ON FLOW OF ELECTRICALLY CONDUCTING FLUIDS OVER A FLAT PLAT IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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Abstract. *The use of a magnetic field to control the motion of electrically conduting fluid is analised. The boundary layer solutions are found for flow over a flat plate when the magnetic field is fixed relative to the plate or to the fluid.The equations are integrated numericaly for the effect of the transverse field on the velocity and temperature profiles and rate of heat transfer. It is concluded that skin friction and the heat transfer rate are reduced when the transverse magnetic field is fixed relative to the plate and increased when fixed relative to the fluid. The total drag is increased in all the cases analised.*

Keywords:*Magneto-Aerodynamic, Plate Flat, Magnetic Field*

1. INTRODUCTION

A number of situations exist in aerodynamics wherein a magnetic or electrical field might be used to relieve high convective heat transfer rates to a wall. Such problems arise in the flow of air in the boundary layer and in the vicinity of stagnation regions of a missile or aircraft moving at supersonic speeds. If the velocity is high enough the air is ionized and, hence, electrically conducting. Others examples are those associated with the flow of combustion products in the propulsion of aircrafts and rockets. There is few theoretical and experimental results available to evaluate the effect of a magnetic field on the drag and heat transfer rate, Dix (1962). An attempt to include all the aerodynamic features of flow of air over or inside a rocket plus the magneto-aerodynamic effects would render the problem so complicated as to make its solution difficult.

2. IMPULSIVE MOTION OF A FLATE PLATE

The impulsive motion of a infinite flate plate in a viscous fluid is used as a model for the boundary layer on a semi infinite flate plate. The advantages of working the problem of the impulsive motion of a flat plate are that it is simple enough to yield a result in closed form and suggests the choice of parameters to be used on the more complicated problem of a semi infinte flate plate.

The velocity profiles will be found for the two cases.In the first case it will be assumed the magnetic lines of force are fixed relative to the plate, and the second case relative to the fluid. In both cases the fluid is assumed uniform density and viscosity, and has the same electrical conductivity throughout. A third case, in which the magnetic field is fixed on the plate and a compensating pressure gradient exit in the fluid, will be shown to be equivalent to the case wherein he magnetic field is fixed relative to the fluid. The fluid and plate will be assumed to be initially at rest. At time t=0, the plate move impulsively with a given velocity.

2.1. Momentum Transport Equations

The momentum transport equation for the flow of a viscous incompressible fluid consisting of a combination of the force terms arising from excess charge density θ, and induced magnetic effect due to the motion of the conducting fluid through magnetic lines of force and the usual Navier Stokes equation is given by:

$$F = \theta\vec{E} + \vec{j}x\vec{B} \tag{1}$$

where the term θE results from the electrostatic force on the excess charges due to the presence of an imposed electric field E. The second term describes the force on the fluid due the interaction of the electric current j, in the fluid and the magnetic induction B. The differential equation in vector notation is given by Dix (1962) :

$$\frac{D\vec{U}}{Dt} - \frac{\theta\vec{E}}{\rho} - \frac{\vec{\mu j}}{\rho}x\vec{H} + \frac{1}{\rho}\nabla p = \nu\nabla^2\vec{U} \tag{2}$$

where ρ is the fluid density,θ is the excess charge density and μ is the magnetic permeability.
The Equation (2) may then be written as:

$$\frac{D\vec{U}}{Dt} + \frac{\alpha B_o^2}{\rho}(\vec{U} - \vec{U}_B) + \frac{1}{\rho}\nabla p = \nu\nabla^2\vec{U} \tag{3}$$

In the development of equation (3), it was assumed that the velocity of the magnetic field is zero. Since only the indutive force is considered, the second term becomes $\alpha B_o^2 / \rho(\vec{U} - \vec{U}_B)$ for a magnetic field in uniform translation, where $\vec{U}_B$ is the velocity of the magnetic field.

2.2. Magnetic Field Fixed Relative to the Plate Flat

At t< 0, the fluid, plate flat and magnetic field are assumed to be everywhere stationary. At time t = 0 and for all later times the plate flat and magnetic field are moving at velocity $u = u_\infty$ Figure 1. The objective is to find the velocity-time history of the fluid.

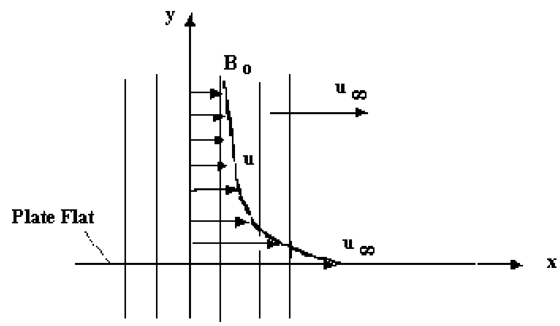


Figure 1 Velocity Profile

The velocity in a given y is constant plane does not change with x. It can then be assumed that the vertical,v, and transversal,w, velocities are zero or negligible together with the pressure gradients in all directions. The Equation (3) then reduces to

$$\frac{\partial u}{\partial t} + \frac{\alpha B_o^2}{\rho} u = \nu \frac{\partial^2 u}{\partial y^2} \tag{4}$$

Since the magnetic field is moving and the fluid is initially at test the relative motion must be accounted for Equation (4) is then:

$$\frac{\partial u}{\partial t} + \frac{\alpha B_o^2}{\rho}(u - u_\infty) = \nu \frac{\partial^2 u}{\partial y^2} \tag{5}$$

The boundary conditions are:

$u = u_\infty$, y=0, t ≥ 0

u = 0, y > 0, t = 0

The Laplace transform, Spiegel (1971) of the velocity u is defined as:

$$\overline{u} = \int_0^\infty e^{-st} u dt \tag{6}$$

Appying the Laplace transformation to the first term in Equation (6) gives:

$$L\left(\frac{\partial u}{\partial t}\right) = \int_0^\infty e^{-st} \frac{\partial u}{\partial t} dt = ue^{-st}\Big|_0^\infty + s\int_0^\infty e^{-st} u dt = s\overline{u}$$

If the others terms of Equation (6) are treated similary, the transformed equation is:

$$s\overline{u} = \nu \frac{d^2\overline{u}}{dy^2} + \frac{m_1 u_\infty}{s} = m_1 \overline{u}$$

or

$$\frac{d^2\overline{u}}{dy^2} = \overline{u}\left(\frac{s + m_1}{\nu}\right) - \frac{m_1 u_\infty}{\nu s} \tag{7}$$

The solution to Equation (7) is the sum of the solution of the homogeneous equation plus the particular solution, that is,

$$\bar{u} = \frac{m_1 u_\infty}{s(s+m_1)} + \frac{s+m_1}{s(s+m_1)}\left[c_1(s)e^{-y\sqrt{\frac{s+m_1}{\nu}}} + c_2 e^{y\sqrt{\frac{s+m_1}{\nu}}}\right] \tag{8}$$

The constante c_2 is chosen equal zero to fit the boundary condition that $\bar{u}$ must be finite at $y=\infty$. The integration contant $C_1(s)$ is found from the boundary contition at y=0,t hat is, on the plate $u_{y=0} = u_\infty$

The Laplace transform of u_∞ is u_∞ / s , Therefore:

$$C_1(s) = \frac{s}{m_1}$$

and

$$\bar{u} = \frac{m_1 u_\infty}{s(s+m_1)} + \frac{u}{(s+m_1)} e^{-y\sqrt{\frac{s+m_1}{\nu}}} \tag{9}$$

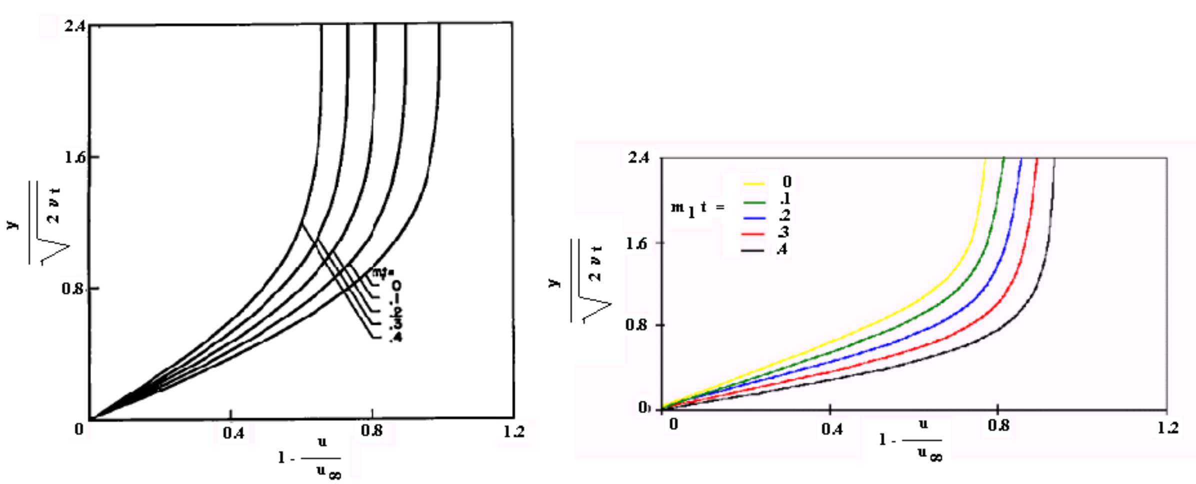
Inverting Equation (9) and combining several terms gives

$$u = u_\infty \left(1 - e^{-m_1 t} erf \frac{y}{2\sqrt{\nu t}}\right) \tag{10}$$

The symbol erf denotes the error function of the argument. The error function is discussed by Carslaw (1947), and Bird et al.(1963) and tabulated extensively by Anon (1954). Velocity profiles are shown in Figure 2 a and b by Pires (2008) and Rossow (1957) . If m_1 is set equal to zero, Equation (10) reduces to the result for the Rayleigh problem, that is

$$u = u_\infty \left(1 - erf \frac{y}{2\sqrt{\nu t}}\right) \tag{11}$$

The velocity is a function of $y / 2\sqrt{\nu t}$ only and hence a single profiles suffices. However, when a magnetic field is acting on the fluid, the velocity profiles are not similar because they change with time according to $e^{-m_1 t}$. The fluid at infinity is accelerted by the magnetic field so that the entire mass of fluid is accelerated by the magnetic field. It is, in fact, accelerated more rapidly than when the only force is the viscous action between layers.



a) b)

Figure 2 Velocity Profiles a) Rossow (1957) b)Pires (2008)

The skin friction coefficient c_f is expressed as:

$$c_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho u_\infty^2 / 2} \tag{12}$$

So that, from Equation (10),

$$c_f = \frac{2v}{u_\infty \sqrt{\pi v t}} e^{-m_1 t} \tag{13}$$

Values of c_f are shown in Figure 3. When m_1 is zero,Equation (13) reduces to the skin friction for the nonelectromagnetic Rayleigh problem.

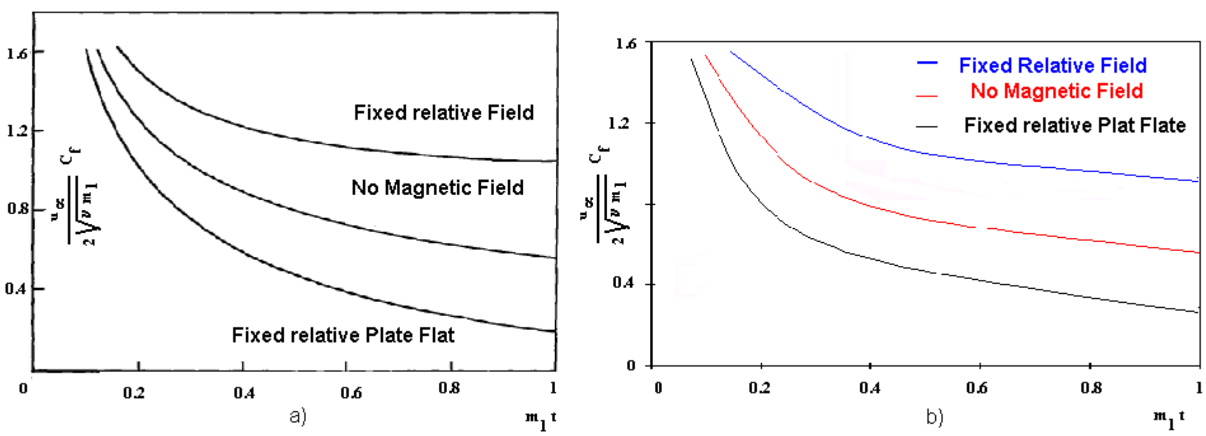


Figure 3 Skin Friction Coefficient values a) Rossow (1957) b) Pires (2008)

The change in the velocity profile is a result of the force exerced by the magnetic field on the fluid.The reaction on the unit generating magnetic field is expressed as:

$$F / unit.area = \int \alpha B_o^2 (u_\infty - u) dy = \alpha B_o^2 e^{-m_1 t} \int_0^{y \to \infty} erf \frac{y}{2\sqrt{vt}} dy \tag{14}$$

The force per unit area goes to infinity as the upper limit of integration goes to infinity because the magnetic field extends undiminushed an infinite distance from the plate.Thereby,na infinite amount of fluid os accelerated,resulting in na infinite force.

2.3.Magnetic Field Fixed Relative to the Fluid

At all times less than zero the fluid,magnetic field and plate flat are assumed to be at rest.At time t=0 the plate flat begins moving with velocity $u = u_\infty$ but the magnetic field remaisn at rest.The differential eqution is the same as Equation (4) because there is no relative motion between the fluid at $y = \infty$ and the magnetic field.

$$\frac{\partial u}{\partial t} + m_1 u = v \frac{\partial^2 u}{\partial y^2} \tag{15}$$

The boundary conditions are

$u = u_\infty$, y=0, $t \geq 0$
u = 0, y > 0, t = 0

Applying the Laplace transformation to Equation (15),yields the trasformed equation

$$v \frac{d^2 \bar{u}}{dy^2} = \bar{u}(s + m_1) \tag{16}$$

The solution to this ordinary differential equation is

$$\bar{u} = C_1(s)e^{-y\sqrt{\frac{s+m_1}{v}}} + C_2 e^{+y\sqrt{\frac{s+m_1}{v}}} \tag{17}$$

where the constant C_2 is set equal to zero because of the requirement of a finite velocity at $y=\infty$.The integration constant, $C_1(s)$,is found from the boundary condition at y=0,t≥0, as

$$C_1(s) = \frac{u_\infty}{s}$$

The Equation (17) can be written as

$$\bar{u} = \frac{u_\infty}{(s+m_1)-m_1} e^{-y\sqrt{s+m_1}\frac{y}{\sqrt{v}}} \tag{18}$$

This equation is inverted as

$$\bar{u} = \frac{u_\infty}{2}\left[e^{-y\sqrt{\frac{m_1}{v}}} erfc\left(\frac{y}{2\sqrt{vt}} - \sqrt{m_1 t}\right) + e^{y\sqrt{\frac{m_1}{v}}} erfc\left(\frac{y}{2\sqrt{vt}} + \sqrt{m_1 t}\right)\right] \tag{19}$$

The velocity is once again dependent on more than one parameter so that a single similar profile cannot be drawn.Several profiles are shown in Figure 4.The fluid at $y=\infty$ is not disturbed in this case because the velocity across the magnetic lines is zero.In the vicinity of the plate the induced magnetic force counteracts the acceleration force of viscosity resulting in an increased rate of shear at the walls as expressed by the skin friction coefficient.

$$c_f = \frac{2v}{u_\infty}\left(\sqrt{\frac{m_1}{v}} erf\sqrt{m_1 t} + \frac{e^{-m_1 t}}{\sqrt{\pi v t}}\right) \tag{20}$$

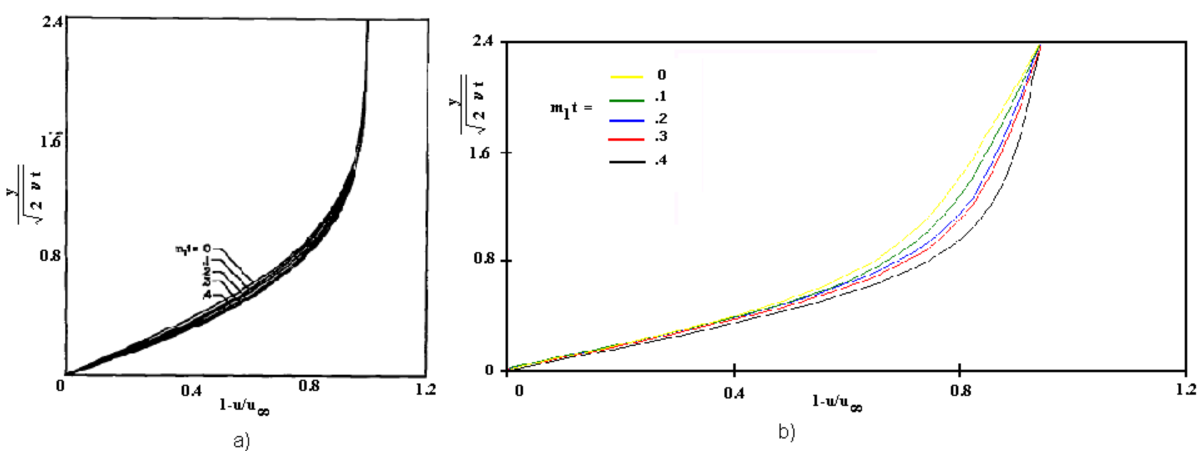


Figure 4 Velocity profiles a) Rossow (1957) b) Pires (2008)

Even at time $t=\infty$,a friction force on the plate flat.Values computed by Equation (20) are shown in Figure 4.
The force on the magnetic field is given by:

$$F\,/\,unit.area = \int_0^\infty \alpha B_0^2 u dy \tag{21}$$

where u is given by Equation (19).The force is finite in this case.

2.4. Temperature Profile

When the pressure gradients are reasoned to be everywhere zero and the two dimensional boundary layer assumptions are made,the differential equation describing the the relationship between the convection and conduction of thermal energy and work done on an eletrically conduting fluid in the presence of a magnetic field is given by Briggs et al.(1970):

$$\rho u C_p \frac{\partial T}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} - \alpha B_0^2 u^2 = \frac{C_p \mu}{Pr} \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (22)$$

If total energy is defined:

$$E = C_p T + \frac{u^2}{2} \quad (23)$$

For case of the magnetic field relative to the plate,the magnetic field lines of force are assumed to be perpendicular to the free stream dirction and to begin at the leading edge of the plate flat. On the basis of boundary layer assumptions Equation (3) reduces to:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + m_1 u = v \frac{\partial^2 u}{\partial y^2} \quad (24)$$

The boundary conditions that u=0 at y=0 and u= u_∞ upstream of the plate.At $y = \infty, \frac{\partial u}{\partial y} = v = 0$ and $\frac{\partial u}{\partial x} = -m_1$

Equations (24) and (23) can be combined to yeld a differential equation for the transport of total energy E,that is,if Equation (24) is multiplied by u ,added to Equation(23),and the prandtl number,Pr,is assumed to be 1.0,

$$u \frac{\partial E}{\partial x} + v \frac{\partial E}{\partial y} = v \frac{\partial^2 E}{\partial y^2} \quad (25)$$

Although the temperature distribution is altered by the presence of the magnetic field because u and v are affected,the total energy of the conducting fluid is not.The kinetic energy removed by the force of the magnetic field is exactly equal to the heat generated by the electric current,independent to the field strengh B_o and the conductivity σ.

A first order estimate of the influence of the magnetic field on the temperature profile can be found.Assuming that:

$$T = T_0(\eta) + T_2(\eta) mx + T_4(\eta) m^2 x^2 + \ldots \quad (26)$$

where T_2,$T_{4\ldots}$ are functions only of $\eta = y\sqrt{u_\infty / vx}$ and $T_1=T_3=T_5\ldots=0$ as in the part the problem dealing with the velocity profile.The derivatives of the temperature with respect to x and y are found as :

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\frac{\partial T}{\partial x} = \frac{1}{x} \left[mxT_2 + 2m^2 x^2 T_4 + \ldots - \frac{\eta}{2} \left(T_0' + mxT_2' + 2m^2 x^2 T_4' + \ldots \right) \right] \quad (27)$$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial y} = \sqrt{\frac{u_\infty}{vx}} \left(T_0' + mxT_2' + 2m^2 x^2 T_4' + \ldots \right) \quad (28)$$

when the various expressions are inserted into Equation (22) and terms containing like powers of mx are equated,the differential equations for T_0 and T_2 are:

$$\frac{T_0''}{Pr} + \frac{T_0' f_0}{2} + \frac{u_\infty^2 (f_0')^2}{Cp} = 0 \quad (29)$$

$$\frac{T_2''}{Pr} + \frac{T_2 f_0}{2} - f_0' T_2 + \frac{3}{2} f_2 T_0' + \frac{u_\infty^2 \left(f_0'^2 + 2 f_0'' f_2'' \right)^2}{Cp} = 0 \quad (30)$$

The boundary conditions are:

$T_0 = T_w$ at $\eta = 0$
$T_0 = T\infty$ at $\eta = \infty$
$T_2'' = T_2' = 0$ at $\eta = \infty$
$T_2 = 0$ at $\eta = 0$

The solution to Equation (29) was first found by Pohlhausen is discussed by Schlichting (1968).The function $T_0(\eta)$ is the temperature profile for a boundary layer on a plate flat and is given by assuming Pr =1.0 :

$$T_0 = T_\infty + \left(T_w - T\infty - \frac{u_\infty^2}{2Cp} \right)(1 - f_0') + \frac{u_\infty^2}{2Cp}\left(1 - f_0'^2\right) \tag{31}$$

The derivte of T_0 to be inserted into Equation (30) becomes:

$$T_0' = -f_0''\left(T_w - T\infty - \frac{u_\infty^2}{2Cp} + \frac{u_\infty^2}{2Cp} f_0' \right)(1 - f_0') + \frac{u_\infty^2}{2Cp}\left(1 - f_0'^2\right) \tag{32}$$

The temperature of the plate flat is assumed to be the same as that of the fluid far from the plate,that is, $T_w = T\infty$ .Equation (30) then becomes Pr =1.0:

$$T_2'' + \frac{T_2'}{2} f_0 - f_0' T_2 + \frac{3}{2} f_2 f_0'' \frac{u_\infty^2}{2C_p}\left(\frac{1}{2} - f_0' \right) + \frac{u_\infty^2}{2C_p}\left(f_0'^2 + f_0'' f_2'' \right) = 0 \tag{33}$$

The Equation (33) was integrated numerically.Several temperature profiles are shown in Figure 5.the quantity of heat transferede to the plate per unit time is:

$$q = k \frac{\partial T}{\partial y})_{y=0} \tag{34}$$

the local convective heat transfer rate is:

$$h = \frac{q}{\frac{u_\infty^2}{2C_p}} = \frac{\rho u_\infty C_p}{2\sqrt{Re_x}} (0.664 - 0.704mx - ... + ...) \tag{35}$$

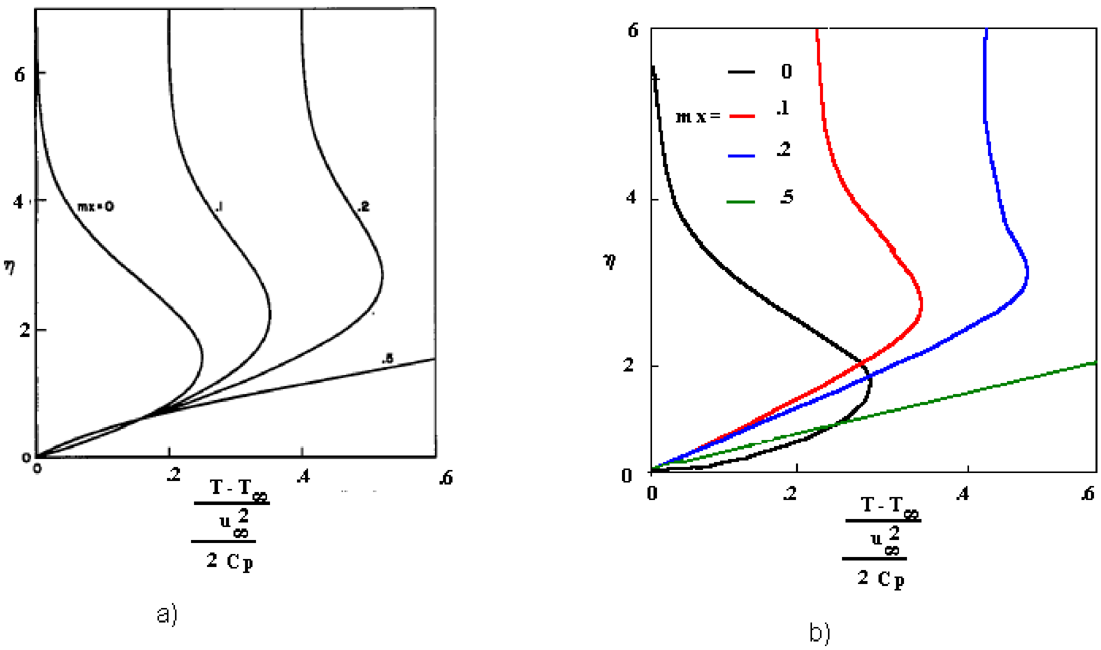


Figure 5 Temperature Profiles a) Rossow (1957) b)Pires (2008)

3.CONCLUSION

The laminar boundary layer solutions presented in this work for the flow of an electrically conducting fluid over a flat plate indicate the changes that will be brought about by a transverse magnetic field. It has been found that the skin friction and heat transfer rate are reduced is there is no relative motion between the plate and the magnetic field. The skin friction and heat transfer rate increased in case where relative motion was assumed. If all cases analysed the total drag is increased.

All the cases analysed, except one, assume the conductivity and magnetic field did not change with distance from the surface. The reduction in heat transfer rate and skin friction for a given value of mx was not the importance of considering the way in which the conductivity magnetic field strength change with distance from the surface.

4. REFERENCES

Anon,1954 "Tables of the Error Function and Its Derivate",National Bureau of Standards Applied Mathematics,ser.41

Bird R.B.,Stewart,W.E.,Lightfoot,E.N. ,1963, "Phenomena Transport",John Wiley & Sons,Inc,New York,EUA.

Briggs,R.J.,Gaugherty,J. And Levy,R.,1970,"Role of Landau Damping in crossed-Field Electron Beams and inviscid Shear flow",Physics of Fluids 13,421

Carslaw,H.S., and Jaeger, J.C. ,1947, "Conduction of Heat in Solids."Clarendon |Press,Oxford.

Dix, M.D.,1962,"The Magnetohydrodynamic Flow Past a Nonconducting Flat Plate in the Presence of a Transverse Magnetic Flow",Technical report 397,Massachusetts Institute Technology,Cambridge –Massachusetts.

Schlichting,H.,1968,"Grenzschicht Theorie"G.Braun,Karlsruhe,Deuschtland

Spiegel, M.R.,1971 "Transformadas de Laplace",Editora McGraw-Hill,Ltda,S. Paulo ,Brazil,344 p.

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