dry matter)
X_{eq}: equilibrium moisture content (g H_2O/g dry matter)
D_{eff}: effective diffusivity of mass (m^2/s)
t: time (s)
L: characteristic length, sample half- thickness (m)

The Tab 1 shows the operational conditions used in the experiments conducted by Kurozawa (2005), and used in this article.

Table 1: Conditions and parameters used in experimental tests of dry mushrooms with a half-thickness L= 2.5×10^{-3} m (Kurozawa, 2005)

Test	**Operating Conditions**	**X_0 (g H_2O/g dry matter)**	**X_{eq} (g H_2O/g dry matter)**	**Time (h)**	**D_{eff} (m^2/s)**
1	45°C e 1.20 m/s	9.8093	0.0317	8	3.88×10^{-10}
2	75°C e 1.20 m/s	9.9688	0.0048	2.5	12.79×10^{-10}
3	45°C e 2.30 m/s	8.4584	0.0265	5	6.91×10^{-10}
4	75°C e 2,30 m/s	9.2295	0.0045	2.5	17.5×10^{-10}
5	40°C e 1.75 m/s	9.7160	0.0544	10	4.14×10^{-10}
6	80°C e 1,75 m/s	7.5560	0.0051	5	14.29×10^{-10}
7	60°C e 1.00 m/s	7.3002	0.0086	7	5.56×10^{-10}
8	60°C e 2.50 m/s	7.1137	0.0103	4	9.49×10^{-10}

The curves presented in Fig. 1, for all the considered tests, show that the moisture content existent at the beginning of the drying process is exponentially reduced until reaching the equilibrium moisture content. Such behavior demonstrates the inexistence of the period of constant drying, thus, the process of drying of the product just happened in the decreasing period of drying, being controlled for the internal diffusion of the liquid until the surface where the evaporation happens.

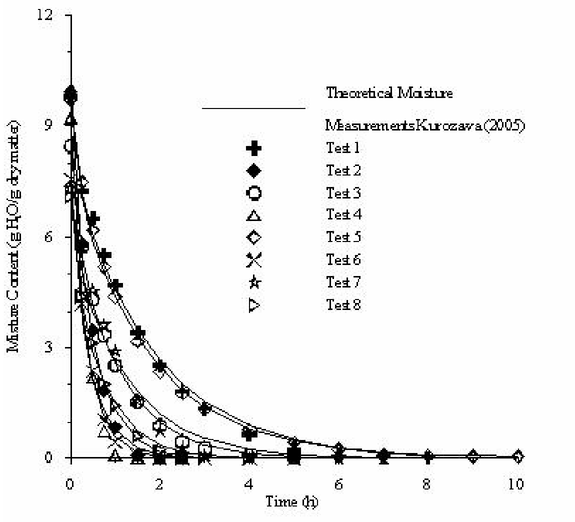

Figure 1: Modelling of drying curves using the diffusional model for fresh mushroom.

A comparative analysis of the curves presented in Fig. 1 it shows that the proposed model presents good agreement with the experimental data and generated ones for it, indicating the possibility of success in the application of the inverse analysis. Several authors (El-Aouar, 2003; Brod, 2003; Park, 2004; Lescano, 2004) had found excellent adjustment of the experimental data of drying to the diffusional model, based on the law of Fick.

3. SENSITIVITY ANALYSIS

The moisture content distribution depends on z, t and also on a number physical parameters. In particular:

$$X = F\left(z, t, X_0, X_{eq}, D_{eff}\right) \tag{6}$$

A sensitivity coefficient is the first derivative of the measured variable in relation to the unknown parameter. Let $\eta(z,t,\mu,\beta)$ be the state variable, $z_i = (z_1, z_2, ..., z_n)$ represents the spatial variables, t the time, $\mu = (\mu_1, \mu_2, ...\mu_q)$ the known parameters and $\beta = (\beta_1, \beta_2, ..., \beta_p)$ the unknown parameters. The sensitivity coefficient at point z_i, time t_n for the parameter β_j is:

$$\chi_j(z_i, t_n, \mu, \beta) = \left.\frac{\partial \eta(z,t,\mu,\beta)}{\partial \beta_j}\right)_{r_i, t_n} = \chi_j\Big)_{i,n} \tag{7}$$

The sensitivity coefficients represent the variation in the state variable due to a change in the value of an unknown parameter. In other words, they verify the influence of the parameter on the moisture content distribution. Therefore, sensitivity coefficients are, in a certain way, "the key of success" to an estimation procedure of the parameter (Beck & Arnold, 1977). Stela et al. (2005) analyzed the calculation and the use of sensitivity coefficients in problems of heat conduction, demonstrating as these supplies fundamental information on the effects of these parameters in the answers of the models.

For the comparison of the sensitivity coefficients which do not have the same units, we use Reduced Sensitivity Coefficients obtained by multiplying the original coefficients by the parameters that they are referrend. In this article the sensitivity coefficients are evaluated to equilibrium moisture content X_{eq} and effective diffusivity of mass D_{eff}.

$$\chi_{X_{eq}} = X_{eq} \frac{\partial X}{\partial X_{eq}} \tag{8}$$

$$\chi_{D_{eff}} = D_{eff} \frac{\partial X}{\partial D_{eff}} \tag{9}$$

3.1. Sensitivity to the Equilibrium Moisture Content

The Fig. 2 contains a graph where finds the temporary evolution of the reduced sensitivity coefficients to the equilibrium moisture content (X_{eq}) for the tests 1, 2 and 3. It is observed that the sensitivity coefficients develop in increasing way with the time until reaching the stability. The curves indicate that the variation of the sensitivity coefficients are more significant when the process is accomplished with smaller temperature values. Analyzing visually the evolution of the coefficients is verified that the processes accomplished with low values of drying temperature supply equilibrium moisture content larger, and the equilibrium condition is reached in bigger interval of time.

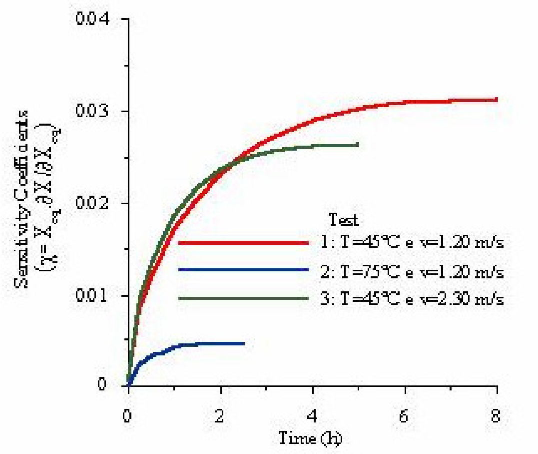

Figure 2: Sensitivity coefficient for X_{eq}, in agreement with each analyzed test.

3.2. Sensitivity to Effective Diffusivity of Mass

The temporary evolution of the sensitivity coefficients to the effective diffusivity of mass, D_{eff}, with the time can be verified in Fig. 3. It is observed that initially the coefficients have a decreasing and negative evolution. Considering the values in absolute terms, the behavior of the curves indicates that the sensitivity coefficient to the effective diffusivity of mass, D_{eff}, reaches its maximum value when the experience is accomplished for larger temperatures of the drying air. This indicates that exists a tendency that the estimate of D_{eff} is better determined if the process to occur in higher temperatures.

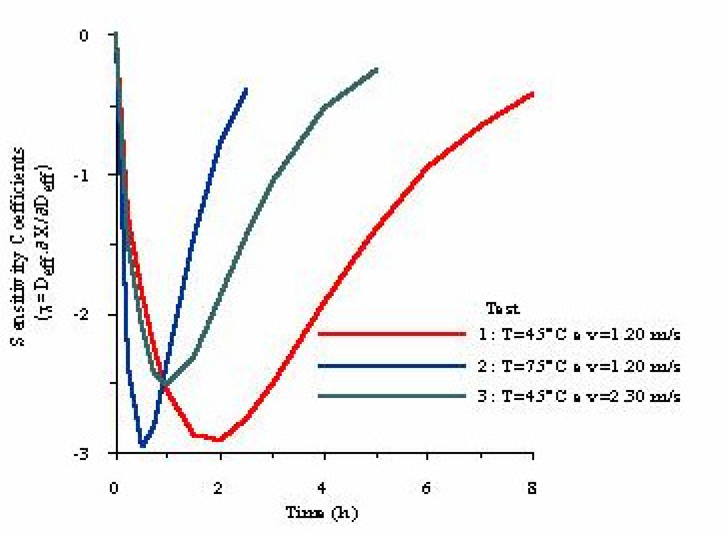

Figure 3: Sensitivity coefficient for D_{eff}, in agreement with each analyzed test.

3.3. Susceptibility of the Model Parameters to the Identification

The Fig. 4 is relative to the tests 2 and present a comparison accomplished among the reduced sensitivity coefficients of the equilibrium moisture content to the parameters equilibrium moisture content, X_{eq}, and effective diffusivity of mass, D_{eff}, with the objective of study the linear dependence between them.

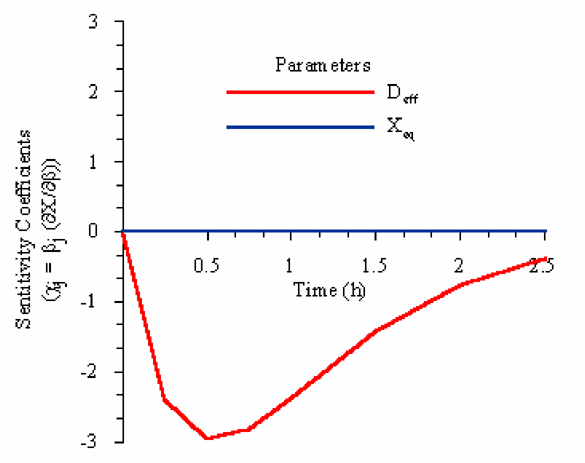

Figure 4: Sensitivity coefficients for X_{eq} and D_{eff}, for second test

It can be verified through a visual analysis of the evolution of those sensitivity coefficients that:

- ✓ The parameters are linearly independent; in other words, the sensitivity coefficients vary in a different manner.
- ✓ The sensitivity to the equilibrium moisture content, X_{eq}, is practically null, during the whole period of the drying. Therefore, it is impossible to estimate this parameter, once the same possesses small sensitivity and variations in this parameter will imply undistinguishable changes to the theoretical model.
- ✓ For effective diffusivity of mass, D_{eff}, considering the values in absolute terms, is observed that the coefficient reaches a maximum value and that value is reduced when the end of the drying is approaching. The sensitivity of this model is high sufficiently, showing that the same have great influence in the profile of the moisture content. In term, of parameter estimation, it indicates that to small changes in the value of this parameter it will affect the model, or in other words, the information in it contained are important.
- ✓ The curves of the reduced sensitivity coefficients still show which the best interval of time to accomplish the estimate of the effective diffusivity of mass, D_{eff}. It is verified that the biggest value in absolute terms is reached with approximately 0.5 h of drying.

Finally, the done observations show that is not possible to estimation the two parameters simultaneously from an only experience.

4. Inverse Problem

For the inverse problem of interest here, the effective moisture diffusivity is regarded as unknown parameter. The choice for this parameter is associated to the analysis of accomplished sensitivity.

The estimation methodology used is based on the minimization of the ordinary least square norm, written as:

$$S(\beta) = [Y - X(\beta)]^T [Y - X(\beta)] \tag{10}$$

where Y is the vector of measured moisture and $X(\beta)$ is the vector of calculated moisture obtained from the solution of the direct problem , $\beta = \beta_1, \beta_2, ..., \beta_p$ is the vector of unknown parameters p. Therefore:

$$[Y - X(\beta)]^T = \left[\left(\vec{Y}_1 - \vec{X}_1\right)\left(\vec{Y}_2 - \vec{X}_2\right)\cdots\left(\vec{Y}_i - \vec{X}_i\right)\right] \tag{11}$$

The Differential Evolution and Levenberg-Marquardt methods were considered for the minimization of the ordinary least square norm, given by Eq. (10).

4.1. Differential Evolution

Differential Evolution (DE) is a population-based stochastic function minimizer (or maximizer) relating to evolutionary computation, whose simple yet powerful and straightforward features make it very attractive for numerical optimization. DE combines simple arithmetical operators with the classical operators of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution.

The different variants of DE are classified using the following notation: DE/ α/ β/ δ, where α indicates the method for selecting the parent chromosome that will form the base of the mutated vector, β indicates the number of difference vectors used to perturb the base chromosome, and δ indicates the recombination mechanism used to create the offspring population. The *bin* acronym indicates that the recombination is controlled by a series of independent binomial experiments.
The variant implemented in this article was the DE/*rand*/1/*bin*, which involved the following steps and procedures:

Step 1: *Parameter setup*
The user chooses the parameters of population size, the boundary constraints of optimization variables, the mutation factor (f_m), the crossover rate (*CR*), and the stopping criterion of maximum number of iterations (generations), K_{max}.
Step 2: *Initialization of an individual population*
Set generation $k = 0$. Initialize a population of $i = 1, .., N$ individuals (real-valued *n*-dimensional solution vectors) with random values generated according to a uniform probability distribution in the *n* dimensional problem space. These initial individual values are chosen at random from within user-defined bounds (boundary constraints).
Step 3: *Evaluation of the individual population*
Evaluate the fitness value of each individual.
Step 4: *Mutation operation (or differential operation)*
Mutation is an operation that adds a vector differential to a population vector of individuals according to the following equation:

$$z_i(k+1) = x_{i,r_1}(k) + f_m \cdot [x_{i,r_2}(k) - x_{i,r_3}(k)] \tag{12}$$

where $i = 1, 2, ..., N$ is the individual's index of population; k is the generation; $x_i(k) = [x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k)]^T$ stands for the position of the *i*-th individual of population of *N* real-valued *n*-dimensional vectors; $z_i(k) = [z_{i1}(k), z_{i2}(k), \ldots, z_{in}(k)]^T$ stands for the position of the *i*-th individual of a *mutant vector*; r_1, r_2 and r_3 are mutually different integers and also different from the running index, *i*, randomly selected with uniform distribution from the set $\{1, 2, \ldots i-1, i+1, \ldots N\}$; $f_m > 0$ is a real parameter called *mutation factor*, which controls the amplification of the difference between two individuals so as to avoid search stagnation and is usually taken from the range [0.1, 1].
Step 5: *Recombination operation*
Following the mutation operation, recombination is applied to the population. Recombination is employed to generate a trial vector by replacing certain parameters of the target vector with the corresponding parameters of a randomly generated donor vector.
For each vector, $z_i(k+1)$, an index $rnbr(\ ) \in \{1,2,...n\}$ is randomly chosen using uniform distribution, and a *trial vector*, $u_i(k+1) = [u_{i1}(k+1), u_{i2}(k+1), \ldots, u_{in}(k+1)]^T$, is generated with

$$u_{ij}(k+1) = \begin{cases} z_{ij}(k+1), \text{ se } randb(j) \leq CR \text{ ou } j = rnbr(i), \\ x_{ij}(k), \text{ se } randb(j) > CR \text{ ou } j \neq rnbr(i), \end{cases} \tag{13}$$

In the above equations, *randb(j)* is the *j*-th evaluation of a uniform random number generation with [0, 1] and *CR* is a *crossover* or *recombination rate* in the range [0, 1]. The performance of a DE algorithm usually depends on three variables: the population size *N*, the mutation factor f_m, and the recombination rate *CR*.
Step 6: *Selection operation*
Selection is the procedure of producing better offspring. To decide whether or not the vector $u_i(k+1)$ should be a member of the population comprising the next generation, it is compared with the corresponding vector $x_i(k)$. Thus, if f denotes the objective function under minimization, then

$$x_i(k+1) = \begin{cases} u_i(k+1), \text{if } f(u(k+1)) \leq f(x_i(k)), \\ x_i(k), \text{otherwise} \end{cases} \tag{14}$$

In this case, the cost of each trial vector $u_i(k+1)$ is compared with that of its parent target vector $x_i(k)$. If the cost, f, of the target vector $x_i(k)$ is lower than that of the trial vector, the target is allowed to advance to the next generation. Otherwise, the target vector is replaced by trial vector in the next generation.
Step 7: *Verification of stop criterion*
Set the generation number for $k = k + 1$. Proceed to Step 3 until a stopping criterion is met, usually K_{max}. The stopping criterion depends on the type of problem.

4.2. LevenbergMarquardt

The Method of Levenberg-Marquardt (LM) (Press et al., 1990) inserts a restriction to the minimization criterion, to get over the instability of the Gauss Method. Based in the criterion of the ordinary least squares, the iterative formula has the following expression:

$$\beta^{(k+1)} = \beta^{(k)} + \left[J^{T(k)} J^{(k)} + \lambda^{(k)} \Omega_m{}^{(k)} \right]^{-1} J^{T(k)} \left(Y - X^{(k)}(\beta) \right) \tag{15}$$

where $\lambda^{(k)}$ is a positive scalar named damping parameter, $\Omega_m{}^{(k)}$ is a diagonal matrix and $J^{(k)}$ is the sensitivity coefficients matrix.

The purpose oh the matrix term $\lambda^k \Omega_m{}^{(k)}$ at Eq.(15) is to damp oscillations and instabilities due to the ill-conditioned character of the problem. With such an approach, the matrix $J^T J$ is not required to be non-singular in the beginning of iterations and the Levenberg-Marquardt Method tends to the Steepest Descent Method. The parameter $\lambda^{(k)}$ is then gradually reduced as the iteration procedure advances to the solution of the parameter estimation problem, and then the Levenberg-Marquardt Method tends to the Gauss Method (Meijas et al, 1999).

The iterative procedure starts with an initial guess, $\beta^{(0)}$, and at each step the vector β is modified until:

$$\frac{\left| \beta_i^{(k+1)} - \beta_i^{(k)} \right|}{\left| \beta_i^{(k)} \right| + \xi} < \delta \text{, for i= 1, 2,} \tag{16}$$

where is δ a small number that must be chosen by the investigator (typically 10^{-3}) and ξ ($<10^{-10}$) prevents overflow if $\beta_i^{(k)} = 0$

Choose a value for $\lambda^{(k)}$, $\lambda^{(0)} = 0,001$. Then:

Step 1: solve the direct moisture content transfer problem give by Eq.(5) with the available estimate $\beta^{(k)}$.
Step 2: compute $S(\beta^k)$ from Eq. (10).
Step 3: compute the sensitivity matrix $J^{(k)}$ and then the matrix $\Omega_m{}^{(k)}$.
Step 4: compute the increments for the unknown parameters by using Eq.(16).
Step 5: compute the new estimate β^{k+1} as $\beta^{k+1} = \beta^k + \Delta\beta^k$.
Step 6: solve now the direct problem Eq. (5) with the new estimate β^{k+1} in order to find $X(\beta^{k+1})$. Then compute $S(\beta^{k+1})$, as defined by Eq. (10).
Step 7: if $S(\beta^{k+1}) \geq S(\beta^k)$, replace $\lambda^{(k)}$ by 10 $\lambda^{(k)}$ and return to step 4.
Step 8:if $S(\beta^{k+1}) < S(\beta^k)$, accept the new estimate β^{k+1} and replace $\lambda^{(k)}$ by 0,1 $\lambda^{(k)}$.
Step 9: check the stopping criteria by Eq. (16).

5. NUMERICAL SIMULATION

The numerical example proposed in this section illustrates the parametric sensitivity analysis as presented in Niliot and Lefèvre (2004).In order to compare the parameter estimation approach results to the experimental results given by Kurozawa (2005), we propose some numerical experiments in the same conditions.

Measurements contained in vector Y are generally not exact. The measurement errors produce on the estimated vector β, amplified by the ill-posed character of the inverse problem. In order to simulate measurements errors (ε_i), an additional Gaussian error of zero mean value and standard deviation σ = 0.05 is added to the moisture content field obtained from direct calculation. Then:

$$\tilde{Y} = \eta_i(\beta) + \varepsilon_i$$

An example without errors, of the numerical and experimental moisture content field is presented in Fig. 5 versus time, for second test. For these results, the maximum values of the residuals were found with the same period of drying for the two methods, being of 0.6947 and 0.7560 gH_2O/g dry matter through Levenberg-Marquardt and Differential Evolution, respectively. The minimum values occurred at different intervals of time, where -0.4382 gH_2O/g dry matter was reached with 1h by Levenberg-Marquardt method and -0.3812 gH_2O/g dry matter happened after 1.5 h of drying using the Differential Evolution.

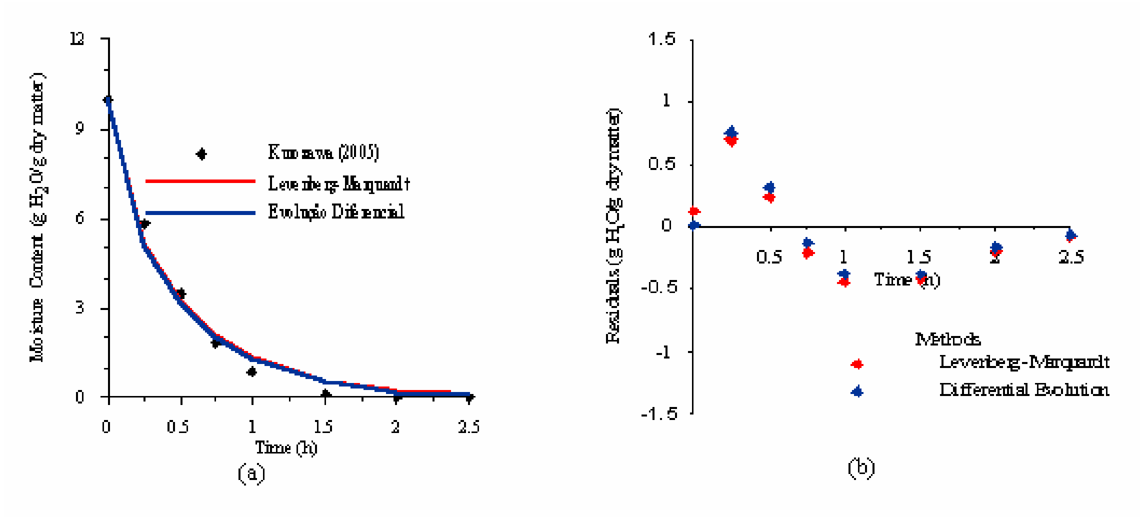

Figure 5: Moisture content (a) and corresponding residuals (b) without errors, for second test

The numerical and experimental moisture content field, selected randomly of the 30 estimations, with Gaussian distribution (with zero mean and standard deviation 0.05) is presented in Fig. 6 versus time, for second test. As it could be predicted, the results are much more stable using good quality measurements (Fig. 5a) than with poor quality measurements (Fig 6 a, b).

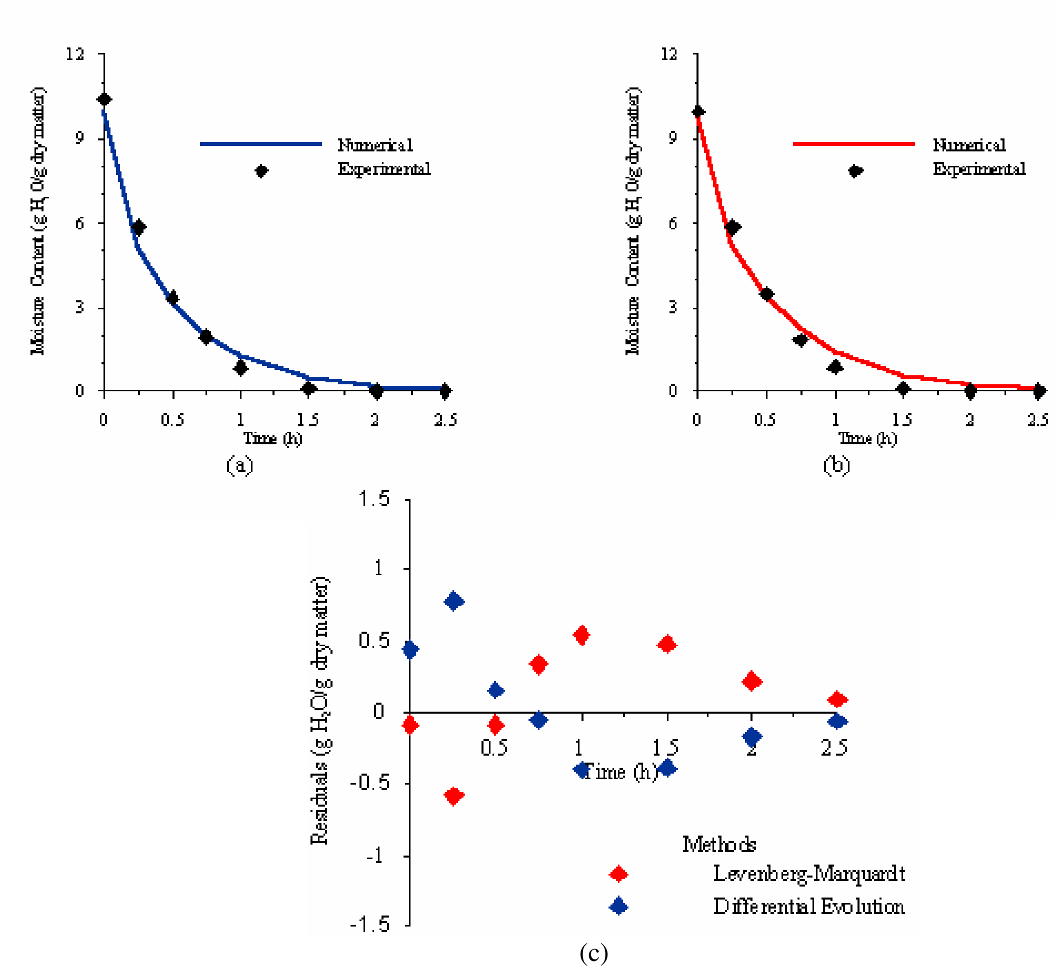

Figure 6: Moisture content and corresponding measurement error for one estimation using σ = 0.05, results for second test: (a) Differential Evolution, (b) Levenberg-Marquardt and (c) Residuals.

Note that the obtained values of the residuals reaches its maxima and minima opposed times, however, it verified that low values, centered around the zero and with a relatively random distribution were found. The results show that it is possible to estimate the variable D_{eff} using both optimization methods, with a good precision.

In Tab 2 the mean values and the standard deviation for D_{eff} and objective function are given. These results show that the D_{eff} obtained for the second, fourth, sixth and eight tests presents the standard deviation bigger than the other tests. This result confirms the sensitivity analysis presented in Fig. 3, where the sensitivity to the effective diffusivity of mass increases when the process to occur in higher temperatures.

Table 2: Results for 30 estimation on a virtual experiment obtained with σ = 0.05.

Test	Levenberg-Marquardt				Differential Evolution			
	$\overline{f}$	σ_f	$\overline{D_{eff}}$	σ_{Deff}	$\overline{f}$	σ_f	$\overline{D_{eff}}$	σ_{Deff}
1	0.18871	0.01711	3.3272×10^{-10}	2.9176×10^{-12}	0.04375	0.00848	4.0745×10^{-10}	5.1655×10^{-12}
2	0.03176	0.00611	12.325×10^{-10}	2.641×10^{-11}	0.13687	0.02224	13.012×10^{-10}	2.0597×10^{-11}
3	0.16264	0.01363	5.8765×10^{-10}	5.4957×10^{-12}	0.06184	0.00600	7.1172×10^{-10}	1.0177×10^{-18}
4	0.04004	0.03744	16.867×10^{-10}	3.6036×10^{-11}	0.12079	0.02027	17.517×10^{-10}	2.8054×10^{-11}
5	0.37935	0.07715	3.1577×10^{-10}	5.026×10^{-12}	0.00308	0.00728	4.1592×10^{-10}	5.4181×10^{-12}
6	0.07087	0.00590	13.599×10^{-10}	3.6613×10^{-11}	0.05303	0.00888	14.377×10^{-10}	2.3083×10^{-11}
7	0.10547	0.00342	5.1719×10^{-10}	2.1441×10^{-12}	0.06410	0.00604	5.9666×10^{-10}	8.1922×10^{-12}
8	0.09789	0.00399	8.69×10^{-10}	4.3999×10^{-11}	0.04931	0.01248	9.5536×10^{-10}	1.5118×10^{-11}

Some results of the Tab 2 are very similar to the experimental results without noise given in Tab 1 showing that optimization methods, Differential Evolution and Levenberg-Marquardt, obtain adequately the solution until with noise in experimental results.

6. CONCLUSIONS

The inverse problem of the estimation of effective diffusivity of mass has been solved using two methods, Differential Evolution and Levenberg-Marquardt. The solution of the direct problem was obtained through the use of the method of separation of variables. Sensitivity analysis is a powerful tool for understanding the physical behavior of the problem and to determine what parameters can be estimated in a single experiment. The residuals obtained showed low values, centered around the zero and with a relatively random distribution. The results shows no significant differences between the effective diffusivity of mass found by Differential Evolution and Levenberg-Marquardt methods and experimental. The determination of thermophysical properties from an inverse method is an attractive technique both from the experimental and methodological point of view, because of its accuracy and short time for parameters estimation.

7. RESPONSIBILITY NOTICE

The authors Cristiane, Viviana and Zaqueu are the only responsible for the printed material included in this paper.

6. REFERENCES

Anderson, B. A. and Singh, R. P., 2006, "Effective Heat Transfer Coefficient Measurement During Air Impingement Thawing Using an Inverse Method", International Journal of Refrigeration, Vol. 29, pp. 281–293.

Arantes, M. B., Oliveira, G. T. S. e Saramago, S. F. P, 2006, "Evolução Diferencial Aplicada à Solução de Alguns Problemas de Engenharia de Produção", FAMAT em Revista, N. 6, pp. 48-61.

Azzous, S., Guizani, A., Jomaa, W., et al., 2002, "Moisture Diffusivity and Drying Kinetic Equation of Convective Drying of Grapes", Journal of Food Engineering, Vol. 55, pp. 323-330.

Beck, J. V., Arnold, K. J., 1977, "Parameter Estimation in Engineering and Science", New York: John Wiley & Sons.

Beck, J. V., 1999, "Sequential Methods in Parameter Estimation" 3 rd International Conference on Inverse Problems in Engineering, Tutorial Session, Port Ludlow, WA.

Brod, F. P. R.; Park, K. J. e Oliveira, de R. A., 2003, "Modelos Matemáticos para Representar a Secagem de Raiz de Chicória", Engenharia Agrícola, Jaboticabal, Vol. 23, No. 2, p p. 354-363.

Colaço, M. J. and Orlande, H. R. B., 2004, "Inverse Natural Convection Problem of Simultaneous Estimation of Two Boundary Heat Fluxes in Irregular Cavities", International Journal of Heat and Mass Transfer, Vol. 47, pp. 1201-1215.

El-Aouar, A. A., Azoubel, P. M. e Murr, F. E. X., 2003, "Drying Kinetics of Fresh and Osmotically Pré-Treated Papaya", Journal of Food Engineering, Vol. 59, pp. 85-91.

Estimate the Thermal Conductivity and Volumetric Thermal Capacity", Journal of Food Engineering, Vol. 67, pp. 361-366.
Huang, Cheng-Hung, Li, Jia-Xun and Kim, S., 2008, "An Inverse Problem in Estimating the Strength of Contaminant Source for Groundwater Systems", Applied Mathematical Modelling, Vol. 32, pp. 417-431.
Kanevce, G. H., Kanevce, L. P., Dulikravich, G. S. and Colaço, M. J., 2003, "An Inverse Method for Drying at High Mass Transfer Biot Number" Proceedings of HT03 ASME Summer Heat Transfer Conference Las Vegas, Nevada, USA, July 21-23.
Kanevce, G. H., Kanevce, L.P., Mitrevski, V. B., Dulikravich, G. S. and Orlande, H.R.B., 2005, "Inverse Approaches to Drying of Bodies with Significant Shrinkage Effects", Proceedings of the 5th International Conference on Inverse Problems in Engineering: Theory and Practice, Cambridge, UK, 11-15th July.
Kurozawa, L. E., 2005, *Efeito das Condições de Processo na Cinética de Secagem de Cogumelo (Agaricus blazei).* Dissertação de M.Sc., FEA/UNICAMP, Campinas, SP, Brasil.
Lescano, C. A; Tobinaga, 2004, "Modelo Codificado e Real para a Difusividade Efetiva da Secagem do Resíduo do Extrato Hidrossolúvel de Soja", Revista Brasileira de Produtos Agro-industriais, Campina Grande, Vol. 6, No. 1, p p. 17-25.
Mariani, V. C., de Lima, A. G. B. and Coelho, L. S., 2008, "Apparent thermal diffusivity estimation of the banana during drying using inverse method", Journal of Food Engineering*,* Vol. 85, pp. 569-579.
Martins, P. C., Porto, P. S. S. e Pinto, L. A. A., 2004, "Estudo das Propriedades Físicas e de Transporte na Secagem de Cebola (*Allium cepa L.*) em Camada Delgada", Ciência e Tecnologia de Alimentos, Campinas, Vol. 24, N. 3, pp. 319-326.
Mejias, M.M., Orlande, H.R.B. and Ozisik, M.N., 1999, "A Comparison of Different Estimation Techniques for the Identification of Thermal Conductivity Components of Orthotropic Solids", 3rd International Conference on Inverse Problems in Engineering, Port Ludlow, USA, pp.1-7.
Mendonça, S. L. R., Celso Filho, R. B. and da Silva, Z. E., 2005, "Transient Conduction in Spherical Fruits: Method to Estimate the Thermal Conductivity and Volumetric Thermal Capacity", Journal of Food Engineering, Vol. 67, pp. 261-266.
Park, K.J.; Tuboni, C. T; De Oliveira, R. A. e Park, K. J. B., 2004, "Estudo da Secagem de Caqui Giombo Com Encolhimento e Sem Encolhimento", Revista Brasileira de Produtos Agroindustriais, Campina Grande, Vol. 6, No. 1, p p. 71-86.
Press, W. H., Teikolsky, S. A., Vetterling, W. T. and Flannery, B. P. ,1990, Numerical Recipes in Fortran, The Art of Scientific Computing, New York, Cambridge University Press, p. 963.
Pijuan, M. N. B., Brod, F. P. R. e Park, K. J., 1997, "Estudio de Secado de Champiñón em Conserva (*Agaricus bisporus*) Utilizando um Secador Vertical", Alimentaria, N. 286, pp. 119-122.
Purcina, L. A. e Saramago, S. F. P., 2007, "Comparação entre Métodos Iterativos Não-Estacionários e Métodos Heurísticos Aplicados à Solução de Grandes Sistemas Lineares", Anais do 8º Congresso Iberoamericano de Engenharia Mecanica Cusco.
Santos, F. R., Preira, F. R. e Carvalho, G. et al., 2002, "A abordagem de Problemas Inversos para a Caracterização Térmica de Materiais". 14. Aug. 2006,< http://www.sbmac.org.br/bol/boletim>
Silva, C. K. F., Silva, Z. E., Andrade, M. P. et al., 2006, "Problema Inverso de Difusão de Massa – Estimação da Difusividade Mássica do Tomate Cereja (*Lycopersicon esculentumvar. cerasiforme*)", Proceedings of the 11th Brazilian Congress of Thermal Sciences and Engineering – ENCIT, Cutitiba, Brazil, paper CIT06-0620.
Simpson, R. and Cortés, C., 2004, "An Inverse Method to Estimate Thermophysical Properties of Foods at Freezing Temperatures : apparent volumetric specific heat", Journal of Food Engineering, Vol. 64, pp. 89-96.
Storn, R., 1997, "Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces". Journal of Global Optimization, 11(4), 341-359.
Storn, R. e Price, K., 1995, "Differential Evolution: A Simple and Efficient Adaptive Scheme for Global Optimization Over Continuous Spaces, Technical Report TR-95-012, International Computer Science Institute, Berkeley, CA, USA.
Vasconcellos, J. F. V., Silva Neto, A. J. e Santana C. C., 2002, "Estimativa do Coeficiente de Difusão e da Isoterma de Adsorção em Processo de Separação de Proteínas". 14. Aug. 2006, http://www.sbmac.org.br/bol/boletim.
Yang, C. e Gao, S., 2007, "An Inverse Analysis to Estimate Thermal Conductivity Components of an Orthotropic Medium. Third International Conference on Natural Computation.
Zueco, J., Alhama, F. and Fernández C.F. G., 2003, "An Inverse Problem to Estimate Temperature Dependent Heat Capacity Under Convection Processes", Heat and Mass Transfer, Vol. 39, pp. 599–604.