THERMAL RADIATION INVERSE ANALYSIS FOR ESTIMATION OF TEMPERATURE FIELD IN 2-D SEMITRANSPARENT MEDIA USING DISCRETE ORDINATES METHOD WITH MULTIDIMENSIONAL SCHEME

Carlos T. Salinas, csalinas_99@yahoo.com, csalinas@unitau.br
Valesca Alves Corrêa, valesca@mec.unitau.br
Department of Mechanical Engineering – University of Taubaté. Rua Daniel Danelli s/n – Taubaté (SP) 12060-440 – Brazil.

Abstract. *Despite the relatively large interest expressed in inverse radiation problems of temperature distributions in media, most of the works use simple spatial interpolation schemes like the step and diamond scheme, and a low order of angular quadratures for the discrete ordinates method (DOM) or the discrete transfer method (DTM). Less accurate solutions were used for the direct problem. This work presents an inverse analysis for temperature field estimation in a two-dimensional gray media with boundary surfaces without external incident radiation; it uses a multidimensional spatial scheme and high order angular quadrature in the discrete ordinates method. The inverse radiation problem considered in this paper is concerned with the estimation of the source term distribution or temperature profile in participating media systems which contain absorbing, emitting, isotropic scattering gray medium from knowing the radiative intensities exiting in some points of boundary surfaces which simulates dates of sensing devises. The radiative properties such as the single scattering albedo and absorption coefficient of the medium are assumed to be uniform everywhere. The boundaries are considered to be transparent. The conjugate gradient method is used to solve the inverse radiation problem for determining the temperature field. The inverse problem is formulated as an optimization problem that minimizes the error between the calculated and the simulated measurement of radiation intensity leaving the media at four points in the walls. The inverse analysis steps are presented. Different angular quadratures are tested; from comparison between these results it is found that the low order angular quadratures do not give accurate estimations. The numerical results are obtained by considering simulated data with and without noise. The temperature field has been estimated with accuracy by using LC11 and Tn6 angular quadratures of DOM.*

Keywords: *radiative transfer, inverse radiation analysis, conjugate gradient method,2-D temperature field, discrete ordinates method.*

1. INTRODUCTION

Radiative heat transfer is the predominant mode of heat transfer in combustion chambers and furnaces. It is a very complicated phenomenon due to the gas it absorbs, emits and scatters in radiation. Since radiation affects the temperature field, accurate knowledge of the gas heat rate is required. In practice, two ways to predict the gas heat rate distribution are commonly used. The first one is to solve the theoretical combustion model or empirical formula that was not able to get an accurate result. The second one is to solve the inverse radiation problem.

Many researchers have reported inverse problems that deal with the prediction of the temperature distribution in a medium from either simulated or experimental radiation measurements. Yi et al. (1992), Li and Ozisik (1992), Siewert (1993, 1994), Liu et al. (1998, 2000), Li (1994, 1997a), have reconstructed the temperature profiles or source terms in a one-dimensional plane-parallel, spherical and cylindrical media by inverse analysis from the simulated data of the radiation intensities exiting at the boundaries. Most of the first works have considered that one-dimensional systems assumed that the bounding surface is transparent or the emissivity of the boundary surface is known. On many occasions, the bounding surface is opaque and its emissivity is unknown; for example, the boundary emissivity of the combustion chamber is changed with the operating condition. Under this condition the unknown temperature profile needs to be estimated simultaneously with the unknown emissivity of the boundary surfaces. Li and Ozisik (1993), Liu et al. (1999), have reconstructed simultaneously temperature profiles and wall emissivity in parallel plane media. The optical thickness, the albedo and the scattering phase function were simultaneously estimated by Neto and Ozisik (1995) in a parallel plane media. Li (1997b) has studied the inverse problem of an unknown source term in a two dimensional rectangular medium with transparent boundaries. Zhou et al. (2000) used an optimization procedure for estimating simultaneously the temperature and scattering albedo profiles.

In most of the above works, the discrete ordinates method (DOM), the discrete transfer method (DTM), the zonal method or the Monte Carlo method were employed to solve the direct and the sensitivity problems.

For a system governed by radiation, the inverse problem is represented by a set of Fredholm equations of the first kind, which are known to be ill-posed (Li, 1997b). If the resulting system is solved by simple techniques, like the Gauss elimination or Gauss-Siedel iteration, the ill-posed character of the governing equations leads to non-physical solutions that are highly affected by small perturbations in the input. Therefore, in order to produce physically reasonable, yet accurate solutions, the system must be regularized. Regularization leads to a set of solutions that ignores some information that is the source of the ill-posed character. Consequently, the solutions are subject to different levels of errors that result from ignored information, and an optimal solution must be sought that satisfies the physical requirements of the problem with acceptable accuracy. There exist a number of regularized solution techniques that have been used for solving similar problems (Ozisik and Orlande, 2000). A review of regularization methods is provided by Hansen (1998).

The inverse radiation problem considered in this paper is concerned with the estimation of the source term distribution or temperature profile in 2-D participating media systems containing absorbing, emitting and scattering gray medium from knowing the radiative intensities exiting in some points of boundary surfaces that simulate data of sensing devises. The radiative properties such as single scattering albedo and scattering phase function of the medium are assumed to be uniform everywhere. The boundaries are considered to be either transparent or opaque. The inverse problem is formulated as an optimization problem and the conjugate gradient method is used for its solution. In this work, the discrete ordinates method is used to solve the direct and the sensitivity problem. A multidimensional high order spatial scheme and different angular quadratures for the discrete ordinates method are used to examine the accuracy of the estimation. Analysis is made of the direct problem, the gradient equation and the sensitivity problem. The procedure for each of these steps is described and then an algorithm for the solution of the inverse radiation problem is presented. Finally, several inverse problems of the source term in two-dimensions are investigated to demonstrate the computational accuracy and efficiency of the inverse analysis method presented in this paper.

2. ANALYSIS

The analysis consists of the direct problem, the gradient equation, and the sensitivity problem. In practical applications of inverse radiation problems the analysis must be based on a more complex model than the one considered in this work, for example, problems with non-gray and scattering medium. The simplified model considered here is to be understood as an attempt to provide a more accurate estimation based on more accurate numerical modeling in the direct and sensitivity problem.

2.1 Direct Problem

It considers the radiative transfer process in an absorbing, emitting, scattering, gray and two dimensional rectangular medium. The boundary surfaces are considered to be transparent. There is no external incident radiation. The radiative properties, such as single scattering albedo (ω) and absorption coefficient of the medium (κ) are assumed to be uniform everywhere. The direct problem of concern here is to find the radiative intensities exiting at the boundaries for the known source term distribution and radiative properties. The radiative transport equation for an absorbing, emitting gray gas with isotropic scattering can be written as Siegel and Howell (1992) did,

$$(\Omega.\nabla)\boldsymbol{I}(\boldsymbol{r},\Omega) = -(\kappa+\sigma)\boldsymbol{I}(\boldsymbol{r},\Omega) + S(\boldsymbol{r}) + \frac{\sigma}{4\pi}\int_{4\pi}\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \text{ with } S(\boldsymbol{r}) = \frac{(1-\omega)\overline{n}^2\overline{\sigma}\boldsymbol{T}^4(\boldsymbol{r})}{\pi} \tag{1}$$

where $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity in $\boldsymbol{r}$, and in the direction Ω; κ is the gray medium absorption coefficient; σ is the gray medium scatter coefficient; and the integration is in the incident direction Ω'. The source term $S(\boldsymbol{r})$ is related to the temperature $T(\boldsymbol{r})$ of the medium by. $\overline{n}$ is the refractive index and $\overline{\sigma}$ is the Stefan Boltzmann constant.

For diffusely reflecting surfaces the radiative boundary condition for Eq. (1) is

$$\boldsymbol{I}(\boldsymbol{r},\Omega) = \varepsilon\boldsymbol{I_b}(\boldsymbol{r}) + \frac{\rho}{\pi}\int_{\boldsymbol{n}.\Omega'<0}|\,\boldsymbol{n}.\Omega'|\,\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \tag{2}$$

where $\boldsymbol{I_b}(\boldsymbol{r})$, is the radiation intensity of the blackbody in position $\boldsymbol{r}$ and at the temperature of the medium; $\boldsymbol{r}$ lies on the boundary surface Γ, and Eq. (4) is valid for $\boldsymbol{n}.\Omega>0$. $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity leaving the surface at the boundary condition, ε is the surface emissivity, ρ is the surface reflectivity and $\boldsymbol{n}$ is the unit vector normal to the boundary surface. In the method of discrete ordinates, the equation of radiation transport is substituted by a set of M discrete equations for a finite number of directions Ω_m, and each integral is substituted by a quadrature series (Fiveland W., 1984),

$$(\Omega_m.\nabla)\boldsymbol{I}(\boldsymbol{r},\Omega_m) = -\beta\boldsymbol{I}(\boldsymbol{r},\Omega_m) + \boldsymbol{S}(\boldsymbol{r}) + \boldsymbol{S}_m \qquad \text{with} \qquad \boldsymbol{S}_m = \frac{\sigma}{4\pi}\sum_{m=1}^{M}\boldsymbol{w}_m\boldsymbol{I}(\boldsymbol{r},\Omega_m) \tag{3}$$

This angular approximation transforms the original equation into a set of coupled differential equations, with $\beta = (\kappa + \sigma)$ as the extinction coefficient; S_m represents the entering scattering source term, $\boldsymbol{w}_m$ are the ordinates weight, M is the number of directions Ω_m of the angular quadrature and $\boldsymbol{I}_m$ is obtained by solving the radiative transport equation in discrete ordinates.

In the Cartesian ordinate system, the two-dimensional radiative transport equation in the $\boldsymbol{m}$ direction for an emitting, absorbing and scattering medium is

$$\mu_m \frac{d\boldsymbol{I}_m}{dx} + \xi_m \frac{d\boldsymbol{I}_m}{dy} = \beta \boldsymbol{I}_m + \boldsymbol{S}(x,y) + \boldsymbol{S}_m \tag{4}$$

where μ_m, ξ_m, are the directional cosine of Ω_m. The boundary condition in discrete ordinates for the case analyzed in this work can be written as

$$\boldsymbol{I}_m = 0; \quad \mu_m > 0 \ \text{ in } x=0; \ \boldsymbol{I}_m = 0; \quad \mu_m < 0 \ \text{ in } x=x_L; \ \boldsymbol{I}_m = 0; \quad \xi_m > 0 \ \text{ in } y=0; \ \boldsymbol{I}_m = 0; \quad \xi_m < 0 \ \text{ in } y = y_L \tag{5}$$

Assuming that the boundary conditions are given, the system of equations is closed and defines an interpolation system relating the intensities at the face to the nodal values. The multidimensional non-linear high order scheme of Balsara (2001), the so-called genuinely multidimensional (GM) scheme, which was used in previous work (Ismail and Salinas, 2004) in radiative transport is used here in order to solve the direct and sensitive problem.

2.2 Inverse Problem

For the inverse problem, the source term distribution or the temperature distribution are unknown, but the other quantities in Eqs. (4) and (5) are known. Measured exit radiative intensities at the center of the wall boundaries are considered available. In the inverse analysis, the source term distribution is estimated by the measured data of exit radiative intensities. The source term can be represented by a polynomial as

$$\boldsymbol{S}(x,y) = \sum_{q=0}^{M} \sum_{r=0}^{N} \boldsymbol{a}_{qr} \boldsymbol{f}_q(x) \boldsymbol{g}_r(y) \tag{7}$$

where $\boldsymbol{f}_q$ (x) e $\boldsymbol{g}_r$ (y) are basic functions, M and N is the order of the source term polynomial expansion. The inverse radiation problem can be formulated as an optimization problem. We wish to minimize the objective function

$$\begin{aligned} \boldsymbol{J}(\tilde{\boldsymbol{a}}) = & \sum_{\mu_i<0} \boldsymbol{w}_i [\boldsymbol{I}_1(\boldsymbol{x}_0, 0{,}5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_1(\mu_i)]^2 + \sum_{\mu_i>0} \boldsymbol{w}_i [\boldsymbol{I}_2(\boldsymbol{x}_L, 0{,}5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_2(\mu_i)]^2 + \\ & \sum_{\xi_i<0} \boldsymbol{w}_i [\boldsymbol{I}_3(0{,}5\boldsymbol{x}_L, \boldsymbol{y}_0, \xi_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_3(\xi_i)]^2 + \sum_{\xi_i>0} \boldsymbol{w}_i [\boldsymbol{I}_4(0{,}5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_4(\xi_i)]^2 \end{aligned} \tag{8}$$

where $Y_1(\mu_i)$, $Y_2(\mu_i)$, $Y_3(\xi_i)$ and $Y_4(\xi_i)$ are measured exit radiation intensities at the boundaries in the points (x, y): (x_0, $y_L/2$), (x_L, $y_L/2$), ($x_L/2$, y_0) and ($x_L/2$, y_L) respectively, where $x_0 = 0$ and $y_0 = 0$; $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$ e $\boldsymbol{I}_4$, are estimated exit radiation intensities at (x_0, $y_L/2$), (x_L, $y_L/2$), ($x_L/2$, y_0) and ($x_L/2$, y_L), respectively, for the estimated vector $\tilde{\boldsymbol{a}} = (a_{00}, a_{10}, \ldots, a_{MN})^T$. Then the problem is to find the vector $\tilde{\boldsymbol{a}}$ which minimizes the function $\boldsymbol{J}$. The computational algorithm of this minimization procedure consists of two main modules: the direct radiation computation and the search modules. In the first module, the Discrete Ordinates Method (DOM) with the multidimensional spatial scheme is employed; while, as previously it was explained and applied in Li and Ozisik (1992), Li and Ozisik (1993), Liu et al. (1999), Liu et al. (2000), Li (1994) and Li (1997b), for the latter module, the Conjugate Gradient Method (CGM) can be used as the basic search method to minimize the function $\boldsymbol{J}$.

2.3 Conjugate Gradient Method of Minimization

The minimization of the objective function with respect to the desired vector is the most important procedure in solving the inverse problem. The Conjugate Gradient Method for determining unknown temperature distribution is used in this work. Iterations are built in the following manner (Li and Ozisik, 1992; Liu et al., 2000): $\boldsymbol{a}^{k+1} = \boldsymbol{a}^k - \alpha^k \boldsymbol{d}^k$, where α^k is the step size, $\mathbf{d}^k$ is the direction vector of descent given by $\mathbf{d}^k = \nabla \boldsymbol{J}(\boldsymbol{a}^k) + \beta^k \mathbf{d}^{k-1}$ and the conjugate coefficient β^k is determined from

$$\beta^k = \frac{\nabla \boldsymbol{J}(\boldsymbol{a}^k)\nabla \boldsymbol{J}^T(\boldsymbol{a}^k)}{\nabla \boldsymbol{J}(\boldsymbol{a}^{k-1})\nabla \boldsymbol{J}^T(\boldsymbol{a}^{k-1})} \qquad \beta^0 = 0 \tag{9}$$

where the row vector $\nabla \boldsymbol{J}$ defined by

$$\nabla \boldsymbol{J} = \left(\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{00}}, \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{10}}, \ldots\ldots.., \quad \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{MN}} \right) \tag{10}$$

is the gradient of the objective function. Its components are defined as

$$\begin{aligned}\frac{\partial \boldsymbol{J}(\tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{qr}} = {} & 2\sum_{\mu_i<0} \boldsymbol{w}_i[\boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_1(\mu_i)]\frac{\partial \boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{qr}} + \\ & 2\sum_{\mu_i>0} \boldsymbol{w}_i[\boldsymbol{I}_2(\boldsymbol{x}_L, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_2(\mu_i)]\frac{\partial \boldsymbol{I}_2(\boldsymbol{x}_L, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{qr}} + \\ & 2\sum_{\xi_i<0} \boldsymbol{w}_i[\boldsymbol{I}_3(0.5\boldsymbol{x}_L, \boldsymbol{y}_0, \xi_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_3(\xi_i)]\frac{\partial \boldsymbol{I}_3(0.5\boldsymbol{x}_L, \boldsymbol{y}_0, \xi_i, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{qr}} + \\ & 2\sum_{\xi_i>0} \boldsymbol{w}_i[\boldsymbol{I}_4(0.5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}_4(\xi_i)]\frac{\partial \boldsymbol{I}_4(0.5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{qr}}\end{aligned} \tag{11}$$

In principle, the step size of the k th iteration α^k can be determined by minimizing the function, $\boldsymbol{J}(\boldsymbol{a}^k - \alpha^k \mathbf{d}^k)$, for the given $\boldsymbol{a}^k$ and $\boldsymbol{d}^k$ in the following manner:

$$\frac{\partial \boldsymbol{J}(\boldsymbol{a}^k - \alpha^k \boldsymbol{d}^k)}{\partial \alpha^k} = 0 \tag{12}$$

Since $\boldsymbol{J}(\mathbf{a}^k - \alpha^k \mathbf{d}^k)$ is the implicit function of α^k, the exact step is difficult to solve. We make the first-order Taylor expansion of the function with respect to α^k. Using Eq. (19), we have in discrete ordinates

$$\begin{aligned}\alpha^k = {} & \Big\{ \sum_{\mu_i<0} \boldsymbol{w}_i[\boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}^k) - \boldsymbol{Y}_1(\mu_i)][\nabla \boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k] + \\ & \sum_{\mu_i>0} \boldsymbol{w}_i[\boldsymbol{I}_2(\boldsymbol{x}_L, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}^k) - \boldsymbol{Y}_2(\mu_i)][\nabla \boldsymbol{I}_2(\boldsymbol{x}_L, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k] + \\ & \sum_{\xi_i<0} \boldsymbol{w}_i[\boldsymbol{I}_3(0.5\boldsymbol{x}_L, \boldsymbol{y}_0, \xi_i, \tilde{\boldsymbol{a}}^k) - \boldsymbol{Y}_3(\xi_i)][\nabla \boldsymbol{I}_3(0.5\boldsymbol{x}_L, \boldsymbol{y}_0, \xi_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k] + \\ & \sum_{\xi_i>0} \boldsymbol{w}_i[\boldsymbol{I}_4(0.5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}}^k) - \boldsymbol{Y}_4(\xi_i)][\nabla \boldsymbol{I}_4(0.5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k \Big\} \\ & \Big/ \Big\{ \sum_{\mu_i<0} \boldsymbol{w}_i[\nabla \boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_{L,}\mu_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k]^2 + \sum_{\mu_i>0} \boldsymbol{w}_i[\nabla \boldsymbol{I}_2(\boldsymbol{x}_L, 0.5\boldsymbol{y}_L, \mu_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k]^2 + \\ & \sum_{\xi_i<0} \boldsymbol{w}_i[\nabla \boldsymbol{I}_3(0.5\boldsymbol{x}_L, \boldsymbol{y}_{0,}\xi_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k]^2 + \sum_{\xi_i>0} \boldsymbol{w}_i[\nabla \boldsymbol{I}_4(0.5\boldsymbol{x}_L, \boldsymbol{y}_L, \xi_i, \tilde{\boldsymbol{a}}^k).\boldsymbol{d}^k]^2 \Big\}\end{aligned} \tag{13}$$

where the row vector

$$\nabla \boldsymbol{I} = \left(\frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_{00}}, \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_1 0}, \ldots\ldots.., \quad \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_{MN}} \right) \tag{14}$$

is the sensitivity coefficient vector, which is essential in the solution procedure of the inverse problem.

2.4 Sensitivity Problem

To obtain the sensitivity coefficients, we substitute Eq. (7) into Eq. (5) and differentiate the direct problem defined by Eqs. (4) and (5) with respect to $\boldsymbol{a}_{qr}$. The equations of sensitivity coefficients can be written as

$$\mu\frac{\partial}{\partial \boldsymbol{x}}\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)+\xi\frac{\partial}{\partial \boldsymbol{y}}\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)+\beta\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)=\boldsymbol{f}_q(\boldsymbol{x})\boldsymbol{g}_r(\boldsymbol{y})+\frac{\omega}{4\pi}\int_{4\pi}\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega')}{\partial \boldsymbol{a}_{qr}}\boldsymbol{d}\Omega' \tag{15}$$

where q = 1, 2,……., M and r = 1, 2,……., N

With the boundary conditions

$$\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)=0 \quad x=0,\quad \mu>0\ ;\quad \left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)=0 \quad x=x_L,\quad \mu<0$$
$$\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)=0 \quad y=0,\quad \xi>0\ ;\quad \left(\frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a}_{qr}}\right)=0 \quad y=y_L,\quad \xi<0 \tag{16}$$

A similar numerical iteration procedure as the direct problem is used for the solution of the sensitivity problem and will not be repeated here. Because the sensitivity coefficient vector $\nabla \boldsymbol{I}$ is independent of the vector $\boldsymbol{a}$, the estimation of the source term distribution is linear, and it is only necessary to solve it once at first.

2.5 Stopping criterion and error estimation

The stopping criterion of the iteration is selected in the following manner: if the problem contains no measurement error, the following condition

$$\boldsymbol{J}(\mathbf{a}^{k+1}) < \delta^* \tag{17}$$

is used for terminating the iterative process, where δ^* is a small specified positive number; otherwise, it is used in the following two conditions

$$\boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}})-\boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1})<\delta_1^* \quad \text{and} \quad \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1)}=8\pi\sigma_0^2 \tag{18}$$

where δ_1^* is a small specified positive number, σ_o is the standard deviation (Li, 1997; Alifanov, 1974). Written in discrete ordinates is

$$\boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1})<\sum_{\mu_m<0}\sigma^2_{0\boldsymbol{W},\boldsymbol{m}}+\sum_{\mu_m>0}\sigma^2_{0\boldsymbol{E},\boldsymbol{m}}+\sum_{\xi_m<0}\sigma^2_{0\boldsymbol{S},\boldsymbol{m}}+\sum_{\xi>0}\sigma^2_{0\boldsymbol{N},\boldsymbol{m}} \tag{19}$$

When Eq. (17) or Eq. (18) is satisfied, then it is used as the stopping criterion. After several numerical experiments, the value of δ_1^* is selected as 10^{-5}. To examine the accuracy of the estimation by using the multidimensional spatial scheme and high order angular quadratures for DOM, the relative error defined as (Li, 1997) will be used.

$$\text{Relative error} = \frac{\boldsymbol{S}_{\boldsymbol{estimated}}(x,y)-\boldsymbol{S}_{\boldsymbol{exact}}(x,y)}{\boldsymbol{S}_{\boldsymbol{exact}}(x,y)} \tag{20}$$

2.6 Computational Algorithm

The computational algorithm for the solution of the inverse radiation problem can be summarized as follow

Step 1. Pick the initial guesses of $\mathbf{a}^o$. Set k = 0.
Step 2. Solve the sensitivity problem and compute the sensitivity coefficient vector $\nabla\boldsymbol{I}$.
Step 3. Solve the direct problem and compute the exit radiation intensities $\boldsymbol{I}_{1,m}(\boldsymbol{a}^k)$, $\boldsymbol{I}_{2,m}(\boldsymbol{a}^k)$, $\boldsymbol{I}_{3,m}(\boldsymbol{a}^k)$ and $\boldsymbol{I}_{4,m}(\boldsymbol{a}^k)$.
Step 4. Calculate the function objective $\boldsymbol{J}(\mathbf{a}^k)$. Terminate the iteration process if the specified stopping criterion is satisfied. Otherwise, go to step 5.
Step 5. Compute the gradient of the function objective $\nabla\boldsymbol{J}(\mathbf{a}^k)$.

Step 6. Knowing $\nabla \boldsymbol{J}(\mathbf{a}^k)$, compute the conjugate coefficient β^k; then compute the direction vector of descent $\mathbf{d}^k$.
Step 7. Knowing $\nabla \boldsymbol{I}, \boldsymbol{I}_{1,m}(\boldsymbol{a}^k), \boldsymbol{I}_{2,m}(\boldsymbol{a}^k), \boldsymbol{I}_{3,m}(\boldsymbol{a}^k)$ and $\boldsymbol{I}_{4,m}(\boldsymbol{a}^k)$, compute the step size α^k.
Step 8. Compute the new estimated vector $\mathbf{a}^{k+1}$.
Step 9. Set k = k +1, and go to step 3.

To start the iteration, the initial guess $\boldsymbol{a}^o = \mathbf{0}$ is used.

3. RESULTS AND DISCUSSION

Based on the theoretical and numerical analysis described earlier, a computer code has been developed to solve the inverse radiation problem of source term in two-dimensional rectangular media from the knowledge of exit radiation at the centers of the boundaries. To examine the accuracy of the method presented in this paper, two different test cases are considered. In the first case, assuming the measurement data exit radiation intensities have no errors, the temperature profile is determined. In the second case, the effects of random measurement errors on the estimation are analyzed. The optical thickness of the slab is chosen to be 1.0. To simulate the measured exit radiation intensities, $\boldsymbol{Y}_i$, containing measurement errors, random errors of standard deviation σ_o are added to the exact exit radiation intensities computed from the solution of the direct problem. Thus we have

$$(\boldsymbol{Y}_i)_{measured} = (\boldsymbol{Y}_i)_{exact} + \sigma_o \zeta \qquad i = 1, 2, 3, 4 \tag{21}$$

where ζ is a normal distribution random variable with zero mean and unit standard deviation. There is a 99% probability of ζ lying in the range $-2.567 < \zeta < 2.567$ (Li and Ozisik, 1992). For all the results presented in this work, it is assumed that the exit radiation intensities are available at the quadrature points for different angular quadratures of DOM. First we consider a source term expressed as a polynomial of degree 4 as in Li (1997),

$$S(x,y) = 1 + 6x + 3y + 4x^2 + 2y^2 + 5x^4y^4, \qquad W/m^2 \tag{22}$$

The estimate values of the source term by inverse analysis are calculated. Figure 1 shows the characteristic convergence for the solution of the inverse problem. The value of δ^* in Eq. (17) is selected after a series of numerical experiments using the graphics of convergence. With no measurement errors, $\sigma_o = 0$, no observable difference could be detected between the exact values of the source term and the estimated values when the LC11 angular quadrature is used, as shown in figure 2.

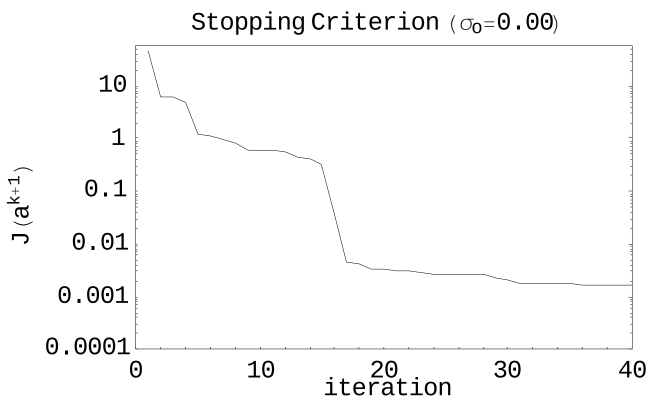


Figure 1. Characteristic convergence profile of the function objective for $\sigma_o = 0$

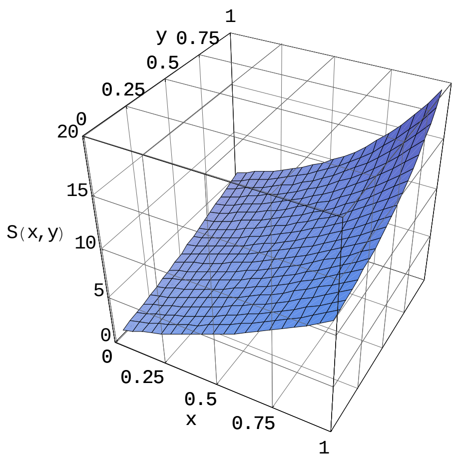


Figure 2. Comparison of exact function for source term S(x,y) and estimation for quadrature LC11, and $\sigma_o = 0$.

Otherwise, when the quadrature S_6 is used, discrepancies are observed with the exact solution. To show the accuracy of the estimation more clearly, the heat flux on the west wall for different angular quadratures is shown in Fig. 3. This heat flux is calculated using the estimated source term. It can be observed that the estimation of heat flux on the west wall agree well with the heat flux calculated using the exact solution when the LC11 quadrature is used and a poor estimation is found when Sn6 quadrature is used. The accuracy of the estimation is sensitive to the angular quadratures used. Estimations when the Tn6 quadrature is used is as accurate as an estimation with LC11 quadrature and is not presented in Fig. 3 for the sake of graphic clarity because the points seem to be the same. In the practical process of measurement and inverse solution, the measured parameters may have random errors. In order to examine the effect of these errors on temperature estimation, it is assumed that the simulated experimental data contain random measurement errors of standard deviation $\sigma_o = 0.03$, $\sigma_o = 0.06$ and $\sigma_o = 0.12$. Different sets of random numbers were used for simulating measurements in each boundary and different sets of random numbers were used to repeat the inverse calculations. To show the accuracy of the estimation more clearly, the heat flux on the west wall for σ_o equal to 0.03, 0.06 and 0.12 is shown in Fig. 4. This heat flux is calculated using the estimated source term. All the estimations in this figure are for the LC11 quadrature. Clearly, the agreement between the exact and the estimated results for the heat flux on the west wall is good for σ_o equal to 0.03 and 0.06. The accuracy decreases when a strong noise with σ_o equal to 0.12 is used, but the results still have good approximation. The value of δ_1^* is selected being equal to 10^{-5} after several numerical experiments using the graphics of convergence. Also, to examine the effects on the accuracy of the estimation, different angular quadratures for DOM were used when σ_o equals 0.03, 0.06 and 0.12 and discrepancies with the exact solution are found as it was shown in Fig. 3; these discrepancies increases as σ_o increases. When low order quadratures like S_4 and S_6 are used, the estimation is less accurate than when the quadratures Tn6 or LC11 is used. This result is not presented here for the sake brevity and also because it is redundant with Fig. 3. To show the accuracy of the estimation more clearly, the relative error defined in Eq. (20) is calculated and the results are compared with existing results reported in Li (1997). Fig. 5(a) shows the relative error of the estimation in this work for σ_o equal to 0.12 and it is compared with the results in Fig. 5(b) taken from the Li (1997) work. Substantially less relative error in our result can be observed and it shows that the use of the multidimensional spatial scheme and high order angular quadrature for DOM permit a more accurate estimation.

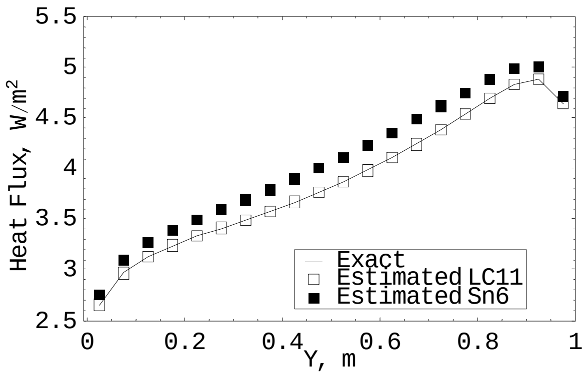


Figure 3. Estimation of the heat flux on west wall using simulated measured exit radiation intensity data with σ_o = 0 and different angular quadratures of DOM.

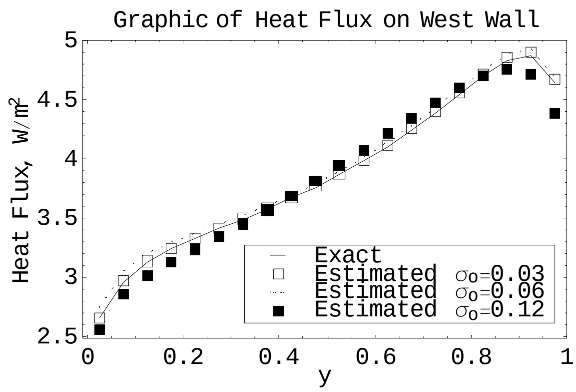
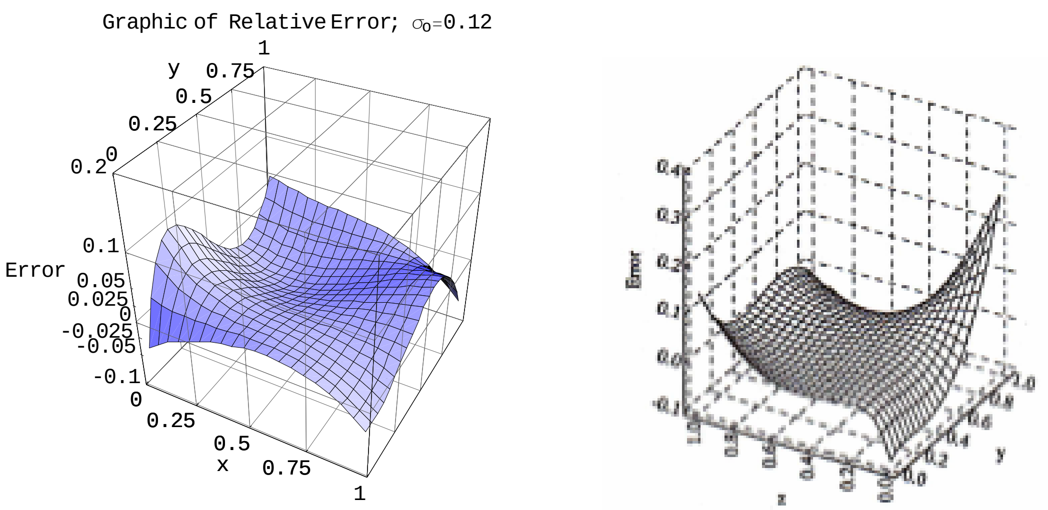


Figure 4. Comparison of the estimation of the source term using simulated measured exit radiation intensity data with angular quadratures LC11, and σ_o = 0.03, 0.06 and 0.12.



(a) this work

(b) taken from Li (1997)

Figure 5. Comparison of the relative error for σ_o = 0.12

4. CONCLUSIONS

An inverse method is presented for estimation of the temperature distribution for a gray emitting, non-scattering, two dimensional rectangular medium. The exit radiation intensities at the center of the bounding surfaces are assumed to be known. The direct and sensitive problems are solved using a high order multidimensional scheme for spatial discretization and the LC11 angular quadrature with 60 directions. The inverse problem is solved by using the conjugate gradient method. Noisy input data have been used to test the accuracy of the method used. The results show that the temperature profile can be estimated accurately even with noise data for a high order angular quadrature Tn6 or LC11 in DOM. However the problem is more sensitive to noise data when a lower order of angular quadratures like S_6 or S_4 is used.

5. ACKNOWLEDGEMENTS

The first author wishes to thank FAPESP for financial support of the young researcher in process 03/12456-7.

6. REFERENCES

Hansen R.C., 1998, “Rank Deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion”, Philadelphia, PA: SIAM.

Fiveland, W.A., 1984, “Discrete Ordinates Solutions of The Radiative Transport Equation for Rectangular Enclosures”, Trans. ASME. J. Heat Transfer, Vol. 106, pp. 699-706.

Ismail, K.A.R. and Salinas, C.T., 2004, “Application of Multidimensional Scheme and the Discrete Ordinate Method to Radiative Heat Transfer in a Two-dimensional Enclosure with Diffusely Emitting and Reflecting Boundary Walls”, JQSRT, Vol. 88, pp. 407-422.

Li, H.Y. and Ozisik, M.N., 1992, “Identification of Temperature Profile in an Absorbing, Emitting and Isotropically Scattering Medium by Inverse Analysis”, J. Heat Transfer, Vol.114, pp 1060-1063.

Li,, H.Y. and Ozisik, M.N., 1993, “Inverse Radiation Problem for Simultaneous Estimation of Temperature Profile and Surface Reflectivity”, J. Thermophys. Heat Transfer, Vol. 7, pp 88-93.

Li,, H.Y., 1994, “Estimation of the Temperature Profile in a Cylindrical Medium by Inverse Analysis”, J. Quant. Spectrosc. Radiative Transfer Vol. 52, pp 755-764.

Li, H.Y., 1997a, “An Inverse Source Problem in Radiative Transfer for Spherical Media”, Numerical Heat Transfer Part B, Vol. 31, pp 251-260.

Li H.Y., 1997b, “Inverse Radiation Problem in Two-dimensional Rectangular Media”, J. Thermophys. Heat Transfer Vol. 11, pp 556-561.

Liu, L.H., Tan, H.P. and Yu, Q.Z., 1998, “Inverse Radiation Problem of Boundary Incident Radiation Heat Flux in Semitransparent Planar Slab with Semitransparent Boundaries”, J. Thermal Sci., Vol. 7, pp 131-138.

Liu L.H., Tan H.P., Yu Q.Z., 1999, “Simultaneous Identification of Temperature Profile and Wall Emissivities in Semitransparent Medium by Inverse Radiation Analysis”, Numerical Heat Transfer Part A, Vol. 36, pp. 511-525.

Liu, L.H., Tan, H.P. Yu, Q.Z., 2000, “Inverse Radiation Problem In One-Dimensional Semitransparent Plane-Parallel Media With Opaque And Specularly Reflecting Boundaries”, J. Quant. Spectrosc. Radiative Transfer, Vol. 64, pp 395-407.

Neto A.J.S., Ozisik M.N., 1995, “An Inverse Problem of Simultaneous Estimation of Radiation Phase Function, Albedo and Optical Thickness”, J. Quant. Spectrosc. Radiat. Transfer, Vol. 53, pp 397-409.

Ozisik, M.N. and Orlande, H.R.B., 2000, “Inverse Heat Transfer”, Taylor and Francis, New York.

Siegel, R. and Howell, J. R, 1992, “Thermal Radiation Heat Transfer”,. 3ª ed., Washington. USA,.

Siewert, C.E., 1993, “An Inverse Source Problem in Radiative Transfer”, J. Quant. Spectrosc. Radiative Transfer, Vol. 50, pp 603-609.

Siewert, C.E., 1994, “A Radiative Transfer Inverse-Source Problem for a Sphere”, J. Quant. Spectrosc. Radiative Transfer, Vol. 52, pp 157-160.

Yi, H.C., Sanchez, R. and McCormick, N.J., 1992, “Bioluminescence Estimation from Ocean in Situ Irradiances”, Applied Optics, Vol. 31, pp 822-830.

Zhou H.C., Yuan P., Sheng F., Zheng C.G., 2000, “Simultaneous Estimation of the Profiles of the Temperature and the Scattering Albedo in an Absorbing, Emitting and Isotropically Scattering Medium by Inverse Analysis”. Int. J. Heat and Mass Transfer, Vol. 43, pp. 4361-4364.

7. RESPONSIBILITY NOTICE