

## INVERSE ANALYSIS APPLIED TO ILLUMINATION DESIGN: DETERMINATION OF OPTIMUM LOCATIONS OF THE LIGHT SOURCES

Fabiano Cassol, [fabiano.cassol@ufrgs.br](mailto:fabiano.cassol@ufrgs.br)

Paulo Smith Schneider, [pss@mecanica.ufrgs.br](mailto:pss@mecanica.ufrgs.br)

Francis Henrique Ramos França, [frfranca@mecanica.ufrgs.br](mailto:frfranca@mecanica.ufrgs.br)

Departamento de Engenharia Mecânica – Universidade Federal do Rio Grande do Sul – UFRGS – Porto Alegre, RS, Brazil

Fabiano Luis de Sousa, [fabiano@dem.inpe.br](mailto:fabiano@dem.inpe.br)

Instituto Nacional de Pesquisas Espaciais – INPE – São José dos Campos, SP, Brazil

Antônio José da Silva Neto, [ajsneto@iprj.uerj.br](mailto:ajsneto@iprj.uerj.br)

Instituto Politécnico, IPRJ – Universidade do Estado do Rio de Janeiro – UERJ – Nova Friburgo, RJ, Brazil

**Abstract.** *This work applies the generalized extremal optimization (GEO) algorithm to solve an illumination design of a three-dimensional enclosure. The illumination design is inherently an inverse problem, in which the design surface is subjected to two conditions – luminous flux and null luminous power – while the light sources are left unconstrained. It is required the determination of the locations and of the luminous powers of the light sources to satisfy the prescribed luminous flux on the design surface. The GEO method is especially advantageous to be applied in complex problems where traditional gradient-based methods may become inefficient, such as when applied to a nonconvex or disjoint design space, or when there are different kinds of design variables in it (in this case, the positions and the power inputs of the lamps). In previous studies of illumination designs, the objective was to determine the luminous power of the light sources whose positions were supposed to be known. The major contribution of the present work is to permit the determination of the optimum luminous powers and of the locations of the light sources.*

**Keywords:** *Inverse analysis, Illumination design, Radiation exchanges, GEO Optimization*

### 1. INTRODUCTION

The first methods for the analysis and design of artificial lighting of environments were established in the beginning of the 20th century, based on the knowledge that the luminous flux on a given working area was not only dependent on the power of the light sources, but also on the absorbing and reflecting effect of the remaining surfaces. New advances provided methods for calculation of light radiation exchanges as well as for the characterization of the light sources behavior. Many studies have been carried out to provide recommending lighting for the many possible applications (Boast, 1953; Mark, 2000). In general, not only the intensity of light (luminous flux intensity) is specified, but also it is required uniformity of the lighting. The major goal of the illumination designer is to determine the positions and powers of the light sources to provide the prescribed luminous flux on the working area.

Among the first works to deal with illumination design, Harrison and Anderson (1916 and 1920) proposed an experimental procedure, the lumen method, in which the luminous flux on a working plane was determined from a combination of a series of proposed assembling of punctual and continuous light sources. Moon (1941) and Moon and Spencer (1946a, 1946b) proposed the interreflection method for the design of three-dimensional rectangular enclosures having any aspect ratio and being formed by diffuse surfaces. The method presented the advantage of allowing the calculation of the reflections of light. Due to the complexities of the required calculations, the method required the use of tables. The lumen method (OSRAM, 2005) is probably the most widely employed for the design of illumination, for its algebraic relations provide a rapid, simple procedure to determine the power of the lamps, although the method lacks on precision. A more elaborate solution can be achieved with the WinElux code (EEE, 2002), which contains a database of different types of lamps. In spite of their widespread use, both the lumen method and the WinElux code are in general not capable of providing solutions that can satisfy uniformity of luminous flux on the design surface (Seewald, 2006). A new approach has been proposed in the works of Smith Schneider and França (2004), Seewald et al. (2006), and Mossi et al. (2007), in which the illumination design is treated as an inverse problem. Starting from the radiation exchange relations within an enclosure, these works proposed a methodology based on fundamental luminous exchange relations, obtaining a luminous flux on the design that satisfied the uniformity and the required intensity. The first and the second works employed the truncated singular value decomposition (TSVD) regularization (Hansen, 1990); the latter applied the generalized extremal optimization (GEO) algorithm (De Sousa et al., 2003).

This paper considers the illumination design of the three-dimensional rectangular enclosure that was studied in Smith Schneider and França (2004) and Mossi et al. (2007). While in those two works, the locations of the light sources were fixed, and the objective was to determine their luminous powers, the present work aims at finding the optimum locations of the lamps. In addition, it is imposed that the luminous power of the light sources are all the same, although

its value is left unperceived and to be determined from the inverse analysis. All the surfaces that form the enclosure are assumed diffuse and having spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum. The results are obtained from the coupling between the forward solution radiation exchanges heat transfer and the GEO algorithm. The proposed methodology is capable of providing satisfactory solutions for the required luminous flux on the design surface.

## 2. PHYSICAL AND MATHEMATICAL MODELING

### 2.1. Luminous flux and thermal radiation

Visible light is contained in the spectrum of thermal radiation, corresponding to wavelengths ranging from 0.4 to 0.7  $\mu\text{m}$ . The luminous flux, in units of lumens/ $\text{m}^2$  or lux, can be related to the thermal radiation flux, in units of  $\text{W}/\text{m}^2$ , by means of the following relation:

$$dq^{(l)} = CV_{\lambda} dq^{(w)} \quad (1)$$

where  $dq^{(l)}$  and  $dq^{(w)}$  correspond respectively to the luminous flux and to the thermal radiation flux for a specific wavelength  $\lambda$ , within an interval  $d\lambda$ ,  $C$  is a conversion factor constant, equal to 683 lumens/W, and  $V_{\lambda}$  is the photopic spectral luminous efficacy of the human eye, which takes into account the human eye sensitivity to the thermal radiation comprehended in the visible region of the spectrum. As shown in Smith Schneider and França (2004) and Mossi et al. (2007),  $V_{\lambda}$  peaks with a value of 1.0 for a thermal radiation in the wavelength of 0.555  $\mu\text{m}$ , and then decay monotonically to zero as the lower and upper limits of the visible region, 0.4  $\mu\text{m}$  and 0.7  $\mu\text{m}$ , are approached.

In general, a source of light is composed by radiation covering the entire range of the visible region. In such a case, Eq. (1) must be applied to each infinitesimal amount of the spectral energy, and then be integrated in the visible spectrum. Details of this procedure can be found in Smith Schneider and França (2004) and Mossi et al. (2007).

### 2.2. Problem definition

A schematic view of a three-dimensional enclosure is shown in Fig. 1, which is formed by surfaces that are diffuse and have spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum. The design surface, where a luminous flux is to be specified, is located on the bottom of the enclosure; the light sources are located on the top surface. The remaining of the enclosure is formed by walls that do not emit but reflect incident light. The length, width and height of the enclosure are designated by  $L$ ,  $W$  and  $H$ , respectively.

Figure 2 shows the division of the enclosure into finite-sized square elements,  $\Delta x = \Delta y = \Delta z$ , in which the luminous energy balance can be applied. In this analysis, it is considered that an uniform luminous flux (in lumens/ $\text{m}^2$  or lux), designated by  $q_{\text{specified}}^{(l)}$ , is specified on the design surface. The problem consists of finding the position of each light source element, and its luminous powers, imposed to be the same for all the light sources.

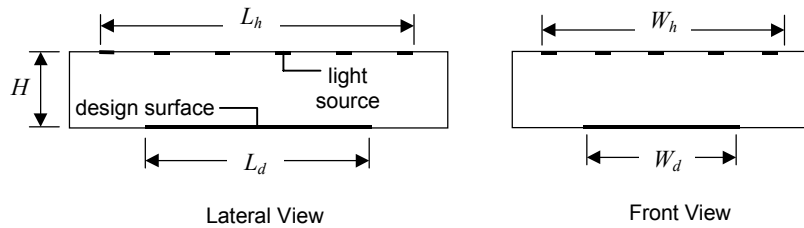


Figure 1. Three-dimensional rectangular enclosure.

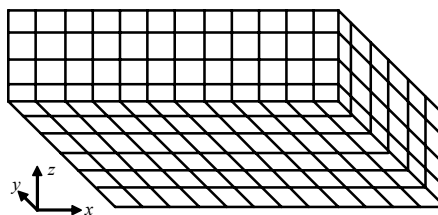


Figure 2. Division of the bottom and two side surfaces of the enclosure into finite size elements.

As shown in Smith Schneider and França (2004) and Mossi et al. (2007), the light energy balance applied to a surface element  $j$  can be expressed in different but complementing forms, as seen below:

$$q_{o,j}^{(l)} = \varepsilon_j e_{b,j}^{(l)} + (1 - \varepsilon_j) \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (1)$$

$$q_{o,j}^{(l)} = q_{r,j}^{(l)} + \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (2)$$

$$q_{i,j}^{(l)} = \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (3)$$

where  $q_o^{(l)}$  is the outgoing luminous flux, in lumens/m<sup>2</sup> or lux, which takes into account both emission and reflection;  $q_r^{(l)}$  is the net luminous flux, in lumens/m<sup>2</sup>, which takes into account emission minus absorption;  $q_i^{(l)}$  is the incident luminous flux, in lumens/m<sup>2</sup>;  $e_b^{(l)}$  is the blackbody luminous power, in lumens/m<sup>2</sup>, which is solely dependent on the temperature;  $F_{j-j^*}$  is the view factor between surface elements  $j$  and  $j^*$ ;  $\varepsilon_j$  is the hemispherical emissivity of the surface in the visible range of the spectrum; finally,  $J$  is the total of elements on the enclosure. In the derivation of Eqs. (1) and (2), it was assumed that the spectral emissivity was independent of the wavelength in the visible region of the spectrum. The extension of the light energy balance to the case in which the spectral emissivity depends on the wavelength was presented in Seewald et al. (2006). Since the objective of this work is mainly the presentation of a methodology for the determination of the optimum location of the light sources, the gray surfaces assumption is adopted for simplicity, but extension to non-gray surfaces is immediate.

In the illumination design, no condition is known for the light source elements, but they need to be found to satisfy the specifications on the design surface. For an element  $jw$  on the side walls, the luminous power is null,  $e_{b,jw}^{(l)} = 0$ , since they do not emit light. For a design surface element  $jd$ , two conditions are specified: the luminous power is also null,  $e_{b,jd}^{(l)} = 0$ , and the luminous flux is equal to  $q_{specified}^{(l)}$ . Depending on the problem, the luminous flux can correspond to either the net or to the incident luminous fluxes,  $q_{r,jd}^{(l)}$  and  $q_{i,jd}^{(l)}$ , respectively. In fact, the combination of Eqs. (1) to (3), with  $e_{b,jd}^{(l)} = 0$ , show that these two quantities are related by  $q_{i,jd}^{(l)} = -q_{r,jd}^{(l)} / \varepsilon_{jd}$ , so prescribing one condition is equivalent to prescribing the other. In this work, it is considered that the prescribed luminous flux is related to the net luminous flux, that is,  $q_r^{(l)} = -q_{specified}^{(l)}$ . Note that the negative signal arises from the adopted convention that the net luminous flux corresponds to emission minus absorption of light. For a surface that is illuminated, it should be negative.

One possible treatment for this problem is to specify the positions as well as the net luminous fluxes,  $q_{r,jl}^{(l)}$ , of the light source elements  $jl$  (alternatively, it could be the blackbody luminous power,  $e_{b,jl}^{(l)}$ , instead of  $q_{r,jl}^{(l)}$ ), and to impose the condition of null luminous power to the elements on the design surface and walls,  $e_{b,jd}^{(l)} = e_{b,jw}^{(l)} = 0$ . Equation (1) is written for each element on the design surface and on the wall, and Eq. (2) is written for each light source element, forming a system of  $J$  linear equations on the  $J$  unknown luminous radiosities of each surface  $j$ ,  $q_{o,j}^{(l)}$ . This system is in general well-conditioned and can be solved by any standard matrix inversion technique, such as Gaussian elimination, or by iterative techniques, such as the Gauss-Seidel. Once the system is solved for the outgoing luminous flux, Eq. (2) is written for each design surface element to determine the net luminous power,  $q_{r,jd}^{(l)}$ , which can be compared to the prescribed luminous power,  $q_{specified}^{(l)}$ . The process is repeated with the placing of the light sources in different positions and specifying a different value for  $q_{r,jl}^{(l)}$ , repeating the process until the conditions on the design surface is attained within a maximum error. Searching through all possible solutions is not a feasible task, unless an efficient searching technique is devised. In this work, this will be done with the aid of the GEO algorithm.

### 3. THE GENERALIZED EXTREMAL OPTIMIZATION ALGORITHM

The generalized extremal optimization (GEO) algorithm (Sousa et al., 2003) is a new evolutionary algorithm devised to improve the Extremal Optimization method (Boettcher and Percus, 2001) so that it could be easily applicable

to virtually any kind of optimization problem. Both algorithms were inspired by the evolutionary model of Bak and Sneppen (1993). Following the Bak and Sneppen (1993) model, in GEO  $L$  species are aligned and for each species it is assigned a fitness value that will determine the species that are more prone to mutate. One can think of these species as bits that can assume the values of 0 or 1. Hence, the entire population would consist of a single binary string. The design variables of the optimization problem are encoded in this string that is similar to a chromosome in a genetic algorithm (GA) with binary representation (see Fig. 2).

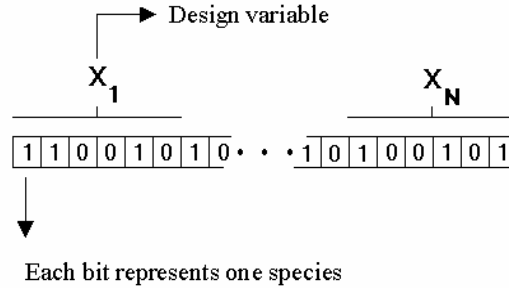


Figure 3. Design variables encoded in a binary string.

To each species (bit) it is assigned a fitness number that is proportional to the gain (or loss) the objective function value has in mutating (flipping) the bit. All bits are then ranked from rank 1, for the least adapted bit, to  $N$  for the best adapted. A bit is then mutated (flipped) according to the probability distribution of the  $k$  ranks, given by:

$$P(k) = k^{-\tau} \quad (4)$$

in which ( $1 \leq k \leq N$ ). In Eq. (4),  $\tau$  is a positive adjustable parameter (for  $\tau \rightarrow 0$ , the algorithm becomes a random walk, whereas for  $\tau \rightarrow \infty$ , we have a deterministic search). This process is repeated until a given stopping criteria is reached and the best configuration of bits (the one that gives the best value for the objective function) found through the process is returned.

The practical implementation of the canonical GEO algorithm to a function optimization problem is as follow:

1. Initialize randomly a binary string of length  $L$  that encodes  $N$  design variables of bit length  $l_j$  ( $j = 1, N$ ). For the initial configuration  $C$  of bits, calculate the objective function value  $V$ .
2. For each bit  $i$  of the string, at a given iteration:
  - 2.1. Flip the bit (from 0 to 1 or 1 to 0) and calculate the objective function value  $V_i$  of the string configuration  $C_i$ .
  - 2.2. Set the bit fitness as  $\Delta V_i = (V_i - V_{ref})$ . It indicates the relative gain (or loss) that one has in mutating the bit, compared to a given reference value ( $V_{ref}$ );
  - 2.3. Return the bit to its original value.
3. Rank the bits according to their fitness values, from  $k = 1$  for the least adapted bit to  $k = L$  for the best adapted. In a minimization problem, higher values of  $\Delta V_i$  will have higher ranking, and otherwise for maximization problems. If two or more bits have the same fitness, rank them randomly with uniform distribution.
4. Choose with uniform probability a candidate bit  $i$  to mutate (flip from 0 to 1 or from 1 to 0). Generate a random number RAN with uniform distribution in the range  $[0,1]$ . If Eq. (9) is equal or greater than RAN the bit is confirmed to mutate. Otherwise, choose a new candidate bit and repeat the process until a bit is confirmed to mutate.
5. Set  $C = C_i$  and  $V = V_i$ , with  $i$  the bit confirmed to mutate in step 4.
6. Repeat steps 2 to 6 until a given stopping criteria is reached.
7. Return  $C_{best}$  and  $V_{best}$ , the configuration of bits associated to best value of objective function, respectively, found during the search. Each time a new configuration of bits is set during the search, it is verified if its associated objective function value is the best found so far or not. If so, they are stored as the current optimal solution found.

Note that in step 4 any bit can be chosen to mutate, but the probability of a given chosen bit be confirmed to mutate is dependent on its rank position. The ones more adapted (with higher rank values) are less prone to have its mutation confirmed and only the least adapted bit (rank = 1) is always confirmed to mutate, if chosen. The probability of mutating the chosen bit is regulated by the adjustable parameter  $\tau$ . The higher the value of  $\tau$ , the smaller the chance of a bit (with rank greater than 1) be mutated. The possibility of making moves that do not improve the value of the objective function is what allows the algorithm to escape from local optima.

In a practical application of the GEO algorithm, the first decision to be made is on the definition of the number of bits that will represent each design variable. This can be done simply setting for each variable the number of bits

necessary to assure a given desirable precision for each of them. For continuous variables the minimum number ( $m$ ) of bits necessary to achieve a certain precision is given by:

$$2^m \geq \left\lceil \frac{(x_j^u - x_j^l)}{p} + 1 \right\rceil \quad (5)$$

where  $x_j^l$  and  $x_j^u$  are the lower and upper bounds, respectively, of the variable  $j$  (with  $j = 1, N$ ), and  $p$  is the desired precision. The physical value of each design variable is obtained through the equation:

$$x_j = x_j^l + (x_j^u - x_j^l) \frac{I_j}{(2^m - 1)} \quad (6)$$

in which  $I_j$  is the integer number obtained in the transformation of the variable  $j$  from its binary form to a decimal representation.

#### 4. SOLUTION PROCEDURE

The problem consists in minimizing the error function  $F$ , which is a measure of the difference between the specified luminous flux on the design surface,  $q_{specified}^{(l)}$ , and the luminous fluxes on the design surface that are obtained from a given choice of configuration and luminous powers of the light sources,  $q_{r,jd}^{(l)}$ . The optimization problem can then be formulated as by the minimization of the function  $F$  below:

$$F = \sqrt{\sum_{jd} \left( |q_{specified}^{(l)}| - |q_{r,jd}^{(l)}| \right)^2} \quad (7)$$

Subject to:

$$i_{x\_low} \leq i_x \leq i_{x\_up} \quad (x \text{ direction restrictions}) \quad (8a)$$

$$i_{y\_low} \leq i_y \leq i_{y\_up} \quad (y \text{ direction restrictions}) \quad (8b)$$

$$q_{r,jh\_low} \leq q_{r,jh} \leq q_{r,jh\_up} \quad (\text{luminous flux restriction}) \quad (8c)$$

where subscripts *low* and *up* indicate the lower and upper limits of each variable, respectively. The variables  $i_x$  and  $i_y$  are indices that define the  $x$  and  $y$  positions of each light source; variable  $Q_{r,jh}$  is the luminous power. These variables will be described below.

To minimize the above relation, the following procedure is followed:

1. Define the required precision,  $p$ , to find minimum number of bits,  $m$ , using Eq. (5);
2. Start with a given set of locations of the light sources and the value of the net luminous flux (assumed to be the same for all light sources);
3. Solve the system of equations described in Section 2 to find net luminous fluxes on the design surface elements,  $q_{r,jd}^{(l)}$ , computing the error function from Eq. (7);
4. Choose a new set of luminous powers on the light sources according to the GEO algorithm;
5. Repeat from step 2 until satisfactory solutions for the locations of the light sources and of the value of the net luminous flux are found.

#### 5. RESULTS AND DISCUSSION

The case considered in this work consists of a three-dimensional enclosure as shown in the schematic of Fig. 1. The aspect ratio of the enclosure base is  $W/L = 0.8$ ; the dimensionless height is  $H/L = 0.2$ . The selection of the other dimensions of the enclosure will require a few considerations. First, the design surface ought not to cover the entire extension of the base, since the portions close to the corners would be mainly affected by the reflections from the side walls, not from the luminous radiation from the light source elements on the top surface. Therefore, the design surface dimensions are taken as  $L_d/L = 0.8$  and  $W_d/L = 0.6$ . The hemispherical emissivities in the visible light region of the design surface, of the light sources and of the walls are  $\varepsilon_d = 0.9$ ,  $\varepsilon_l = 0.9$  and  $\varepsilon_w = 0.5$ , respectively

The boundary conditions are: for the elements on the design surface and on the wall, the luminous emissive power is zero,  $e_{b,jd}^{(l)} = e_{b,jw}^{(l)} = 0$ ; the locations of the light sources as well their net luminous flux (the same for all) will be sought with the aid of the GEO algorithm to assure the expected dimensionless net luminous flux (defined as  $Q_{r,jd} = q_{r,jd}^{(l)} / q_{specified}^{(l)}$ ) is  $Q_{r,jd} = -1.0$ , within some acceptable error.

Figure 5 shows the division of the bottom (or top) surface into 15 and 12 elements in the  $x$  and  $y$  directions. It results that the position of each light source can be specified by varying integer indices  $i_x$  and  $i_y$  in the intervals  $[1, 15]$  and  $[1, 12]$ , respectively, in accordance with Eqs. (8a) and (8b). For the dimensionless net luminous flux of the light sources ( $Q_{r,jh} = q_{r,jh}^{(l)} / q_{specified}^{(l)}$ ), the chosen interval is  $[0, 50]$ , in accordance with Eq. (8c). The precision  $p$  for this problem was chosen of 0.1. To allow a comparison with the solutions presented in Smith Schneider and França (2004) and Mossi et al. (2007), it is considered that a total of 10 light sources are used. Therefore, the error function defined by Eq. (7) depends on a total of 21 variables: the two integer indices  $i_x$  and  $i_y$  for each of the 10 light sources, plus the dimensionless net luminous flux  $Q_{r,jh}$ . If the powers of the light sources were allowed to be different, the problem would involve 30 variables (the twenty position indices and the ten different powers). This would probably lead to a better solution in terms of the minimization of the error function  $F$ , but would require more computation time.

Since the performance of the GEO algorithm is dependent on the parameter  $\tau$ , it was first performed a study to determine its optimum value for the present problem. With the aforementioned definitions, the GEO algorithm was run 10,000 times for the evaluation of the error function  $F$ . Fifty independent runs were made for each algorithm. The results are shown in Fig. 4(a). From this results, it was selected the value of  $\tau = 1.75$  for all the other runs, because this value showed the lowest values for the error function  $F$ . However, the stopping criterion was modified, so that more time was given to the algorithm to search for the design space for the optimum solution.

The values of the error function towards the global minimum can be seen in Fig. 4(b). This results shown the best results for the minimization of function  $F$ , for which there is a correspondent set of the unknown parameters ( $i_x, i_y, Q_{r,jh}$ ). As can be seen, it is important to increase the number of runs in order to find better results for the function  $F$ . The higher the number of variables, the larger the number of runs needed to reach a satisfactory result.

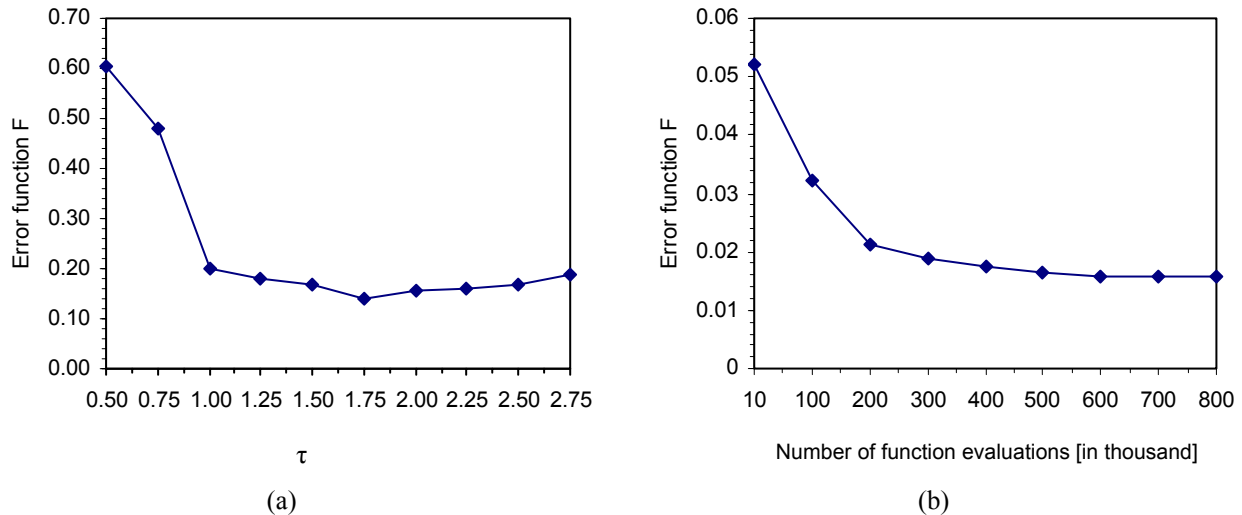


Figure 4. (a) Average of best results for fifty independent runs of GEO for different  $\tau$ . Each run stopped after 10,000 evaluations of the objective function  $F$ . (b) Lowest values of  $F$  for different numbers of evaluations with  $\tau = 1.75$ .

The positions of the light sources, found with the GEO algorithm, are shown in Fig. 5(a). The light sources are indicated by the circular dots (distributed on the top surface). The shaded area represents the design surface at the bottom surface. Due to the problem symmetry, indicated by the dashed lines, only a quarter of the domain needs to be solved ( $0 \leq x/L \leq 0.5, 0 \leq y/L \leq 0.4$ ). These results can be compared with the light sources configuration recommended in Smith Schneider and França (2004) and Mossi et al. (2007), shown in Fig. 5(b), in which the positions were not found from an optimization, but with the identification of the points of local maximum light powers when the entire top was covered with light sources. In their solutions, the design problem was solved with the TSVD (truncated singular value decomposition) and with GEO regularization method, considering that the positions of the light sources were fixed, but the values of their net luminous fluxes were left free to vary among each other. All other conditions were similar. Table 1 compares the three solutions.

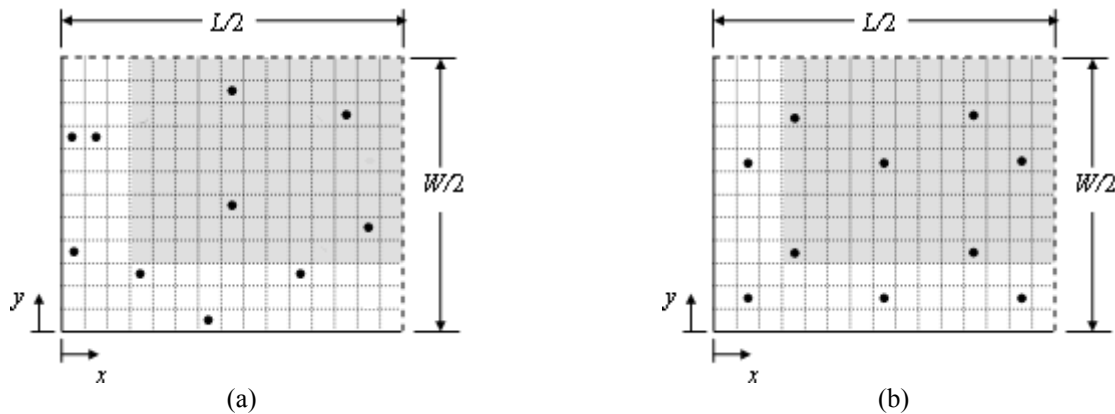


Figure 5. Locations of the design surface (shaded area) and light source elements (circular dots) in one quarter of the bottom and top: (a) present work (optimization); (b) from Smith Schneider and França (2004) and Mossi et al. (2007). Dashed lines indicate symmetry.

Table 1: Required dimensionless net luminous flux on the light source elements

$jl$	Present work			Mossi et al. (2007)			Smith Schneider and França (2004)		
	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$
1	1	4	24.1399	2	2	50.0000	2	2	17.4009
2	1	9	24.1399	2	8	10.0000	2	8	11.7953
3	2	9	24.1399	4	4	12.3529	4	4	31.1068
4	4	3	24.1399	4	10	37.4510	4	10	36.6861
5	7	1	24.1399	8	2	31.3725	8	2	35.6220
6	8	6	24.1399	8	8	21.7647	8	8	20.8525
7	8	11	24.1399	12	4	18.8235	12	4	15.7124
8	11	3	24.1399	12	10	31.1765	12	10	31.2191
9	13	10	24.1399	14	2	25.0980	14	2	24.5422
10	14	5	24.1399	14	8	7.4510	14	8	9.1672

Figure 6 presents the resulting net luminous flux distribution on the design surface for the solution obtained in the present work, and in the works of Mossi et al. (2007) and Smith Schneider and França (2004). All the solutions were capable of satisfying the net luminous on the design surface (specified as  $Q_{r,jd} = -1.0$ ) with an error of 3.0% or less, which would be very difficult to obtain using a trial-and-error approach. This indicates the usefulness of the inverse analysis as a designing tool for illumination systems.

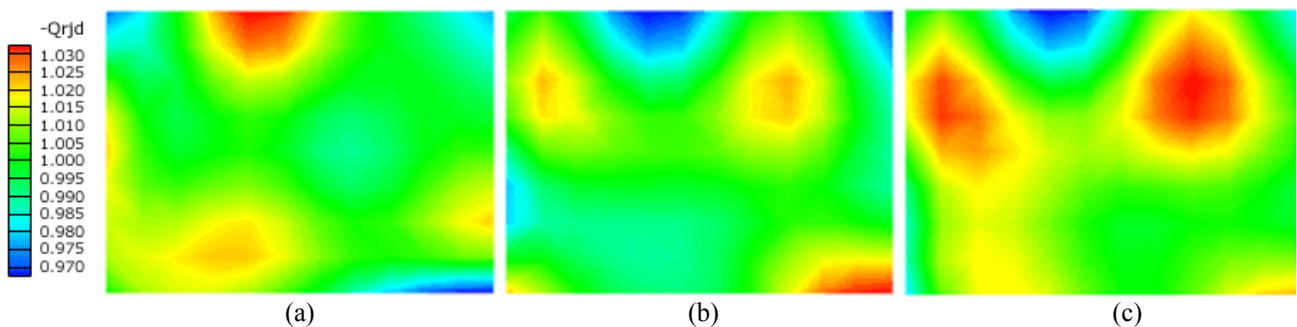


Figure 6. Dimensionless net luminous flux on the design surface. (a) present work; (b) from Mossi, 2007; (c) from Smith Schneider and França (2004).

Table 2 presents the values of the error function defined by Eq. (7), but computed with the dimensionless net luminous fluxes, for the three net luminous flux distributions on the design surface that are shown in Fig. 6. The result obtained with the new configuration of the light sources, found with the use of the GEO algorithm, was the one with the smallest value of the error function. It is expected that the error will be still smaller if the powers, as in the other previous studies, of the light sources is allowed to be different from each other.

Table 2: Minimized error function (in dimensionless form)

	$F = \sqrt{\sum_{jd} \left( q_{specified}^{(l)} - q_{r,jd}^{(l)} \right)^2}$
Present work	0.01573
Mossi et al. (2007)	0.01983
Smith Schneider and França (2004)	0.01754

## 7. CONCLUSIONS

In this paper, the generalized extremal optimization (GEO) algorithm was applied to an illumination design. Its application to a real design problem highlighted its characteristic of being easy to implement and effective to find satisfactory solutions for complex design problems. This method was applied to an inverse design problem in which the positions and the luminous powers of the light source elements were determined to satisfy a specified uniform luminous flux on the design surface. Despite the GEO algorithm requiring a larger computational effort, as typical of stochastic methods, it allows finding a larger amount of satisfactory solutions, and in general can be extended more easily to non-linear problems than it would be possible with direct regularization methods. As possible next steps, the proposed inverse design analysis can be applied to consider the effect of external illumination, to include surfaces that present both specular and diffuse reflection characteristics, to take into account the directional and/or wavelength dependency of the surface emissivities, and to consider the problem of finding the optimum number of the light sources.

## 8. ACKNOWLEDGEMENTS

FC thanks CNPq due to its financial support by means of a Masters Degree scholarship. FHRF and AJSN thank CNPq for research grants 304535/2007-9 and 300171/97-8, respectively.

## 9. REFERENCES

- Bak, P. and Sneppen, K., 1993, "Punctuated Equilibrium and Criticality in a Simple Model of Evolution", *Physical Review Letters*, Vol. 71, Number 24, pp. 4083-4086.
- Boettcher, S. and Percus, A.G., 2001, "Optimization with Extremal Dynamics", *Physical Review Letters*, Vol. 86, pp. 5211-5214.
- Boast W.B., 1953, *Illumination Engineering*, McGraw-Hill, New York.
- EEE, 2002, "Empresa de equipamento elétrico SA". Águeda, Portugal. [www.eee.pt](http://www.eee.pt)
- Hansen, P. C., 1990, Truncated SVD Solutions to Discrete Ill-Posed Problems With Ill-Determined Numerical Rank, *SIAM J. Sci. Statist. Comput.*, 11, pp. 503-518.
- Harrison, W., and Anderson, E. A., 1916, "Illumination Efficiencies as Determinated in an Experimental Room," *Trans. Illum. Eng. Soc.*, 11, pp 67-91.
- Harrison, W., and Anderson, E. A., 1920, Coefficients of Utilization, *Trans. Illum. Eng. Soc.*, 15, pp 97-123.
- Mark, S., 2000, *The IESNA Lighting Handbook*, Illuminating Engineering Society of North America, New York.
- Moon P., 1941, "Interreflections in Rooms," *J. Optical Soc. Am.*, 31, pp 374-382.
- Moon, P., and Spencer, D. E., 1946a, "Light Distribution in Rooms," *J. Franklin Inst.*, 242, pp 111-141.
- Moon, P., and Spencer, D. E., 1946b, "Light Design by the Interreflection Method," *J. Franklin Inst.*, 242, pp 465-501.
- Mossi, A. C., De Souza, F. L., Silva Neto, A. J., Schneider, P. S., França, F. H. R., 2007. "Application of the Generalized Extremal Optimization (GEO) Algorithm in an Illuminatuon Inverse Design". In: 19th International Congress of Mechanical Engineering, COBEM. Brasília.
- OSRAM, 2005, "OSRAM – Luz, Lâmpadas, Reatores e Sistemas de Iluminação", [www.osram.com.br](http://www.osram.com.br)
- Smith Schneider, P., and França, F. H. R., 2004, "Inverse Analisis Applied to an Illumination design". Proceedings of ENCIT 2004, CIT04-0306. ABCM, Rio de Janeiro.
- Seewald, A., 2006, "Projeto Inverso em Cavidades Radiantes com Superfícies Não-Cinzas. Uma Abordagem para Projetos de Iluminação", *Dissertação de Mestrado*, Departamento de Pós-Graduação em Engenharia Mecânica (PROMEC), UFRGS.
- Seewald, A., Smith Schneider, P. S., and França, F. H. R., 2006, "Inverse Design of an Illumination System Having Non-Gray Walls. Proceedings of ENCIT 2006, CIT06-0364. ABCM, Curitiba.
- De Sousa, F. L., Ramos, F. M., Paglione, P., and Girardi, R. M., 2003, "New Stochastic Algorithm for Design Optimization", *AAIA Journal*, Vol. 41., No. 9., pp. 1808-1818.

## 9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.