RADIATIVE TRANSFER IN A MEDIUM WITH REFRACTIVE INDEX AND SPECULAR REFLECTION AT THE BOUNDARIES

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Abstract. *An analysis of the Fresnel reflection interfaces has been performed at a slab containing an anisotropic medium and a refractive index ratio. The incident radiation is normal onto surface medium. This analysis represents a coating on an opaque substrate. The substrate can reflected the radiation speculary or diffusilly. Numerical results are used to illustrate the hemispherical reflectance and analysis the influence of radiative properties. The radiative properties are the optical thickness, the albedo, the refractive index and the parameters of the phase function.*

Keywords*: radiative transfer, semitransparent media, coatings, discrete ordinate method*

1. INTRODUCTION

In recent years, various works had had been carried on radiation heat transfer to diverse applications such as: combustion engines, boilers, furnaces, rocket engines and many other examples that involve high temperatures.

Many of this works are carry on the Radiative Transfer Equation (RTE) solutions to the high porosity medium with the radiative index close to unity and, consequently, its interface is not considerate. However, many usual materials like coatings, glasses and thin films present refractive index higher than unity, n>1.

Conventional paints transmit or absorb most of the incident infrared radiation. A coating can be optimized to produce interesting effects in point of the thermal performances. Respecting the color of the coating on the visible range the scatter on the infra-red radiation can be changed in order to improve the reflection. This effect can be applied to automobiles, oil and gas tanks and building coatings. In some countries, minimum building separation distances must be respected by consideration of the fire radiant flux (Berdahl, 1995).

The radiative behavior to the medium n>1 needs especial interface conditions must be used to take in account the refractive index (Wu *et al*., 1994, Liou and Wu, 1996 and Hottel *et al*., 1968). Furthermore, an especial discretization to use the Discrete Ordinate Method (DOM) must be employed to consider the changes of the radiation direction; even so the radiation beam has a normal incidence.

Hottel *et al.* (1968) analyzed the effects of Fresnel reflection at the interface of a slab containing anisotropic scatter medium using the DOM. In 1970, Hottel *et al.* compared the measurements of monodisperse polystyrene spheres confined between two parallel glass slides with the values predicted from Mie theory. Orel et al. (1990) measured the solar absorptance by FTIR spectroscopy of paint coatings for solar collectors. Wu *al.* (1994) analyzed a radiative transfer problem in an isotropic media with no unit refractive index and Fresnel Boundaries. They used a set of DOM quadratures to treat the strongly angular dependence of the radiative intensity around the critical angles.

Oppenheim and Feiner (1995) investigated the polarized IR reflectivity of painted and rough surfaces. Values of the bidirectional reflectance function are measured to sandblasted aluminum, concrete, painted metal and asphalt surfaces. In the same year, Berdahl presented a study on the pigments to reflect the infrared radiation form fire. He used the Mie theory to analyze the radiative transfer by the particles of titanium dioxide, iron oxide, chromium oxide, and silicon, with particle diameters in the 1 to 2 μm.

Shah and Adroja, 1996, measured the diffuse reflectance of titanium dioxide pigment dispersions in visible region. They used the Mie theory to predict the reflectance. Theoretical and experimental values of the reflectance are compared with reasonably agree.

Liou and Wu, 1996, analyzed the radiative transfer in a two-layer scattering medium with Fresnel conditions. They used the DOM with composite quadratures to take in account the refractive index differences. They results presented the effects of albedos and refractive indices.

More recently, Abdallah and Le Dez (2000) analyzed by the ray-tracing method the intensity of radiation emitted from a non scatter semitransparent plate with a refractive index varying with position. Lemonnier and Le Dez (2002) analyzed the same problem now using the DOM getting good agreement. In this case the directions of radiation propagation in the medium vary greatly due to variable refractive index. Lacroix *et al.* (2002) analyzed a similar problem, but now consider the conduction heat transfer coupling. Garcia *et al*. (2008) studied the Fresnel boundary and interface conditions for multilayered media. They use an "analytical" discrete-ordinates method - ADO method – presented by Barichello and Siewert (1999).

In this work it is analyzed an anisotropic medium with Fresnel reflection interfaces and different refractive index ratios. The incident radiation is normal onto surface medium. This analysis represents a coating on an opaque substrate. The substrate can reflect the radiation speculary or diffusely. Numerical results are used to illustrate the hemispherical reflectance and analysis the influence of radiative properties. The radiative properties are the optical thickness, the albedo, the refractive index and the parameters of the phase function. Forthcoming, this model will be used to identify the radiative properties.

2. RADIATIVE TRANSFER EQUATION

The Radiative Transfer Equation (RTE), which describes the variation of the spectral radiation intensity, *I*, (in a solid angle Ω, function of optical depth τ) in an absorbing-emitting-scattering medium, can be written as:

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = (1-\omega) I_o(T) + \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')\,\mathrm{p}(\mu',\mu)\,\mathrm{d}\mu' \tag{1}$$

where ω is the albedo, p is the phase function, and I_o is Planck's blackbody function (in order to simplify the notations, the spectral subscript λ is not considered in the text). These properties are those of a pseudo-continuum medium equivalent, in terms of radiative transport, to the real dispersed material. The boundary conditions assumed a normally incident collimated beam onto the coating/substrate that can be have specular or diffuse reflection. The boundary conditions can be expressed like:

$$\begin{cases} \mathrm{I}(\tau = 0,\mu) = \mathrm{I}_C\,; & \mu_o < \mu < 1 \\ \mathrm{I}(\tau = 0,\mu) = 0\,; & 0 < \mu < \mu_o \\ \mathrm{I}(\tau = \tau_o,\mu) = 2\rho_1 \int_0^1 I(\tau_o,\mu)\mu'\,\mathrm{d}\mu' + 2\rho'_1\, I(\tau_o,\mu) & -1 < \mu < 0 \end{cases} \tag{2}$$

where I_C is the radiative intensity of the incident beam with a divergence angle, θ_o ($\mu_o = \cos\theta_o$), ρ_1 and ρ'_1 is the diffuse and specular reflectivity on the substrate, respectively. The reflection on the coating surface is not taking in account on this analysis.

Equation (2) represents the physical conditions show in Fig. 1.

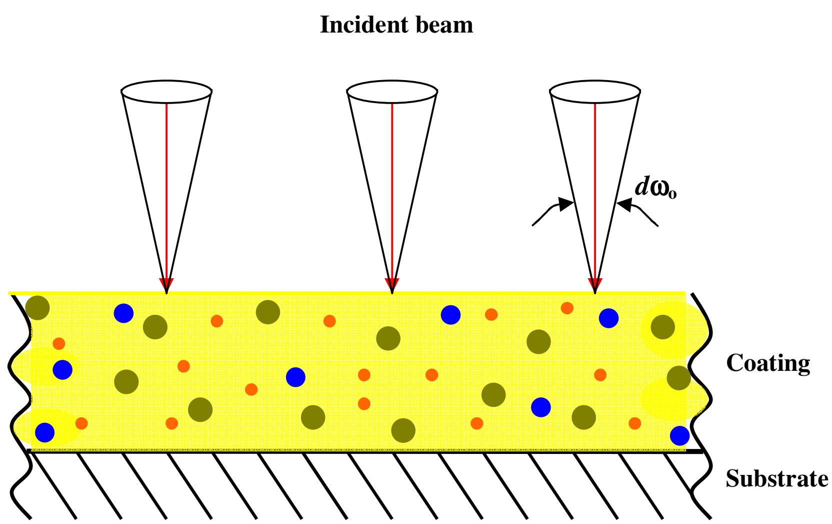

Figure 1. Physical conditions.

The RTE in a scattering medium has been studied, analytically and numerically in astrophysics, atmospheric, heat transfer and, more recently, in medical applications. However, analytical solutions are not always possible, and the numerical solutions must be employed. Some assumptions can be adopted to facilitate the solution of these problems, for example, homogeneous media, isotropic scattering, one-dimensional geometry, constant radiative properties, etc.

Numerical methods such as the DOM (Chandrasekhar, 1960) have been used on radiative transfer problems when the analytical solutions are not available.

2.1. Discrete Ordinate Method

The DOM was initially used by Schuster (1905) and Schwarzschild (1906) for studying radiative transfer in stellar atmospheres (Siegel and Howell, 2002), and after, Chandrasekhar (1960) extended the formulation to astrophysics problems. Carlson and Lathrop (1968) had developed a solution to the neutrons transport equation.

The majority of works use the RTE formularization presented by Chandrasekhar (1968) and Özisik (1973). These techniques of solution of the ETR can be found in Moura *et al*. (1997 and 1998). The RTE solution by DOM is constituted of two stages: i) an angular discretization, where the integral term of RTE is substituted by a radiative intensities weighted sum of the angular directions. In this way, the integro-differential equation is transform on a set of first-class ordinary partial equations; ii) a space discretization, considering control volume, for solution of partial equations. To a "cold" media (no self-emission media), Eq. (1) can be re-write as:

$$I_{i+1/2,j}=\frac{1}{\left(1+f\alpha_j\right)}\left[f\alpha_j\frac{\omega}{2\beta}\left[\sum_{n=1}^{N}w_n\left(p_{nj}I_{i+1/2,n}\right)\right]+I_{i,j}\right] \tag{3}$$

where $i+1/2$ represent the control volume center coordinate, f is the interpolation function that can be: upwind, linear, integral or exponential, w is the weight and α_j is:

$$\alpha_j=\frac{\Delta\tau_{i+1/2}}{\mu_j} \tag{4}$$

where $\Delta\tau$ is the optical thickness of the control volume.

Traditionally in literature there are two different materials class considerations involving refractive index change solutions: metallic and dielectric materials (Özisik, 1973, Modest, 1993, McCluney, 1994 and Siegel and Howell, 2002). All these solutions are obtained from Maxwell equations, Planck (1914) and Hulin *et al*. (1998). In this case the coating can be considered like a dielectric material. This method was compiled by Moura (2003) and involves the use of Snell equation to obtain the angular quadratures of DOM due to the refractive index considerations. Even when there is a normal incidence onto the sample the solid angle of incident beam changes. This must be considered to the quadrature setup and the use of classic quadratures, like Gauss and Radau, are not possible. Moura (1999) shows that the small variations in the solid angle of the incident beam in relation can result great errors on phase function identification. In this way, it is recommendable to use adaptive quadratures in respect on solid angle of the incident beam, or either, to a refractive index medium, either possible to directions $\mu=\pm1$ calculate the respective weights.

Wu *et al*. (1994) and Liou and Wu (1996) had used a linear transformation to correct the quadratures to refractive index the directions μ ($\mu=\cos\theta$) and the quadratures pounds, w, are written as:

$$\mu'_j=\left(\mu_j+1\right)/2 \qquad w'_j=w_j/2 \tag{5}$$

$$\int_0^1 f(\mu')d\mu'=\int_{-1}^{1} f\left(\frac{\mu+1}{2}\right)d\mu \tag{6}$$

The divergence angle, θ'_2, inside the medium is obtained by the incident divergence angle, θ_2, by the follow equation:

$$\theta'_2=arcsen\left(\frac{sen\,\theta_2}{n_2}n_1\right) \tag{7}$$

3. RESULTS

A previous validation of the numerical model used in this work was performed by Moura (2003) to a medium without substrate. Figure 2 show the agreement with Hottel *et al.* (1968) results. The hemispherical transmittance function represents the total radiative energy reflected by the media.

To this analysis the hemispherical bidirectional reflectance and transmittance is calculated by Eqs. (8) and (9), respectively:

$$R_{eb}=\frac{\sum_{\mu<0} w_n \mu_n I_{i,n}}{I_o \mathrm{d}\omega_o} \tag{8}$$

$$T_{eb}=\frac{\sum_{\mu>0} w_n \mu_n I_{i,n}}{I_o \mathrm{d}\omega_o} \tag{9}$$

where $I_o \mathrm{d}\omega_o$ is the incident radiative beam onto the slab with a divergence angle, $\mathrm{d}\omega_o$.

Figures 3 to 7 presented the analysis to different values of refractive index, albedo, optical thickness and phase function. Figures 3 to 6 consider the substrate with specular reflection and Fig. 7 analyzes the diffuse refection by the substrate.

The phase function has been approximated by a combination of two Henyey-Greenstein (HG) functions coupled with an isotropic component (Nicolau, 1994):

$$p(\theta_o)=f_1 f_2 p_{HG,g_1}(\theta_o)+(1-f_1)f_2 p_{HG,g_2}(\theta_o)+(1-f_2) \tag{10}$$

where the parameters g_1 and g_2 govern the shape of HG functions ($p_{HG,g1}$ and $p_{HG,g2}$) in the forward and backward directions. f_1 is the weighting factor between forward and backward anisotropy in the phase function, f_2 is the weighting factor between anisotropic and isotropic scattering.

Figure 3 shows the hemispherical bidirectional function to albedo, ω=0.95 and the phase function g_1=0.84, g_2=-0.6, f_1=0.9 and f_2=0.95. The hemispherical reflectance decrease with the refractive index and to optical thickness up to 10 this parameter keeps constant. This limit will be defined as optical thickness limit (OPL point).

Figure 4 presents the same analysis, except to albedo that changes to ω=0.75. The refractive index influence decrease and the OPL point is around τ_o=5. Decreasing the albedo to ω=0. 5 this effect is more important, Fig. 5, and the OPL point is around τ_o=3.

Figure 6 present the effect of a new phase function g_1=0.84, g_2=-0.6, f_1=0.7 and f_2=0.95 to albedo ω=0.75. A weak reduction on the OPL point can be observed in respect to Fig. 4 analysis.

Figure 7 presents the effect of the diffuse refection by the substrate, considering the first phase function and albedo ω=0.75. Once again it is not expressive when compared to Fig. 4. In the real cases the substrate has comportment between specular and diffuse reflection. With these results, it can be observed that the more important parameter to hemispherical reflectance variation is the albedo.

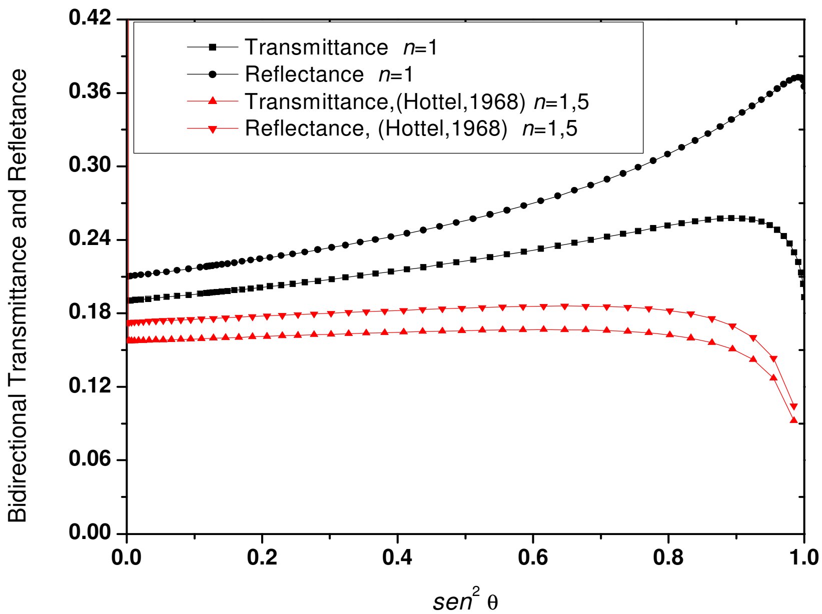

Figure 2. Bidirectional transmittance and reflectance to an isotropic medium, ω=0.9 and τ_o=1.

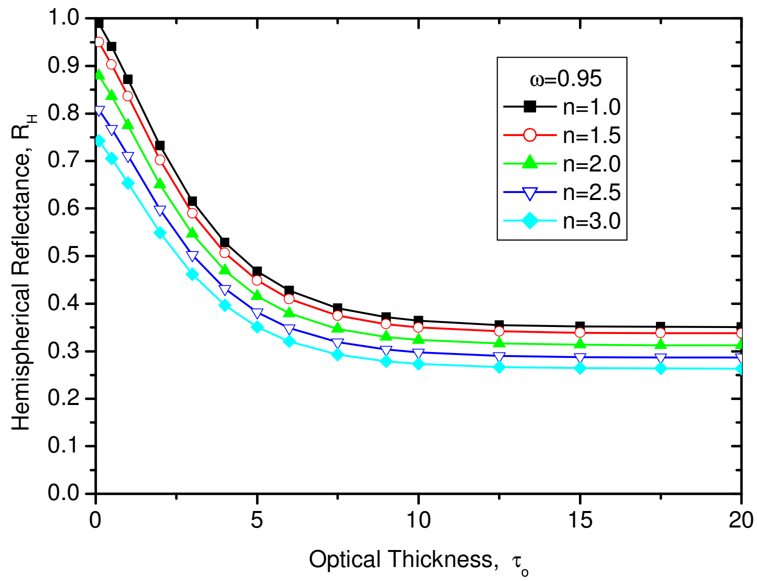

Figure 3. Hemispherical reflectance function of optical thickness to ω=0.95.

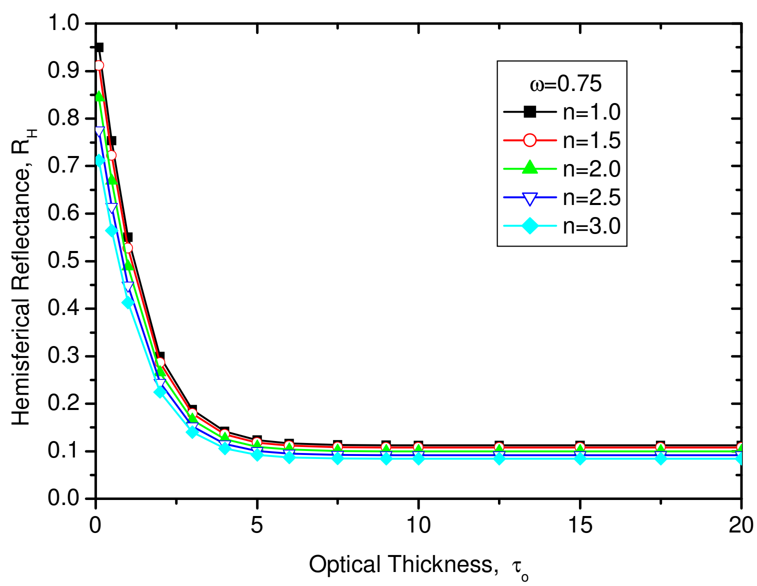

Figure 4. Hemispherical reflectance function of optical thickness to ω=0.75.

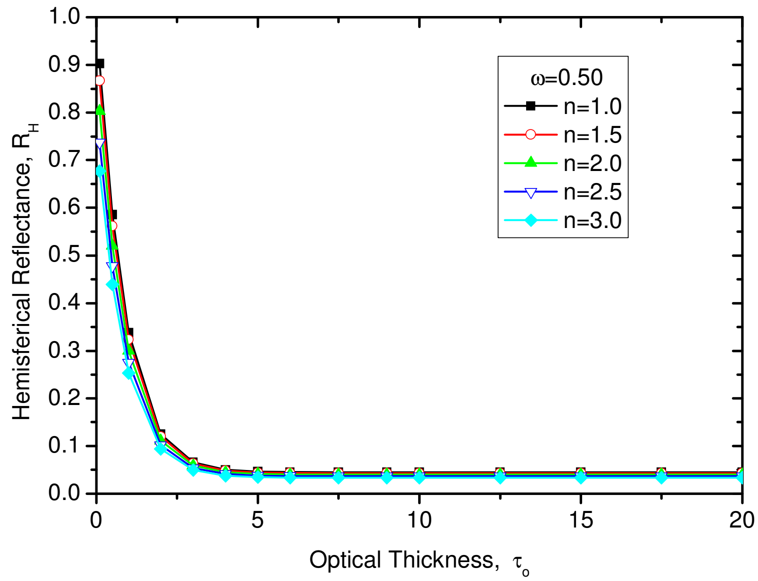

Figure 5. Hemispherical reflectance function of optical thickness to ω=0.50.

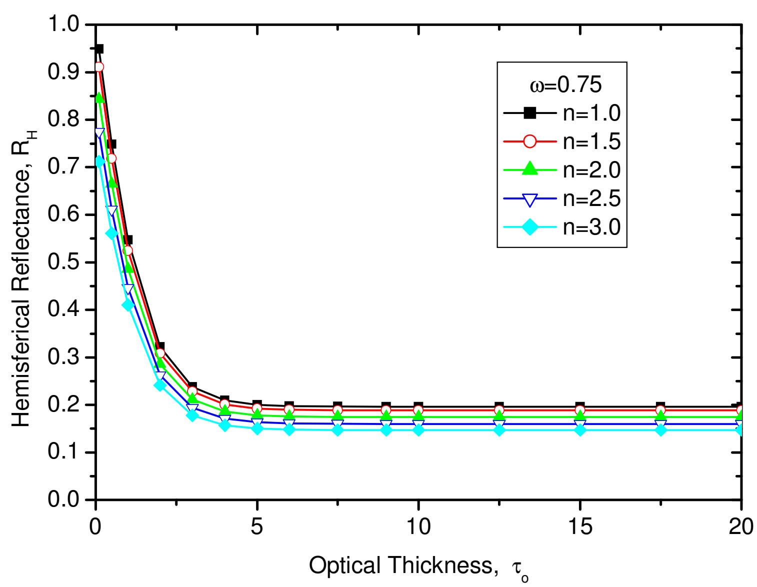

Figure 6. Hemispherical reflectance function of optical thickness to ω=0.75. Phase function

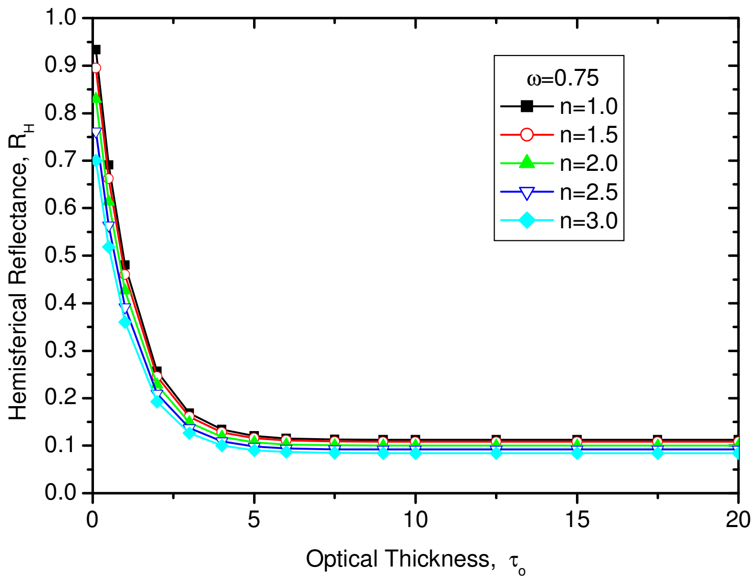

Figure 7. Hemispherical reflectance function of optical thickness to ω=0.75. Diffuse reflection on face 2.

4. CONCLUSION

In this work an analysis of the Fresnel reflection interfaces has been performed at a slab containing an anisotropic medium and a refractive index ratio. This analysis is specially focused on coatings on opaque substrate hemispherical reflectance comportment. The analyses were performed to different values of the analysis to different values of refractive index, albedo, optical thickness, surface condition and phase function. It can be observed that the more important parameter to hemispherical reflectance variation is the albedo. Forthcoming, this model will be improved to use to identify the radiative properties.

3. ACKNOWLEDGEMENTS

The author acknowledges the financial support gives by CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) grant APQ n° 479005/01-0.

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