# FLUID FLOW AND HEAT TRANSFER AROUND AN OSCILLATING CIRCULAR CYLINDER USING A PARTICLE METHOD

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Abstract. The relationship between wake structure and heat transfer for a transverselly oscillating circular cylinder in a uniform freestream flow is analyzed. Discrete heat particles are generated close to a circular cylinder surface in addition to nascent vortex elements. The vortex method is extended in order to simulte mumerically the convection and diffusion heat transfer. The influence of the frequency and amplitude oscillation on the aerodynamics loads and on the heat transfer are discussed and the preliminary numerical results are presented.

Keywords: vortex and heat element method, heat transfer, heaving, aerodynamics loads, bluff body

## **1. INTRODUCTION**

Particle methods are an alternative to traditional mesh-based methods for solving differential equations. Applications arise in many fields including astrophysics (gravitational interaction), chemistry, materials science, and plasma dynamics (electrostatic interaction), and fluid dynamics (vortex interaction).

The vortex methods have been developed and applied for analysis of complex, unsteady and vortical flows in relation to problems in a wide range of industries, because they consist of simple algorithm based on physics of flow (Kamemoto, 2004). The essentials of vortex methods (Chorin, 1973; Leonard, 1980; Sarpakaya, 1989; Lewis, 1999; Alcântara Pereira *et al.*, 2002; Stock 2007) are: (i) numerical technique to solve the Navier-Stokes equations; (ii) suitable for Direct Simulation and Large-Eddy Simulation; (iii) uses vorticity (curl of the velocity) as a variable; (iv) computational elements move with the fluid velocity. Therefore, the vortex methods offer a number of advantages over the more traditional Eulerian schemes: (i) computational elements only where vorticity is non-zero; (ii) no grid in the field; (iii) only 2D grid on vehicle surface; (iv) boundary conditions in the far field automatically satisfied.

On the other hand, there are only a few examples of the simulations of vorticity and heat transport using a particle method. Understanding heat transfer from transversely oscillating circular cylinder in a uniform freestream flow is an important and challenging engineering problem. Vortex-induced vibration is known to occur for long, cylindrical elements in tube-bank heat exchangers. This makes it important to understand how oscillations affect the heat transfer so that equipment can be properly designed. Many areas of fluid mechanics are involved in understanding this type of flow. Convective heat transfer, fluid-structure interactions, separated flows and vortex dynamics are all involved in relating cylinder oscillations to heat transfer.

In order to handle fluid flow and heat transfer by the vortex method, the following models are required (Ogami, 2001): (i) the modeling of discretization of heat distribution into the heat particles; (ii) the modeling of the process in which the vortex is generated by the effect of a heat; (iii) the modeling of the diffusion process of heat and vortex. Ghoniem and Sherman (1985) investigated one-dimensional heat diffusion using random walk scheme (Chorin, 1973, 1978). They present a complete analysis of heat particles with different properties and the vorticity generation due to the heat transfers process. Ghoniem et al. (1988) and Zhang and Ghoniem (1993) handle shear layers and a rise of a plume in two-dimensions. The diffusion process was simulated using the core spreading method (Leonard, 1980) and the density difference was considered, although heat transfer was not. Smith and Stansby (1989) and Stansby and Dixon (1983) used a hybrid method to analyze the vorticity and the heat transport around a circular cylinder. Using the vortex in cell method incorporate with the random walk model, they introduced both vortex and temperature particles according to the similarity of equations of vorticity transport and energy. The vorticity generation due to heat and natural convection are not accounted for. Kamemoto & Miyasaka (1999) used the core spreading model to simulate the forced convection heat transfer around a circular cylinder at high Reynolds numbers. Discrete heat elements with thermal core were introduced in the thin thermal layer along the body surface. Although they made an approximation that the temperature in the thermal layer was constant along the normal direction, the time-averaged Nusselt number distribution showed reasonable agreement with that of experiment (Igarashi, 1984). Alcântara Pereira and Hirata (2003) extended the vortex method to take into account the convection and diffusion heat transfer. Discrete heat particles were generated close to a circular cylinder surface in addition to nascent vortex elements. The unsteady flow and heat transfer were simulated around a circular cylinder in a uniform flow. The result of the surface and time-averaged Nusselt number at constant surface temperature showed reasonable agreement with that of experiment. The generation of vortex due the heat was not analyzed.

While it is evident from a review of the literature that the wake structure is the connection between oscillations and heat transfer, the mechanism of this connection is not understood. In addition, it is not known how the cylinder oscillations determine the wake structure or what other factors, if any, are involved in this process. Pottebaum (2003) presented a series of experiments in order to understand the relationship between wake structure and heat transfer for a transversely oscillating circular cylinder in cross-flow and explored the dynamics of the vortex formation process in the wake. The experiments were carried out in a water tunnel at a Reynolds number of 690. It was found that wake structure and heat transfer both significantly affect one other. The wake mode, a label indicating the number and type of vortices shed in each oscillation period, is directly related to the observed heat transfer enhancement. The cylinder's transverse velocity was shown to influence the heat transfer by affecting the circulation of the wake vortices. For a fixed wake structure, the effectiveness of the wake vortices at enhancing heat transfer depends on their circulation. Also, the cylinder's transverse velocity continually changes the orientation of the wake with respect to the freestream flow, thereby spreading the main source of heat enhancement – the vortices near the cylinder base – over a larger portion of the cylinder surface. Previously observed heat transfer enhancement associated with oscillations at frequencies near the natural shedding frequency and its harmonics were shown to be limited to amplitudes of less than about 0.5 cylinder diameters. A new phenomenon was discovered in which the wake structure switches back and forth between distinct wake modes. Temperature induced variations in the fluid viscosity are believed to be the cause of this mode-switching. It is hypothesized that the viscosity variations change the vorticity and kinetic energy fluxes into the wake, thereby changing the wake mode and the heat transfer coefficient. This discovered underscores the role of viscosity and shear layer fluxes in determining wake mode, potentially leading to improved understanding of wake vortex formation and pinch-off process in general. The heat transfer is also affected by aspect ratio for oscillation conditions characterized by weak synchronization of the wake to the oscillation frequency. Additional research into mode-switching needs to be performed. Identifying the criteria for the occurrence of mode-switching would reveal a great deal about wake formation processes.

Recently, Recicar *et al.* (2006) handled vortex elements to deals with the analysis of a circular cylinder oscillating around a fixed position which is located in an incoming uniform flow with constant velocity; to simplify matters the oscillatory motion was restricted to heave. The numerical experiments were carried out at a Reynolds number of  $1.0 \times 10^5$ . Due to the alternate vortex shedding the lift coefficient oscillated, around zero, during the numerical simulation; the amplitude of the lift coefficient oscillation was increased with the cylinder oscillation keeping, however, the mean value almost identically to zero. It is also possible to identify three different types of flow regime as the cylinder oscillation frequency increases. The first type – Type I - is observed for low frequency range of the cylinder oscillation; in this situation the Strouhal number remains almost constant. Type I is followed by an intermediate range of frequency – Type II, the transition regime - where apparently the shedding frequency does not correlate to the frequency of the cylinder oscillation. Finally in Type III – high frequency of cylinder oscillation – the vortex shedding frequency is locked-in with the cylinder oscillation frequency (as will be plotted later in "Fig. 2").

In this paper, the vortex method (Recicar *et al.*, 2006) is employed to simulate numerically the unsteady flow and heat transfer (Alcântara Pereira and Hirata, 2003) around and oscillating circular cylinder which moves with constant velocity in a quiescent Newtonian fluid. The two-dimensional aerodynamics characteristics are investigated at a Reynolds number of  $1.0 \times 10^5$ . The main purpose of present numerical study is the investigation of the relationship between near-wake structure and heat transfer for the Type I of flow regime (Recicar *et al.*, 2006).

The authors of this paper has developed the present vortex method to simulate the macro scale phenomena, therefore the smaller scale ones are taken into account through the use of a second order velocity function (Alcântara Pereira et *al.*, 2002). In the present paper, the effects of small scale are not still considered.

# 2. GENERAL FORMULATION AND NUMERICAL METHOD

The purpose of this study is to understand heat transfer from transversely oscillating circular cylinder in cross-flow. It is intuitive to believe that oscillating a cylinder in a fluid will increase its heat transfer from the surface. Herein we consider the incompressible flow of a Newtonian fluid around a moving circular cylinder in an unbounded two-dimensional region. Also for all flows considered temperature variation has not a negligible impact on the flow field.

"Figure 1" shows the incident flow, defined by free stream speed U with constant temperature  $T_{\infty}$  and the domain  $\Omega$  with boundary  $S = S_1 \cup S_2$ ,  $S_1$  being the body surface at constant temperature  $T_w$  and  $S_2$  the far away boundary. The cylinder moves to the left with constant velocity; an oscillatory motion with finite amplitude A and constant angular velocity  $\lambda$  is added to body motion. In this figure the (x,o,y) is the inertial frame of reference and the ( $\eta$ ,O, $\xi$ ) is the coordinate system fixed to the cylinder; this coordinate system oscillates around the x-axis as  $y_0=Acos(\lambda t)$ .

The evolution of such a fluid is governed by the following relations for conservation of mass, momentum and energy respectively

 $div \mathbf{u} = 0$ 

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla \mathbf{p} = \upsilon \nabla^2 \mathbf{u} + \mathbf{f}$$
<sup>(2)</sup>

$$\frac{\partial \mathbf{T}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{T} = \frac{\mathbf{k}}{\rho \mathbf{c}_{p}} \nabla^{2} \mathbf{T} \,. \tag{3}$$

In the equations above **u** is the velocity vector field, p is the pressure, **f** is body force,  $\upsilon$  is the fluid kinematics viscosity coefficient, k is the thermal conductivity and  $\rho c_p$  is the volumetric heat capacity (k/ $\rho c_p$  is the thermal diffusivity). "Equation (3)" is according Boussinesq's approximation.



Figure 1. Transversely oscillating cylinder in uniform freestream flow

In this formulation it is necessary to solve for both the pressure, temperature and two velocity components in order to evolve the flow. This can complicate computational approaches and it proves favorable to consider the evolution equation for the curl of the velocity, the vorticity, instead

$$\boldsymbol{\omega} = \nabla \times \mathbf{u} \ . \tag{4}$$

For two-dimensional flow, the vorticity vector has only one component and thus can be expressed as a scalar field  $\omega(x,y)$ .

Taking the curl of "Eq. (2)" and applying "Eq. (1)" and the two-dimensionality constraint yields

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = \upsilon \nabla^2 \omega + \nabla \times \mathbf{f}$$
(5)

which is the evolution of the scalar vorticity field.

In order to determine a flow from the vorticity equation, "Eq. (5)", it is necessary to find a velocity field in terms of the vorticity field. To do so, consider decomposing the velocity vector into two fields

$$\mathbf{u} = \nabla \times \mathbf{\psi} + \nabla \phi \tag{6}$$

which  $\psi$  is the vector streamfunction and  $\phi$  is the velocity potential. Now consider the curl of "Eq. (6)"

$$\boldsymbol{\omega} = \nabla \times \nabla \times \boldsymbol{\psi} + \nabla \times \nabla \boldsymbol{\phi} = -\nabla^2 \boldsymbol{\psi} + \nabla (\nabla \cdot \boldsymbol{\psi}) \tag{7}$$

Note that the velocity potential serves to include components of the velocity field which cannot be represented in the vorticity field. For two-dimensional flow, "Eq. (7)" simplifies to

$$\omega = -\nabla^2 \psi \,. \tag{8}$$

The Green's function for the two-dimensional Laplacian can be now convolved with the stream-function to give a vorticity-based representation of the streamfunction as follows

$$\Psi(\mathbf{x}) = -\frac{1}{2\pi} \iint \ln \left| \mathbf{x} - \mathbf{x}' \right| \omega(\mathbf{x}') d\mathbf{x}'$$
(9)

Now "Eq. (9)" can be used in "Eq. (6)" to yield the relation commonly known as the Biot-Savart law

$$\mathbf{u}(\mathbf{x}) = -\frac{1}{2\pi} \iint \frac{(\mathbf{x} - \mathbf{x}') \times \omega(\mathbf{x}') \hat{\mathbf{z}}}{\left|\mathbf{x} - \mathbf{x}'\right|^2} d\mathbf{x}' + \nabla\phi$$
(10)

The foundation of vortex methods rests on the use of "Eq. (5)" and "Eq. (10)" to track a fluid based on the evolution of this vorticity field. The vorticity equation is free of the computational instabilities associated with the convective term. In the inviscid case without body forces and frame of reference acceleration

$$\frac{D\omega}{Dt} = 0.$$
(11)

Computational simulation requires the discretization in space and time of "Eq. (5)" and "Eq. (10)". Particles strengths remain constant to satisfy "Eq. (11)", so

$$\frac{\mathrm{d}x_{i}}{\mathrm{d}t} = u(x_{i}) \text{ and } \frac{\mathrm{d}\Gamma}{\mathrm{d}t} = 0, \qquad (12)$$

where the amount of vorticity carried by a given particle is termed its circulation and represented by  $\Gamma$ .

The vorticity convection is governed by "Eq. (11)" and the velocity field is given by (Recicar et al., 2006)

$$\mathbf{u}(\mathbf{x},\mathbf{t}) = \mathbf{u}\mathbf{i}(\mathbf{x},\mathbf{t}) + \mathbf{u}\mathbf{b}(\mathbf{x},\mathbf{t}) + \mathbf{u}\mathbf{v}(\mathbf{x},\mathbf{t}) . \tag{13}$$

Note that "Eq. (3)" gives the law that the temperature distribution, T, moves both with the convection velocity. Vortex elements and discrete heat elements distributed in the flow field are followed during numerical simulation according to the first order Euler scheme. It is clear that the energy equation, "Eq. (3)", has the similar form to the vorticity transport equation, "Eq. (5)". This suggests that the energy equation can be solved in an analogous way using the random walk method to the motion of the heat elements to account for diffusion (Chorin, 1973).

In this paper, the temperature  $T_w$  is considered constant around the body surface, see "Fig. 1". The heat transport from the body surface to the fluid nearby the body surface is determined by the temperature gradient at the surface. The effect of buoyancy is not considered here because our study is focused on the forced convection heat transfer (Alcântara Pereira and Hirata, 2003). The surface heat flux is determined by Fourier's Law

$$\dot{q} = -\lambda \frac{dT}{dn}$$
(14)

where n denotes the normal direction to the surface and  $\lambda$  is the thermal conductivity of fluid. The heat quantity transfered from the surface (j-th panel with length  $\Delta S_i$ ) to the k-th nascent heat element is given by

$$\Delta Q_{j} = \alpha \Delta t \, \frac{\left(T_{w} - T_{j}\right)}{\varepsilon} \Delta S_{j} \tag{15}$$

in which  $\alpha = \nu/Pr$  (Pr is Prandtl number) and  $\varepsilon$  is the displacement normal to the straight-line panel.

The temperature distribution T(z) results from the contribution of all the heat particles in the field

$$T(z) = \sum_{j} \frac{\Delta Q_{j}}{\pi \sigma_{T}^{2}} \exp\left[-\frac{(z - z_{j})^{2}}{\sigma_{T}^{2}}\right]$$
(16)

where  $\sigma_T$  is the core radius of the heat particles

The pressure calculation starts with the Bernoulli function, defined by Uhlman (1992) as

$$Y = p + \frac{u^2}{2}, \ u = |\mathbf{u}|.$$

$$\tag{17}$$

Kamemoto (1993) used the same function and starting from the Navier-Stokes equations was able to write a Poisson equation for the pressure. This equation was solved using a finite difference scheme. Here the same Poisson equation was derived and its solution was obtained through the following integral formulation (Shintani and Akamatsu, 1994)

$$H\overline{Y_{i}} - \int_{S_{1}} \overline{Y}\nabla G_{i} \cdot \boldsymbol{e}_{n} dS = \iint_{\Omega} \nabla G_{i} \cdot (\boldsymbol{u} \times \boldsymbol{\omega}) d\Omega - \upsilon \int_{S_{1}} (\nabla G_{i} \times \boldsymbol{\omega}) \cdot \boldsymbol{e}_{n} dS$$
(18)

where H is 1.0 inside the flow (at domain  $\Omega$ ) and is 0.5 on the boundary S<sub>1</sub>. G<sub>i</sub> =  $(1/2\pi)\log R^{-1}$  is the fundamental solution of Laplace equation, R being the distance from ith vortex element to the field point.

It is worth to observe that this formulation is specially suited for a Lagrangian scheme because it utilizes the velocity and vorticity field defined at the position of the vortices in the cloud. Therefore it does not require any additional calculation at mesh points. Numerically, "Eq. (18)" is solved by mean of a set of simultaneous equations for pressure  $Y_i$ .

#### 4. RESULTS AND DISCUSSIONS

When the Vortex Method is applied to heat-fluid motion, it becomes evident that the procedure is very sensitive to the numerical parameters involved. The main influences are: the non-dimensional time step, t\*=tU/d (where d is the cylinder diameter); distance of release particles from surface; particles blob radius and time increment. Numerical simulations for flow and heat transfer was performed around a circular cylinder at Re=  $1.0 \times 10^5$  and Pr= 0.71. The number of source panels around a circular cylinder was M= 100, the dimensionless time was  $\Delta t$ = 0.05. A constant temperature T<sub>w</sub>= 363 K on the body surface as a boundary condition, and the freestream temperature was set to T<sub>∞</sub>= 293 K. The standard numerical strategy is to represent the vorticity in the fluid domain by a large number N of small discrete vortices  $\Delta \Gamma_k$ . The numerical analysis is conducted over a series of small discrete time steps  $\Delta t$  for each of which a discrete vortex element  $\Delta \Gamma_k$  is shed from each body surface element. The intensity  $\Delta \Gamma_k$  of these newly generated vortices is determined using the no-slip condition. The core radius was  $\sigma_0$ = $\epsilon$ =0.0032d for nascent vortex elements and  $\sigma_T$ = $\epsilon$ =0.09d for nascent heat elements.

Recicar *et al.* (2006) showed that in general the high Reynolds number simulations agree quite well the low Reynolds number vortex synchronization regions devised by Williamson and Roshko (1988). "Figure 2" shows that the fluid flow develops according to three possible modes. In this paper we concentrate attention for the Mode I, which is defined by low values of St<sub>b</sub> (low frequency of oscillations) – the flow behaves much like if the body were fixed and the amplitude of the cylinder oscillation, A, has negligible influence on the vortex shedding frequency f. The Strouhal number assume values very close to 0.20, that is  $S_t=O(0.2)$  as  $f_b\rightarrow 0$ .

"Figure 3" shows time histories of drag coefficient,  $C_D$  and lift coefficient,  $C_L$ , of the circular cylinder. Both values of drag and lift are defined by the sum of pressure forces. "Figure 3(a)" indicate that the fluctuation of  $C_D$  have double frequency that of  $C_L$ , because it fluctuate once for each of upper and lower shedding.

"Figure 4(a)" shows temperature distribution for circular cylinder at t= 26.0. This instant is defined in "Fig. 3(a)" and represent high pressure distribution on the rear part of the cylinder when the drag curve assumes a maximum value. To confirm the applicability of the present method to simulation in the separated flow region, the heat transfer from transversely oscillating circular cylinder in cross-flow was simulated and the result was compared with "Fig. 4(b)" at t= 24.4. The result of the simulation at t=24.4 seems a higher transport of heat in the near wake when compared with stationary cylinder, "Fig. 4 (a)". The value of fluctuating Nusselt number will be carried out.

From the study of the heat convection from a cylinder as function of forced oscillation frequency and amplitudes, we need to investigate the relation between the incensement of natural vortex shedding frequency and heat transfer.

Pottebaum (2003) founded that wake structure and heat transfer both significantly affect one another. The wake mode, a label indicating the number and type of vortices shed in each oscillation period, is directly related to the

observed heat transfer enhancement. The dynamics of the vortex formation process, including the trajectories of the vortices during roll-up, explain this relationship.



Figure 2. Strouhal number behavior as a function of the body oscillation frequency (Recicar et al., 2006)

According to the Pottebaum (2003), the cylinder's transverse velocity was shown to influence the heat transfer by affecting the circulation of the wake vortices. For a fixed wake structure, the effectiveness of the wake vortices at enhancing heat transfer depends on their circulation. Also, the cylinder's transverse velocity continually changes the orientation of the wake with respect to the freestream flow, thereby spreading the main source of heat transfer enhancement – the vortices near the cylinders base – over a large portion of the cylinder surface.



Figure 3. Time histories of  $C_D$  and  $C_L$  of a circular cylinder for Re=1.0×10<sup>5</sup>

Because the distributed vorticity and heat of the mainstream flow has been replaced in the numerical model by two clouds of particles, the CPU time for particle-particle interaction turns expensive. No attempts to simulate the flow for M greater than 100 were made since the operation count of our algorithm is proportional to the square of N. As M increases N also tends to increase, and the computational efforts becomes expensive. This is a major source of difficulties, and it can only be handled through the utilization of faster schemes for the induced velocity calculations,

such as the multipole technique (Greengard and Rokhlin, 1987) and/or parallel computers to run long simulations (Takeda *et al.*, 1999). "Figure 5" shows the distribution of 48800 vortex elements at t=24.4.

Finally, the results are promising and encourage performing additional tests in order to explore the phenomena in more details.



(a) t=26.0: A=0 and  $\lambda$ =0

(b) t= 24.4: A=0.15 and  $\lambda$ =0.30





Figure 5. Distribution of vortex elements for Re=  $1.0 \times 10^5$  (t = 24.4, when A=0.15 and  $\lambda$  = 0.30).

# **5. CONCLUSIONS**

In the present study, a vortex and heat element method was presented for the analysis of unsteady heat transfer in a flow around a stationary and oscillating body. The calculated value for aerodynamics forces around a circular cylinder showed good agreement with the data from the literature. The differences encountered in the comparison of the results simulation with the experimental are attributed mainly to the inherent three-dimensionality of the real flow. Use of a larger number of panels distributed on the body surface can also improve the results, but for this is necessary a larger number of vortex and heat elements in the cloud and consequently a larger computational effort. The present calculation required 28 h of CPU time in a PENTIUM II/400 Mhz. The use of a fast summation scheme to compute the velocities of the particles, such as the multiple expansions, allows an increase in the number of vortex elements and heat elements and a reduction of the time step, which increases the resolution of the simulation to a reduction of the CPU time, which allows a longer simulation time to be carried out. The main objective of the work with the implementation of a vortex and heat particles method for the analysis of unsteady and forced-convective heat transfer in a flow around a stationary and oscillating body has been achieved. The present methodology, therefore, is able to provide good estimates for Strouhal number, lift and drag coefficients, pressure distribution and time-averaged Nusselt number (Alcântara Pereira and Hirata, 2003), and is able to predict the flow correctly in a physical sense. As a future work, the effect of buoyancy will be carried out. A new method to simulated diffusion will be carried out (Rossi, 2006).

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