FILM COOLING TEMPERATURE ESTIMATION OF A 200N HYDRAZINE THRUSTER BY AN INVERSE PROBLEM APPROACH

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Abstract. *The heat flux and the temperature distribution on the film cooling near the internal wall of a thruster are difficult quantities to estimate and measure. In the present paper, we describe an inverse problem approach to the estimation of the temperature distribution on the film cooling of a thruster without regenerative cooling. The external wall temperature along the thruster is considered as a known input, and is the target function to be retrieved by optimization of the film cooling profile parameters, which in turn defines the temperature distribution on the film. The inverse problem is solved by using a hybrid version of the Generalized Extremal Optimization (GEO) and Evolution Strategies (ES) algorithms linked with the INPE´s thermal software (PCTER). The profile was obtained for steady-state conditions and the Boltzmann's type profile parameters, which define the film cooling temperature profile, are the design variables. Results using simulated data showed that this approach was efficient in recuperating those parameters. The approach presented in this work can be used on the design of thrusters with lower wall temperatures, which is a desirable feature of such devices.*

Keywords: *film cooling, thruster, optimization, generalized extremal optimization, inverse problem.*

1. INTRODUCTION

The development of low thrust bipropellant rocket engines presents a difficult problem regarding the heat load on the thruster wall. Since the ratio of wall surface area to propellant mass flow rate is inversely proportional to the engine thrust and chamber pressure, the regenerative cooling of the wall is not a viable option for engines with thrust lower than approximately 50 kN (this limit depends on the type of propellants and chamber pressure).

For most propellant pairs, the adiabatic combustion temperature for stoichiometric mixture ratio, is much higher than the allowable temperature limit of the chamber material.

The designers task is to find a compromise solution between the energetic efficiency (near stoichiometric mixture ration) and wall heat load (which requires a fuel or oxidizer rich film near the wall).

The heat load on the thruster wall depends on the temperature of the combustion products in the gas film near the wall, and the heat transfer coefficient. The determination of these parameters by analytical methods is subject to very large errors due to difficulties in modeling the atomization, mixing and combustion processes inside the chamber. The experimental measurement of the heat transfer coefficient and near wall gas film temperature is also difficult to perform.

In this paper we describe a method to estimate the near wall gas film temperature from the measured outside wall temperature during the wall heating transient. The success of the method depends on the hypothesis that the aero-thermodynamic process inside the chamber is much shorter than the wall heating transient.

The near wall film temperature profile is approximated by a Boltzmann type equation. The heat transfer coefficient is calculated by a Bartz equation (Dieter and David, 1992). A hybrid of two evolutive optimization algorithms is used to determine the coefficients of the Boltzmann equations that best match the outside wall temperature profile during the initial thermal transient.

The test article is a 200 Newton thrust bipropellant engine, under development at INPE, for use in apogee accelerator block and launch vehicle roll control. The propellant pair used is Nitrogen Tetroxide and Monomethyl Hydrazine. The o/f mixture ratio is varied on the range of 0.6 to 1.3. The adiabatic combustion temperature for this mixture ration goes from 2000 K to 2500 K. The thrust chamber was manufactured in Inconel 600 and the working temperature limit is 1300 K.

The external wall temperature time history used to compare the results is simulated using the INPE's thermal software (PCTER) which a know problem.

2. THE 200N HYDRAZINE THRUSTER

The thruster in development at INPE (Fig. 01) have 200N of impulse and is intended for use in the apogee acceleration block of geostationary satellites.

The thruster wall is machined in Inconel 600. The total length of the thrust chamber is 210mm. The combustion chamber is cylindric with an internal diameter of 42mm. The fuel is monomethyl hydrazine and the oxidizer is nitrogen tetroxide. For an O/F mass ration of 1 the adiabatic temperature of the combustion products for this propellant pair is 2360 K.

The thruster will be fire-tested in a test stand with altitude simulation. The outer wall temperature is measured in a discreet number of points with type K thermocouples. An infrared camera will be used to monitor the entire external wall temperature.

The thermal protection mechanism for the thruster wall is a film cooling on the inside and thermal radiation on the outside. To be able to determine with good precision the heat load on the inside of the wall we must know the temperature of the gas layer near the wall. The heat transfer convective coefficient is determined in internal surface by Bartz`s equation (Dieter and David, 1992). The direct measurement of these quantities, or its determination from analytical models, is very difficult to be realized.

The determination of the heat load on the inner surface, from the temperature profile of the outer surface of the wall leads to an inverse problem. An attempt at solving this inverse problem using the GEO+ES hybrid algorithm is the essence of the work presented here.

The knowledge of the heat load on the thruster chamber wall is very important to determine a safe range of operating parameters of the thruster regarding mixture ratio, amount of fuel used in the film cooling and mechanical properties of the wall material at high temperature. The thermal model and the optimization algorithm has been used to simulate this temperature using thermal balance and comparing the external wall temperature simulated with the measured in the infrared camera.

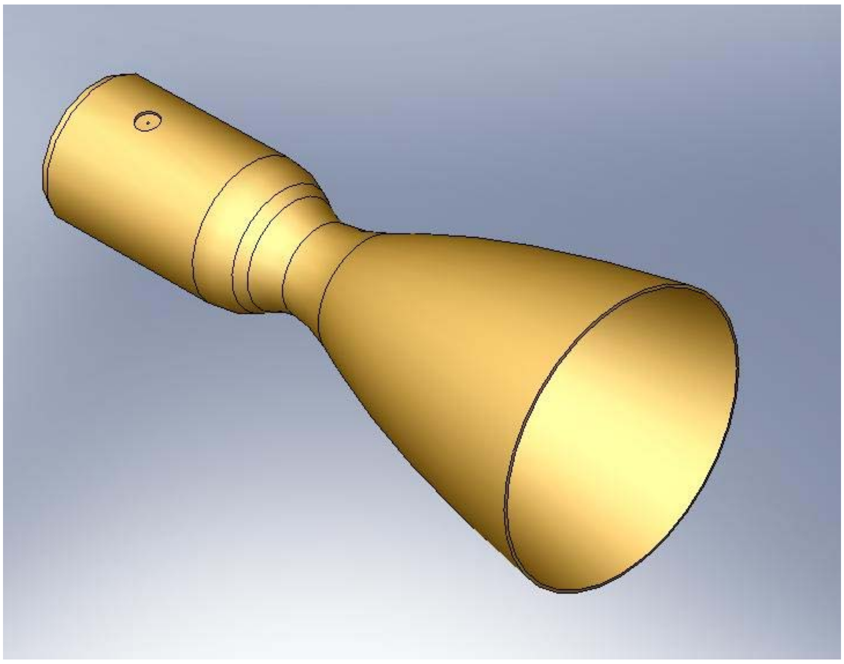

Figure 1. Perspective view of the INPE 200N thruster

3. THERMAL MATHEMATICAL MODEL

The thruster thermal model used in this work was constructed using a lumped parameter network formulation (Gilmore, 1994). In this method, the thruster is divided into a number of nodes, which are assumed isothermal. A thermal network is drawn connecting the nodes. A governing thermal energy expression is written for each node, resulting in a system of coupled equations whose solution yields the temperature of nodes. For the 200N hydrazine thruster, the thermal model was constructed using INPE´s PCTER thermal software package (Cardoso *et al.,* 1990).

To apply the optimization technique, a lumped parameter model was constructed for the thruster using 160 nodes, as shown on Fig. 2 and Fig. 3. As the thruster has radial symmetry along the longitudinal axis, only half of the longitudinal section needs to be modeled and the resulting model is bidimensional.

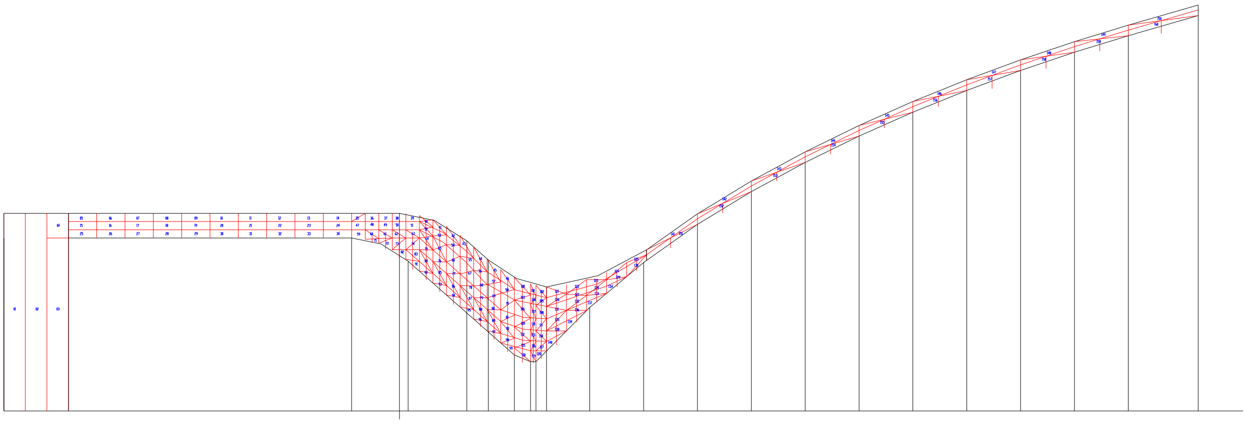

Figure 2. Geometric distribution of the nodes

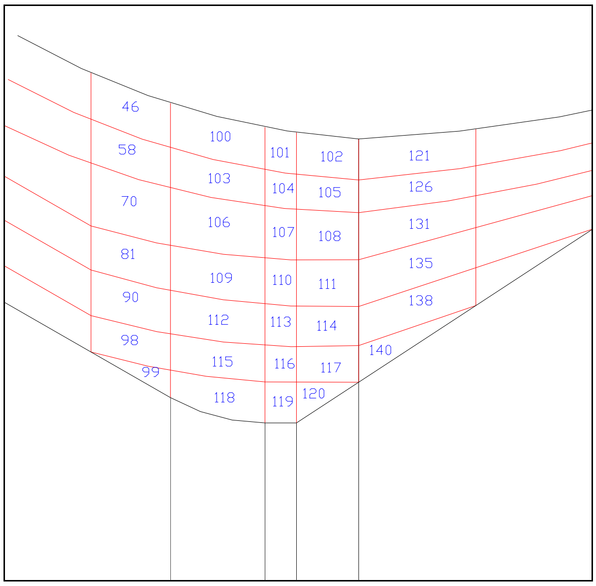

Figure 3. Zoom of the throat of geometric distribution of the nodes

Heat from the hot gases inside the thruster is exchanged by radiation and convection with the thruster's internal wall, then is transferred by conduction to the external surface of the thruster and then to space by radiation. (external boundary conditions temperature).

Using the lumped parameter representation (Gilmore, 1994) and assuming steady state conditions, the heat balance at each one of the nodes takes the form of the following system of equations:

$$\overbrace{k\frac{dT}{dx}+k\frac{dT}{dy}}^{(1)}+\overbrace{\sum_{j=1}^{n+1}R_{ji}\sigma\varepsilon\left(T_j^4-T_i^4\right)}^{(2)}+\overbrace{h_i dT_i}^{(3)}=0 \qquad (1)$$

Where labels (1), (2) and (3) in Eq. (1), are the conductive, radiative and convective terms, respectively. k is the thermal conductivity, h is the thermal convective factor, R is the geometric configuration factor between nodes, ε is the emissivity of the material, σ is the Stefan-Boltzmann constant, T is the temperature of the node, x is the longitudinal distance along the thruster and y is the radial distance of the thruster.

As is usual in this type of thruster, the propellant is injected from the periphery of the injector plate, as a way to create a thin cooling propellant film on the inner surfaces of the thruster, protecting them. The film temperature profile along the length of thruster is approximated by a Boltzmann type equation (Koslov and Hinckel, 1998) in the following form.

$$y = \frac{A_1 - A}{1 + e^{(x - x_o)/dx}} + A \qquad (2)$$

Where y is the film temperature, x is the longitudinal distance along the thruster, x_0 locates the average temperature between A and A_1 temperature valor, dx is the longitudinal distance which shows how fast is the change from the point where the film cooling temperature goes from level A_1 to level A. A and A_1 is the asymptotic temperatures.

When the set of parameters $\{x_0,dx,A_1,A_2\}$ is known, the film cooling temperature profile is obtained and the external wall temperature can be calculated using the direct model. When the set of parameters $\{x_0,dx,A_1,A_2\}$ is not known, but experimental data about the temperatures on the outer surface of thruster is available, then it is possible to formulate an optimization problem in which $\{x_0,dx,A_1,A_2\}$ are the unknowns and the objective is to find the values of them that lead to the best match between the observed data (measured temperatures) and the corresponding data obtained by solving Eq. 1. This is the inverse problem. In this paper, INPE PCTER thermal software package (Cardoso *et al.*, 1990) is used for solving Eq. 1.

4. OPTIMIZATION ALGORITHM

In the following, ES and GEO stand alone algorithms are briefly described, being followed by the description of the GEO + ES hybrid.

4.1 ES Algorithms

ES is an optimization technique based on the ideas of adaptation and evolution, being another member of the family of the Evolutive Algorithms (EA) (Eiben and Smith, 2003). ES methods use real coded vectors (non binary) and mutation mainly, among others, as operators. As it is common in EA, the operators are applied in order: recombination, mutation, evaluation of the adaptation function and natural selection. Applying this loop one time is called a generation, and it is repeated until a stop criterion is reached.

The mutation, in its simpler version, is obtained by adding to each design variable vector component values coming from the same Gaussian distribution. The size or intensity of this mutation, i.e., the standard deviation σ of the Gaussian distribution, usually vary during the search, evolving together with the design variables, in a process known as self-adaptation. The self-adaptation is perhaps the main contribution from ES methods to the evolutionary algorithms.

According to Eiben and Smith (2003), the essential characteristics present in the ES are: (i) ES methods are, in general, used for optimizing problems with continuous design variables; (ii) Mutation is the main operator used to generate new individuals; (iii) Design variables mutation is implemented by adding noise coming from a Gaussian distribution and; (iv) The parameters that control the mutation are modified along the algorithm execution.

4.2 GEO Algorithms

Generalized Extremal Optimization (GEO) is a recently developed evolutionary algorithm (De Sousa et al., 2003) that has been successfully applied to real world optimal design problems.

In Galski (2006), four improvements for the canonical GEO algorithm were suggested. One of them, called GEO_4, uses real coded variables instead of the binary ones used by the canonical GEO.

All GEO versions and ES share the characteristic of using mutation as the main operator to generate individuals. The GEO_4 version is the only one that shares with the ES also the characteristic of using real instead of binary codification of the design variables. By this reason and also because of its good performance results with test functions, in terms of convergence properties, GEO_4 was chosen to form the GEO + ES hybrid to be presented here.

4.3 GEO + ES hybrid algorithm

In the ES, the parameter σ defines the size of the mutations that affect **X**, the design variables vector and, as a consequence, defines also the locality of the search (Eiben and Smith, 2003). In GEO_4, the locality of the search is defined by the parameter b (Galski, 2006). This way, by analogy to the ES, the idea is to apply to the parameter b a variation mechanism similar to that used in the ES for the parameter σ. However, preliminary tests using function tests (Galski, 2006) have shown that the best way of mutating b is not by multiplication of a random variable with lognormal distribution, as ES do, but by the addition of a random variable with a Gaussian distribution. Then:

$$b^n = b^{n-1} + gauss(\delta, \alpha) \qquad (3)$$

Where the subscripts n-1 and n refers to the value of b before mutation and after mutation, respectively.

Besides the learning rate, α, Eq. 3 presents yet the parameter δ, that is the mean of the Gaussian distribution used to mutate δ. It indicates the bias imposed in the mutation of b. If δ =0, there is no bias. Imagining a search starting with

$b=b_{MIN}$ (>1), then, using $\delta > 0$ generates, to the end of many generations, a schedule with stochastically increasing values for b, even that not monotonic ones (except if $\alpha = 0$). Since that low values for b impose a sparse search and high values for b impose a local search, then a scheduling with increasing values for b means a search that starts sparse, when $b \sim b_{MIN}$, and that ends local, when b>>1. The idea behind using $\delta \neq 0$ is to allow a better tuning flexibility to the algorithm at the expenses of tuning one more algorithm parameter. In the cases where this flexibility is not wanted or needed, it is just can be used the same idea adopted by the vast majority of the ES, i.e., to set $\delta = 0$.

Regarding the way of muting **X**, the same sistematic adopted for GEO_4 is used (Galski, 2006).

An important change introduced in GEO + ES, regarding GEO_4, is the use of a selection by elitism instead of stochastic selection with τ. Selection by elitism, in case of GEO_4, implies only selecting the best among all the L individuals generated on each iteration, not requiring, this way, any additional parameter.

The steps of the GEO + ES hybrid used in this paper are given by:

1. Initialize randomly with uniform distribution between $\mathbf{X_{min}}$ and $\mathbf{X_{max}}$ a vector **X** containing the N design variables. Calculate the objective function F(**X**), do $\mathbf{X_{best}}=\mathbf{X}$ and save $F(\mathbf{X_{best}})$.
2. Define the number of mutations l_j, $j \in \{1,2,...,N\}$ for each variable X_j, such that $\sum_j l_j = L$, being L the total number of mutations acting on the N design variables. Define values for the parameters δ and α. Define values for the limits b_{MIN} and b_{MAX}, of the base, with $b_{MIN}>1$. Do $b=b_{MIN}$.
3. Calculate the vector **ε**, where $\varepsilon_j \equiv (X_{MAXj} - X_{MINj})/(b^{l_j}-1)$ and j is the variable index, i.e., $j \in \{1,2,...,N\}$. The element ε_j defines the mutation resolution of the j-th design variable. By resolution understand the least value to be added to or subtracted from X_j.
4. Do F(X)ref= F(X), F(Xbest)ref= F(Xbest).
5. For each design variable $j \in \{1,2,...,N\}$ of vector X, do:
  a. Draw from uniform distribution a value for $c \in \{0, 1\}$.
  b. For each mutation $i \in \{1,2,..., l_j\}$ of the variable X_j, do:
    b.1. Calculate the mutation size $m=b^{(i-1)}\varepsilon_j$ and the mutation sign $s=(-1)^{(i-c)}$, where c is the value obtained in the step 5.a.
    b.2. Mutate X_j. First, do $X_{aux}=X_j$. Then, do $X_j=X_j+s \cdot m$, generating a mutated vector $\mathbf{X_i}$. Verify the limits: If $X_j>X_{MAXj}$ or $X_j<X_{MINj}$, then draw a new $X_j \in [X_{MINj}, X_{MAXj}]$ with uniform distribution and do $m= abs(X_j-X_{aux})$ and $s=(X_j-X_{aux})/m$.
    b.3. Calculate the objective function value $F(\mathbf{X_i})$. Atribute to the mutation i an adaptation value $\Delta F(\mathbf{X_i})=F(\mathbf{X_i})-F(\mathbf{X_{best}})_{ref}$, that indicates the gain or loss the objective function has if the mutation i occurs, when compared with the best objective function value found up to the previous iteration. Next, if $F(\mathbf{X_i})<F(\mathbf{X_{best}})$ then do $F(\mathbf{X_{best}})=F(\mathbf{X_i})$ and $\mathbf{X_{best}}=\mathbf{X_i}$.
    b.4. Return **X** to its non mutated condition: do $X_j= X_j-s \cdot m$.
  c. Choose, within the l_j mutations generated in the step 5.b, the one with the minor $\Delta F(\mathbf{X_i})$. Save the respective X_j at the j-th element of the vector $\mathbf{X_c}$.
6. Perform the mutations of the present iteration: do $\mathbf{X}=\mathbf{X_c}$, where $\mathbf{X_c}$ is the resulting vector from the execution of the step 5.c for each variable $j \in \{1,2,...,N\}$.
7. Verify the base adaptation: Calculate the new F(**X**) value. If $F(\mathbf{X}) \geq 0.99 \cdot F(\mathbf{X})_{ref}$ then do b=b+y, with y=gauss(δ,σ), where gauss(δ,α) is the value drawn from an uniform distribution with mean δ and standard deviation α. Verify base limits: If $b>b_{MAX}$ or if $b<b_{MIN}$, do $b=b_{MIN}+gauss(0,1)\cdot(b_{MAX}- b_{MIN})$.
8. Repeat the steps 3 to 7 until a given stop criterion be satisfied.
9. Return $\mathbf{X_{best}}$ and $F(\mathbf{X_{best}})$.

When GEO + ES is compared to the canonical GEO, the main differences can be summarized by:
- Real valued vector instead of a binary string is used to represent the design variables;
- Mutations on the design variables are done by magnitudes (any base) instead of bits (base 2);
- The number of mutations is dissociated from the numeric precision of the design variables;
- Absence of the τ parameter: Selection is performed by elitism, instead of stochastically;
- Adaptedness of the base b, which controls both the locality and the stochasticity of the search;
- Existence of three adjusting parameters: α, the learning rate, δ, the learning bias, and lj, the number of mutations.

5. OPTIMATION PROBLEM FORMULATION

Mathematically, the optimization problem described in the last paragraph of Section 3 is stated as follows:

$$Minimize \quad F(\mathbf{X}) = \left\| \mathbf{T}s(\mathbf{X}) - \mathbf{T_D} \right\|_2 \tag{4}$$

$$Subject\ to: \quad \mathbf{X_{MIN}} \leq \mathbf{X} \leq \mathbf{X_{MAX}} \tag{5}$$

Where the objective function, F(**X**), is the Euclidian norm of the difference vector between the calculated and the given temperatures on the external surface of the thruster wall. $\mathbf{T_S}$ is the given temperature profile and T_D is the calculated one. The vector **X** is the design variables vector, i.e., $\mathbf{X} = [x_0, dx, A_1, A_2]$, that must remain between the side limits $\mathbf{X_{MIN}}$= [1.0, 1500.0, 0.19, 0.005],and $\mathbf{X_{MAX}}$.= [40.0, 2400.0, 0.57, 0.3].

6. RESULTS

In order to evaluate the performance of the GEO + ES hybrid in the solution of the optimization problem described in section 5, the set {A,A1,x0,dx}={23.0, 2087.0, 0.41, 0.238} was considered and experimental data was synthetically generated using the PCTER software, as real experimental data on the outer wall temperature was not available. In this way, the optimal solution is known beforehand.

Three mutations per variable were used, so lj=l=3 and L=12. The limits for varying b were set to b_{MIN}=1.05 and b_{MAX}=10. The values of δ=0.0 and of α=0.3 were used. After that, GEO + ES was run 10 times, each one with a different and random starting point. For each run, the limit of 5×10^4 evaluations of F(**X**) was used as the stopping criterion. Each run took approximately 3.6 hours on an AMD Athlon (1.1GHz) PC computer with 896MB of RAM memory.

The ten solutions found with the help of GEO + ES plus the Boltzmann exact solution are presented in Table 1. As can be seen from the Table, the greatest parameter value variation occurred for the parameter A.

Table 1. Inverse problem solutions

Run number	X1 (A)	X2 (A1)	X3 (x0)	X4 (dx)	F(**X**)
1	15.37	2092.6	0.4096	0.2403	5.10
2	27.82	2089.0	0.4119	0.2382	1.42
3	20.83	2089.9	0.4105	0.2398	1.80
4	30.81	2081.3	0.4110	0.2372	1.56
5	27.16	2089.7	0.4122	0.2386	1.21
6	37.89	2080.4	0.4123	0.2355	2.28
7	29.40	2090.2	0.4125	0.2383	3.10
8	26.51	2092.2	0.4126	0.2392	1.70
9	34.49	2082.3	0.4122	0.2363	1.99
10	26.94	2090.9	0.4125	0.2390	1.54
BOLTZ.	**23.00**	**2087.0**	**0.4100**	**0.2380**	**0.00**

Figure 7 shows the 10 solutions retrieved by the GEO + ES hybrid, in terms of the resulting film cooling temperature profiles, identified as C01 to C10. For each one, the graph gives also the respective F(**X**) final value. The original temperature profile is also presented, identified as "BOLTZMANN". As can be seen, all the solutions match very well the original profile, up to a point that they can't be seen isolatedly, meaning that the optimization algorithm was able to solve the inverse optimization problem very well for all the ten runs. It indicates that the algorithm is not sensitive to the random starting point used to start the search. This is also confirmed by the numerical values obtained for F(**X**). Even the worst one, that happened to be C01, had F(**X**)=5.10ºC.

Considering that F(**X**) is, in fact, a difference vector composed of 41 elements, and if it is assumed that the differences have homogeneous distribution among all the 41 elements, then, it would mean a discrepancy of just 5.10/ $\sqrt{41} \approx$ 0.8ºC per element (sensor) for the worst match and only 1.21/ $\sqrt{41} \approx$ 0.19ºC per element (sensor) for the best match.

Figure 8, by its turn, shows the complete profile of temperatures. For the external wall and for the film cooling two temperature profiles are presented. One given by the Boltzmann original parameters (exact solution) and the other given by the best solution found obtained by GEO + ES (run number 5). As can be observed in Fig. 8, the two temperature profiles coming from the optimization problem solution fit quite well the corresponding original temperature profiles.

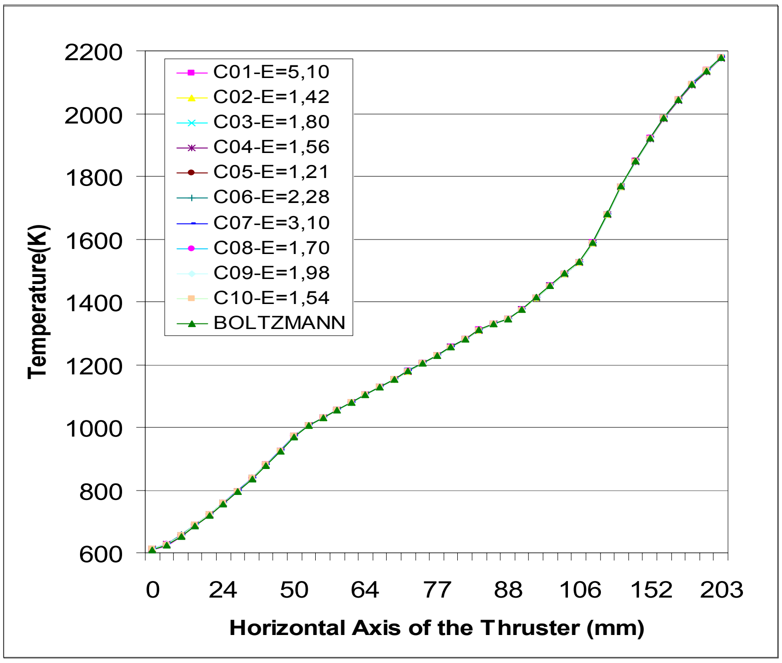

Figure 7. Steady state film cooling temperature distribution.

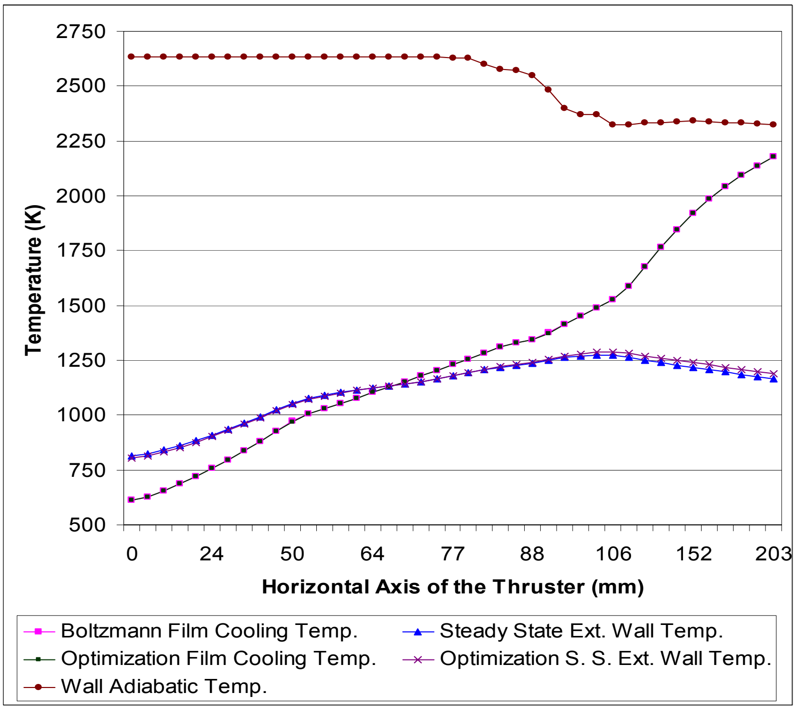

Figure 8. Original and optimized external wall temperature.

6. CONCLUSIONS

In this paper the temperature distribution of the film cooling near the internal wall of a thruster was estimated using an inverse design technique. A newly proposed hybrid evolutionary algorithm, called GEO_4 + ES, conjugated with INPE´s PCTER thermal software package was used to obtain the temperature in the film cooling, by estimating the Boltzmann equation parameters , that resulted in a target external wall temperature profile on the thruster. A very good accuracy for the recuperated external temperature profile, compared with the target one, was obtained. The results indicated that GEO + ES was able to solve the problem very well and in a consistent way, independently of the starting point randomly used by the algorithm to start the search. In a following step of this research, the methodology developed here will be used to estimate the film cooling profile using experimental temperature data on the outer surface of a real thruster.

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