Inverse Design of Thermal Systems with Non-Gray Walls

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Abstract. *This work investigates the application of the inverse analysis to a thermal process in a three-dimensional rectangular enclosure having non-gray surfaces. The problem consists of finding the power of the heating elements, located on the top of the enclosure, that satisfies a prescribed uniform heat flux on the design surface, located on the bottom surface. The solution assumes that all the surfaces emit and reflect diffusely, but the hemispheric spectral emissivities are dependent on the wavelength. As a result, the inverse analysis is described by a system of non-linear equations that is expected to be ill-conditioned since it involves the solution of a Fredholm-type integral equation of the first kind. To deal with the ill-conditioned system of equations, the Truncated Singular Value Decomposition (TSVD) regularization method is applied. The non-linear problem is tackled by an iterative approach, which is based on first assuming a uniform distribution of the luminous energy in the spectrum bands, and then correcting the distribution according to the emission characteristics of the heat sources. The results will include the heat input required in the heaters and the errors of the inverse solution for different levels of regularization.*

Keywords: *Thermal processing, non-linear inverse problems, thermal radiation, TSVD regularization*

1. INTRODUCTION

Thermal processing of materials, such as metals and silicon wafers, requires controlled heating of materials. In such processes, not only the uniformity of the temperature is required but also that it follows a specified time-dependent curve. This can be achieved only by means of a carefully controlled heat flux on the surface of the processed material, as established by the energy balance, so in fact both the temperature and heat flux are imposed. The design problem consists of finding the heating conditions in the system such that the time-dependent temperature can be achieved.

Two methodologies can be applied for the solution of the design problem. The first one, the forward design, is based on the conventional approach of setting one single condition to every region of the system. For instance, the positions and temperatures of the heating devices are specified, together with the temperature of the design surface, and then it is verified if the specified heat flux on the design surface is attained. This approach will probably require a large number of attempts before reaching a satisfactory answer. In the inverse design, the power inputs and/or position of the heating devices are directly determined from the two conditions imposed on the design surface, avoiding the trial-and-error steps of the forward approach. The inverse model allows the prescription of two conditions in some boundaries, while other boundaries are left unconstrained. For problems that involve thermal radiation heat transfer, this type of formulation is described by a Fredholm integral-type equation of the first kind, known to result in ill-posed problems that can be solved only by means of regularization methods (Hansen, 1998]). Despite of the difficulties, there has been a considerable advance in the inverse design, including solutions that involve combined-mode heat transfer and transient processes. Some of the key advances in the inverse design in radiative systems can be found in França et al. (2002), França and Howell (2006), Daunn et al. (2006) and Mossi et al. (2008).

Despite the considerable advances in the inverse design, there are still important areas that have not been explored. All the above solutions considered that the surfaces were gray absorbers and emitters, which allows treating thermal radiation as a linear problem on the unknown radiosities of the surface elements. However, there are a number of cases in which the wavelength dependence of the thermal radiation cannot be neglected, which requires the integration of the thermal radiation in the wavelength spectrum or, when it is possible, setting the radiative balance in all the bands where the emissivities of the surfaces can be assumed uniform. In the inverse design analysis, there is the additional difficulty that it is unknown how the prescribed heat flux is distributed in the wavelength. A similar problem has been considered in Seewald et al. (2006), who considered illumination design of a space formed by walls that were not gray in the visible region of wavelength spectrum.

This paper considers the inverse design of a three-dimensional rectangular enclosure formed by surfaces that are diffuse but not gray. The objective is to find the power input distribution in the heater located at the top of the enclosure so that both the prescribed temperature and heat flux are achieved in the design surface. The energy transport is governed by thermal radiation, which is solved through the discretization of the radiative terms of the energy equation in wavelength bands where the emissivities of all surfaces can be assumed uniform. The resulting system of equations, assembled for each band, is expected to be ill-conditioned, since it arises from the discretization of a Fredholm-type integral equation of the first-kind. The set of equations is solved by first relating the known temperatures and heat fluxes of the design surface elements directly to the unknown radiosities of the heater elements. The ill-conditioned nature of the system is treated by means of the truncated singular value decomposition (TSVD).

2. PROBLEM DEFINITION AND FORMULATION

Figure 1 presents a schematic view of a three-dimensional enclosure, which is formed by non-gray, diffuse surfaces. The space inside the enclosure is filled with a transparent medium, so heat is transported solely by thermal radiation exchanges among the surfaces. The design surface and the heater are located on the bottom and top of the enclosure. The remainder of the enclosure is formed by walls that are isolated, albeit not ideally, from the outside. The length, width and height of the enclosure are L, W and H.

As depicted in Fig. 2, the enclosure is divided into finite-size square elements, $\Delta x = \Delta y = \Delta z$, to which the energy balance is applied. The elements in the design surface, heater and wall are designated by *jd*, *jh* and *jw*, respectively. When a general relation applies to any kind of surface element, the index *j* will be used.

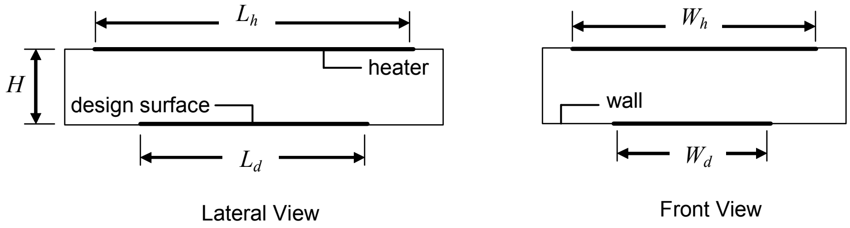


Figure 1. Schematic of the radiative enclosure

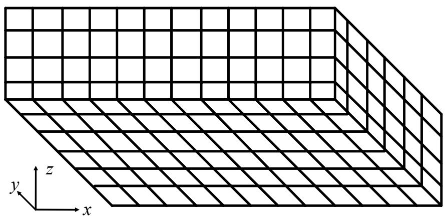


Figure 2. Division of the bottom and two side surfaces of the enclosure into finite size elements

It is considered that both the temperature and the heat flux are imposed on the design surface elements, while no boundary condition is imposed on the heater elements. In fact, the conditions in the heater elements are to be found so that the specifications on the design surface can be attained. Since heat transfer is governed by thermal radiation, the heat flux corresponds to the (net) radiative heat flux, a balance between the emission and absorption by each surface element.

2.1. Forward formulation

In the forward formulation, one thermal condition, either the temperature or the net heat flux, is specified on every surface element that forms the enclosure. It is considered here that the temperature is the imposed condition; in turn, the radiative heat flux is to be determined from the radiative balance in each element. For surfaces having spectral emissivities that vary with the wavelength, the radiative balance must be set for each wavelength, and then the total quantities are obtained from the integration of the spectral quantities in the entire wavelength spectrum.

The spectral radiosity of a surface element, $q_{o,\lambda,j}$, is given by:

$$q_{o,\lambda,j} = \varepsilon_{\lambda,j} e_{\lambda b,j} + (1-\varepsilon_{\lambda,j}) \sum_{j'=1}^{J} F_{j-j'} q_{o,\lambda,j'} \tag{1}$$

The first and the second terms of the right-hand side correspond to the spectral emissive power of the surface element and to the reflection of the spectral irradiation. In Eq. (1), $e_{\lambda b,j}$ is the blackbody spectral emissive power at the temperature of the surface element, T_j; $\varepsilon_{\lambda,j}$ is the spectral hemispherical emissivity of element *j*; and $F_{j-j'}$ is the view factor between surface elements *j* and *j*'. Assuming the surfaces to be diffuse, the spectral hemispherical emissivity and absorptivity are equal so that $(1-\varepsilon_{\lambda,j})$ is actually the spectral reflectivity.

The total radiosity is found from the integration of $q_{o,\lambda,j}$ over the entire spectrum:

$$q_{o,j} = \int_{\lambda} q_{o,\lambda,j} d\lambda \quad (2)$$

The integration in the spectrum can be considerably simplified if the spectrum can be divided into L bands $\Delta\lambda_l$ where the spectral emissivities of all surfaces can be assumed constant. This is exemplified in Fig. 3(a), which shows the division of the spectrum in three bands, but the present methodology can be used for the division of the spectrum in as many bands as it is necessary to assure the non-variation of all the spectral emissivities in each band. The partial radiosity, $q_{o,\Delta\lambda,j}$, are defined as the integral of $q_{o,\lambda,j}$ over the band $\Delta\lambda_l$:

$$q_{o,\Delta\lambda,j} = \int_{\Delta\lambda_l} q_{o,\lambda,j} d\lambda \quad (3)$$

Combining Eqs. (1) and (3) leads to:

$$q_{o,\Delta\lambda_l,j} = \varepsilon_{\Delta\lambda_l,j} f_{\Delta\lambda_l,jd} e_{b,j} + (1 - \varepsilon_{\Delta\lambda_l,j}) \sum_{j'=1}^{J} F_{j-j'} q_{o,\Delta\lambda_l,j'} \quad (4)$$

in which $e_{b,j}$ is the total blackbody emissive power of the surface element, given by σT_j^4, with σ representing the constant of Stefan-Boltzmann; $\varepsilon_{\Delta\lambda_l,j}$ is the spectral emissivity of surface element j in the band $\Delta\lambda_l$; and $f_{\Delta\lambda_l,jd}$ is the fraction of the emissive power of the blackbody at the temperature T_{jd} within the band $\Delta\lambda_l$, which is solely dependent on T_{jd} and the location of $\Delta\lambda_l$ in the spectrum. Equation (4) can be written for each band $\Delta\lambda_l$ to form a system of linear equations on the unknown partial radiosities $q_{o,\Delta\lambda_l,j}$. The numbers of equations and of unknowns are both the same and equal to $J = JD+JH+JW$; in addition, the system of equations is well behaved, and can be solved with any standard method such as the Gaussian elimination or Gauss-Seidel iterative method.

Solving the systems for all the bands $\Delta\lambda_l$ allows the determination of the partial radiosities, $q_{o,\Delta\lambda_l,j}$, of all surface elements j. Next step is to determine the total net radiative heat flux on every surface element. The spectral net radiative heat flux, $q_{r,\lambda,j}$, can be obtained from a balance between the radiosity and the irradiation in surface element j:

$$q_{r,\lambda,j} = q_{o,\lambda,j} - \sum_{j'=1}^{J} F_{j-j'} q_{o,\lambda,j'} \quad (5)$$

Integrating Eq. (5) over the band $\Delta\lambda_l$ allows establishing a relation for the partial radiative heat flux, $q_{r,\Delta\lambda_l,j}$:

$$q_{r,\Delta\lambda_l,j} = q_{o,\Delta\lambda_l,j} - \sum_{j'=1}^{J} F_{j-j'} q_{o,\Delta\lambda_l,j'} \quad (6)$$

Since, at this point, the partial radiosities $q_{o,\Delta\lambda_l,j}$'s are known, the partial radiative heat flux can be readily found from Eq. (6). Finally, the net radiative heat flux is found from the integration of $q_{r,\lambda,j}$ over the entire spectrum or from the summation of $q_{r,\Delta\lambda_l,j}$ in all bands $\Delta\lambda_l$:

$$q_{r,j} = \sum_{l=1}^{L} q_{r,\Delta\lambda_l,j} \quad (7)$$

The determination of the net radiative heat flux from Eq. (7) completes the analysis with the forward formulation. In problems in which both the temperature and the heat flux are imposed on the design surface, using the forward formulation requires that the temperatures of the heater elements are guessed until the desired net heat flux is attained on the design surface. This procedure can require a very large number of attempts to achieve a satisfactory solution, if it is ever achieved at all.

2.2. Inverse Formulation

The inverse formulation allows the specification of both the temperature and the radiative heat flux on the design surface, while the heater surface is left unconstrained. Typically, one condition is known for the wall surface, either the temperature or the radiative heat flux. It is considered here that the temperature is known.

As with the forward formulation, the radiative balances can be applied to each band $\Delta\lambda_l$ to take the advantage of the spectral emissivities $\varepsilon_{\Delta\lambda_l,j}$ being constant. One difficulty that arises is that, while the prescribed heat flux, q_{design}, is known, the partial radiative heat flux in each band, $q_{r,\Delta\lambda,jd}$, is unknown. As will be seen, the knowledge or at least a estimative of $q_{r,\Delta\lambda,jd}$ is required to establish the relations of the inverse formulation.

The total and the partial radiative heat fluxes are related by the following relation:

$$q_{design} = \sum_{l=1}^{L} q_{r,\Delta\lambda_l,jd} \tag{8}$$

Considering as a first approximation that the radiative heat flux is equally distributed in each band $\Delta\lambda_l$, one finds that:

$$q_{r,\Delta\lambda_l,jd} = \frac{q_{design}}{L} \tag{9}$$

In general, the above relation is not correct, but allows the start of the inverse calculation. It will be shown later how to improve the estimation until the correct values of $q_{r,\Delta\lambda,jd}$ are obtained.

Starting with the estimated values of $q_{r,\Delta\lambda,jd}$, Eqs. (4) and (6) can be combined to determine the partial radiosity of each design surface from its emissive power and partial radiative heat flux:

$$q_{o,\Delta\lambda_l,jd} = f_{\Delta\lambda_l,jd} e_{b,jd} - \frac{(1-\varepsilon_{\Delta\lambda_l,jd})}{\varepsilon_{\Delta\lambda_l,jd}} q_{r,\Delta\lambda_l jd} \tag{10}$$

Therefore, the above relation allows the direct determination of the partial radiosities of the design surface elements. Next, Eq. (10) is applied to Eq. (4) for each design surface element, and then rearranged to render the relations for the partial radiosities of the heater elements, $q_{o,\Delta\lambda_l,jh}$.

$$\sum_{jh=1}^{JH} F_{jd-jh} q_{o,\Delta\lambda_l,jh} = (q_{o,\Delta\lambda_l,jd} - q_{r,\Delta\lambda_l,jd}) - \left(\sum_{jd'=1}^{JD} F_{jd-jd'} q_{o,\Delta\lambda_l,jd'} + \sum_{jw=1}^{JW} F_{jd-jw} q_{o,\Delta\lambda_l,jw} \right) \tag{11}$$

It should be noticed that employing Eq. (4) to establish the relations for $q_{o,\Delta\lambda_l,jh}$ is not advantageous here, since the temperatures (and the blackbody emissive powers) of the heater elements are unknown. Instead, the relations for $q_{o,\Delta\lambda_l,jh}$ are "borrowed" from the design surface, as given by Eq. (11). As for the wall elements, for which the temperatures are assumed to be known, Eq. (4) takes the following form:

$$q_{o,\Delta\lambda_l,jw} = \varepsilon_{\Delta\lambda_l,jw} f_{\Delta\lambda_l,jw} e_{b,jw} + (1-\varepsilon_{\Delta\lambda_l,jw}) \left(\sum_{jd=1}^{JD} F_{jw-jd} q_{o,\Delta\lambda_l,jd} + \sum_{jh=1}^{JH} F_{jw-jh} q_{o,\Delta\lambda_l,jh} + \sum_{jw'=1}^{JW} F_{jw-jw'} q_{o,\Delta\lambda_l,jw'} \right) \tag{12}$$

Equations (11) and (12) form a system of linear equations on the unknown radiosities of the heater and wall elements, $q_{o,\Delta\lambda_l,jh}$ and $q_{o,\Delta\lambda_l,jw}$. One difficulty is that the numbers of equations and of unknowns, (*JD*+*JW*) and (*JH*+*JW*), respectively, are not necessarily the same. The second difficulty is that the system is typically ill-conditioned, so even when the numbers of equations and unknowns are the same they cannot be solved by the standard methods, requiring regularization to lead to meaningful results. The methodology for the solution of the system is discussed in Section 4, which presents the TSVD regularization.

Once the system of equations are solved for the unknown partial radiosities for each band $\Delta\lambda_l$, the emissive power of the heater elements can be determined from rearranging Eq. (4):

$$e_{b,jh}^{(\Delta\lambda_l)} = \frac{1}{\varepsilon_{\Delta\lambda_l,jh} f_{\Delta\lambda_l,jh}} \left[q_{o,\Delta\lambda_l,jh} - (1-\varepsilon_{\Delta\lambda_l,jh}) \left(\sum_{jd=1}^{JD} F_{jh-jd} q_{o,\Delta\lambda_l,jd} + \sum_{jh'=1}^{JH} F_{jh-jh'} q_{o,\Delta\lambda_l,jh'} \sum_{jw=1}^{JW} F_{jh-jw} q_{o,\Delta\lambda_l,jw} \right) \right] \tag{13}$$

Equation (13) allows the determination of the emissive power of a given heater element from the radiative balance in every band $\Delta\lambda_l$ that spans the spectrum. However, when applying Eq. (13) for all the bands, the obtained values of the emissive power will not necessarily be the same, as required by the very definition of the emissive power, which is solely dependent on the absolute temperature of the heater element, $e_{b,jh} = \sigma T_{jh}^4$. In fact, the inverse analysis has been based so far on the assumption that the net radiative heat flux on the design surface is equally distributed in all bands, as given by Eq. (9), which results that the radiative balances in the bands are decoupled from each other. However, each band affects the other in the global radiative balance, an effect that can be accounted in the following way.

First, for each heater element, it is defined an average emissive power $\bar{e}_{b,jh}$ from the emissive power $e_{jh}^{(\Delta\lambda_l)}$, computed from the radiative balance in each band $\Delta\lambda_l$ in Eq. (13):

$$\bar{e}_{b,jh} = \sum_{l=1}^{L} f_{\Delta\lambda_l,jh} e_{jh}^{(\Delta\lambda_l)} \tag{14}$$

Equation (14) assures that the emissive power of the heater element corresponds to the summation of the emissive power of the heater element from each band $\Delta\lambda_l$, $f_{\Delta\lambda_l,jh} e_{jh}^{(\Delta\lambda_l)}$. Note that $\bar{e}_{jh} = e_{jh}^{(\Delta\lambda_l)}$ ($l = 1,\ldots L$) when the values of emissive powers are the same for all the bands. From the average emissive power, one can then compute an average absolute temperature for each heater element:

$$\bar{T}_{jh} = \left(\bar{e}_{b,jh} / \sigma\right)^{1/4} \tag{15}$$

Next, the forward formulation of Section 2.1 is run setting, as the boundary conditions, the prescribed temperature for the design surface and the walls, and the average temperatures as computed from Eq. (15) for the heater elements. The solution of the forward formulation will lead to the partial radiative heat flux on each design surface element, to be designated as $\bar{q}_{r,\Delta\lambda_l,jd}$. Finally, a new proposition can be made for the distribution of the partial radiative heat flux on the design surface as an improvement of the uniform distribution of Eq. (9), based now on a proportionality between the partial radiative heat fluxes that arises from the newly computed average temperatures on the heater elements:

$$q_{r,\Delta\lambda_l,jd} = \frac{\bar{q}_{r,\Delta\lambda_l,jd}}{\sum_{l=1}^{L} \bar{q}_{r,\Delta\lambda_l,jd}} q_{design} \tag{16}$$

With the new values of the partial radiative heat fluxes on the design surface elements, $q_{r,\Delta\lambda_l,jd}$, the inverse analysis described by Eqs. (10) to (16) is rerun until convergence on each $q_{r,\Delta\lambda_l,jd}$ is achieved.

3. TSVD REGULARIZATION

Based on the procedure proposed in França et al. (2002), the following approach is adopted. The partial radiosities of the wall elements are initially neglected in equation (11). Therefore, the unknowns are only the partial radiosities of the heater elements, $q_{o,\Delta\lambda_l,jh}$. Once equation (11) is written for each of the *JD* elements that form the design surface, a system with *JD* equations will be formed. The unknowns are the outgoing luminous fluxes on the *JH* heater elements. Therefore, the number of equations and the number of unknowns are not necessarily the same, unless *JD* = *JH*. In addition, since the problem corresponds to a discrete form of a Fredholm-type integral equation of the first-kind, one should expect the system of equations to be ill-conditioned.

The solution is achieved by means of the following procedure. First, the partial radiosities of the wall elements, $q_{o,\Delta\lambda_l,jw}$, is set equal to zero. Then, the system of equations formed by equations (11) allows the determination of the radiosities of the heater elements, $q_{o,\Delta\lambda_l,jh}$. Next, equation (12) is applied to each wall element *jw* to form a system of equations on $q_{o,\Delta\lambda_l,jw}$. Once the system is solved, the newly computed $q_{o,\Delta\lambda_l,jw}$ are inserted into the system of equations formed by equations (11), and the procedure is repeated until convergence is achieved. Finally, with the converged values of $q_{o,\Delta\lambda_l,jh}$, equation (14) is applied to the emissive power $e_{jh}^{(\Delta\lambda_l)}$ of each band $\Delta\lambda_l$.

The above procedure involves the solution of a system of linear equations on the emissive powers of the heater elements for each band $\Delta\lambda_l$, as formed by equation (11), which can be represented by:

$$\mathbf{A}\cdot\mathbf{x}=\mathbf{b}(\mathbf{x}) \tag{17}$$

where matrix **A** is formed by the view factors between the design surface and the heaters elements, F_{jd-jh}; vector **x** represents the unknown outgoing luminous fluxes on the light sources, $q_{o,\Delta\lambda_l,jh}$; and vector **b** contains the terms on the right-hand side of equation (11).

The system of equations represented in Eq. (17) is ill-conditioned, so the truncated singular values decomposition (TSVD) is employed for its solution. First, matrix **A** is decomposed into three matrices:

$$\mathbf{A}=\mathbf{U}\cdot\mathbf{W}\cdot\mathbf{V}^{\mathbf{T}} \tag{18}$$

where **U** and **V** are orthogonal matrices, and **W** is a diagonal matrix formed by the singular values w_n. As a consequence, the solution vector **x** can be computed by:

$$\mathbf{x}=\sum_{n=1}^{JL}\left(\frac{b_m\cdot u_{mn}}{w_n}\right)\mathbf{v}_n \tag{19}$$

Typically, the singular values w_n decay continuously to very small values, which causes **x** to be formed by unrealistic numbers. The TSVD regularization consists of eliminating from Eq. (19) the terms related to the p largest singular values. The solution is the vector **x** with the smallest norm subjected to minimum deviation $|\mathbf{A}\cdot\mathbf{x}=\mathbf{b}|$. Another useful feature of the TSVD method is that it can also be applied to the situation where the numbers of unknowns and equations are not the same, as will be shown in the results section. Due to the need for regularization of the system of equations, an exact solution is not expected. The following procedure is used for the verification of the solution. Once the radiative heat fluxes on the heater elements are obtained, a forward problem is solved where the radiative heat fluxes on the heater elements are imposed together with the temperatures of the design surface and walls. The radiative heat flux on each element jd of the design surface is then calculated, and compared to the specified heat flux by:

$$\gamma_{jd}=\left|\frac{q_{design}-q_{r,jd}}{q_{design}}\right| \tag{20}$$

Once γ_{jd} is calculated for each element jd in the design surface, the arithmetic average and the maximum errors, γ_{avg} and $\gamma_{\max}$, and can be readily found.

4. RESULTS

The case considered in this work is a three-dimensional enclosure, with the following aspect ratios: W/L = 0.8, H/L = 0.2 (see Fig. 1). The selection of the other dimensions of the enclosure requires a few additional considerations. First, the design surface ought not to cover the entire extension of the base, since the portions close to the corners would be mainly affected by the reflections from the side walls, not from the direct irradiation from the heater elements on the top surface. Therefore, the design surface dimensions are chosen to be L_d/L = 0.8 and W_d/L = 0.6. The heater elements are allowed to cover the entire top surface: L_d/L = 1.0 and W_d/L = 0.8.

The dimensionless heat flux on the design surface is specified as $Q_{r,jd}=q_{design}/\sigma T_{ref}^4=-1.0$, where T_{ref} = 800 K. The negative signal arises from the adopted convention that net radiative heat flux *out* of the surface is positive. The imposed dimensionless temperatures ($t = T/T_{ref}$) of the elements of the design surface and walls are $t_{jd}=t_{jw}=0.375$. The spectral hemispherical emissivities of the surfaces are divided in three bands as shown in the Fig. 3(a): $\Delta\lambda_1\le 5.0\,\mu\text{m}$, $5.0\,\mu\text{m}\le\Delta\lambda_2\le 10.0\,\mu\text{m}$, and $\Delta\lambda_3\ge 10.0\,\mu\text{m}$. As seen, for the elements on the design surface: $\varepsilon_{\Delta\lambda_1,jd}=0.5$, $\varepsilon_{\Delta\lambda_2,jd}=0.7$, and $\varepsilon_{\Delta\lambda_3,jd}=0.9$; for the elements on the wall: $\varepsilon_{\Delta\lambda_1,jw}=0.8$, $\varepsilon_{\Delta\lambda_2,jw}=0.7$, and $\varepsilon_{\Delta\lambda_3,jw}=0.9$; for the heater elements: $\varepsilon_{\Delta\lambda_1,jh}=0.5$, $\varepsilon_{\Delta\lambda_2,jh}=0.5$, and $\varepsilon_{\Delta\lambda_3,jh}=0.7$.

The procedure outlined in Section 2 and 3 is then applied to determine the net radiative heat flux on the heater elements. The singular value decomposition of matrix **A** led to the singular values shown in Fig. 3(b). Since it is formed by the view factors between elements on the design surface and on the heater surface, matrix **A** and its singular values remain the same for all bands. This allows the application of the same regularization to the system of equations for the three bands. As seen in Fig. 3(b), the singular values decay to very small values, down to 10^{-9}. This causes the components of vector **x,** given by Eq. (9), to present very large, unrealistic values. Figures 4(a) and 4(b) present the dimensionless heat flux on the heater elements ($Q_{r,jh} = q_{r,jh} / \sigma T_{ref}^4$), and on the design surface ($Q_{r,jd} = q_{r,jd} / \sigma T_{ref}^4$), for $p = 4$. As seen, the dimensionless heat flux on the design surface presents values that vary from -1.04 to -0.96, while the specified value is -1.0. The average error of the heat flux on the design surface, as described by Eq. (20), is of 2.2%. Figures 5(a) to 5(c) show the dimensionless partial heat fluxes on the heater for the three bands ($q_{r,\Delta\lambda_l,jh} / \sigma T_{ref}^4$). As seen, the energy is not uniformly concentrated in the bands, being more concentrated in the first band, $\Delta\lambda_1 \leq 5.0\,\mu\text{m}$. The fraction of the total energy from the heater elements in bands 1, 2 and 3 correspond to 61%, 27% and 12%.

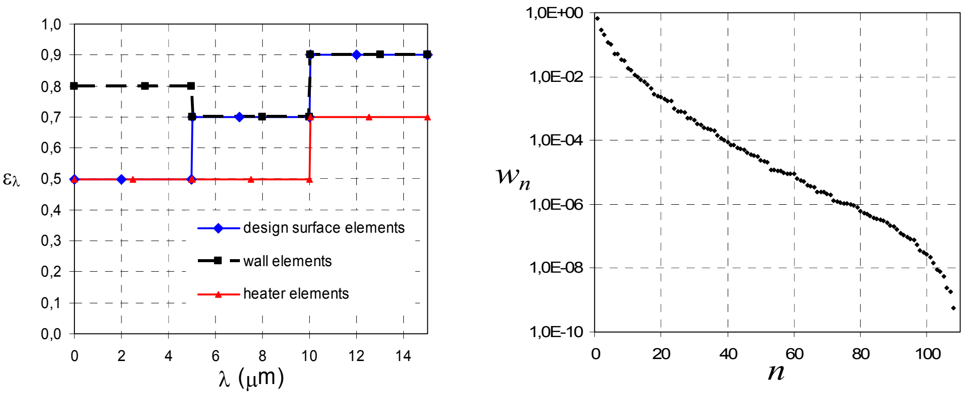


Figure 3. (a) Spectral emissivities of the surfaces; (b) Singular values of the matrix **A** in Eq. (17).

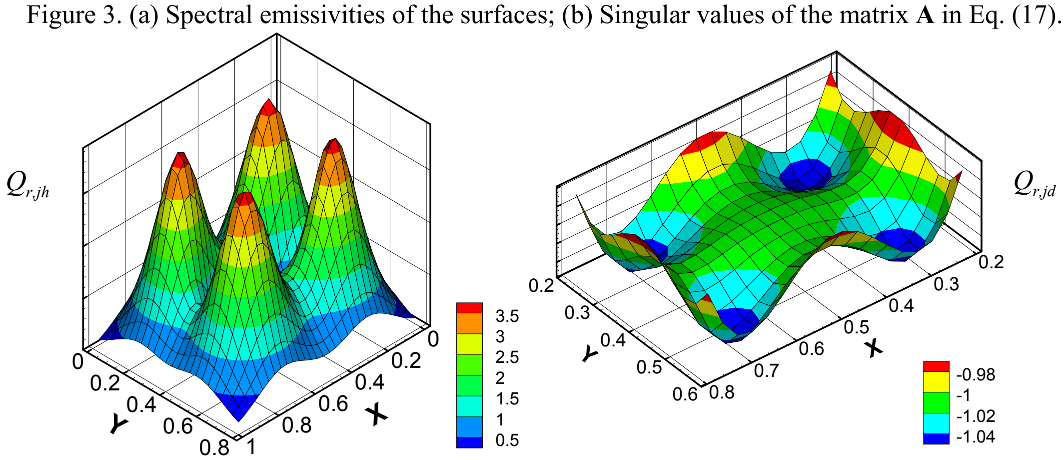
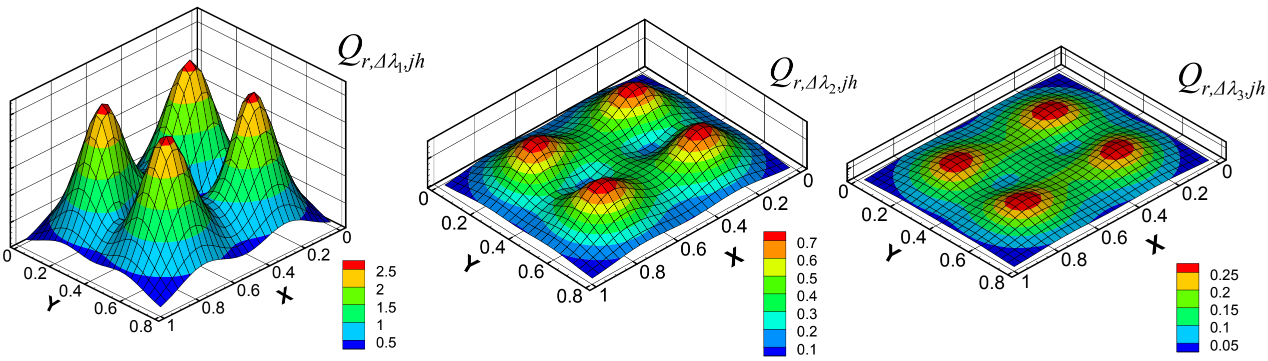


Figure 4. (a) Total heat flux of heater elements; (b) Total heat flux of design surface.



(a) **(b)** **(c)**

Figure 5. Partial heat fluxes on the heater elements for (a) band 1; (b) band 2; and (c) band 3.

5. CONCLUSIONS

This work considered the inverse design of a three-dimensional radiative enclosure in which uniform temperature and heat flux were imposed on the design surface. The problem consisted of determining the powers of the heater elements that were capable of satisfying the two conditions imposed on the design surface. The major contribution of the present work was present a methodology that considers enclosures formed by non-gray surfaces. The solution was based on the application of the radiative balance in each wavelength band in which the spectral emissivities of the surfaces can be assumed independent of the wavelength. One particular aspect of this problem is that, while the total radiative heat flux on the design surface is known, the partial amounts were unknown. To overcome this difficulty, it was proposed an iterative procedure in which the partial radiative heat fluxes were guessed, and the correction was based on determining an average blackbody emissive power for each heater element from the emissive powers obtained from the solution for each band. The core of the solution was formed by a system of equations relating the unknown partial radiosities of the heater elements to the prescribed conditions on the design surface. As usual in the inverse design approach, the resulting systems of equations were ill-conditioned, which are known to require special solution techniques. The solution was accomplished with the truncated singular valued decomposition (TSVD) regularization. In the present case, only a few terms of the exact SVD series that form the solution were kept in the summation.

For the example presented in this paper, the proposed inverse analysis was capable of providing the prescribed radiative heat flux on the design surface within an average error of about 2.0 %. While the proposed methodology starts with the assumption that the amount of radiative energy in all bands are the same, the iterative procedure proved to be effective to allow the amount of energy to vary from band to band, depending on the spectral emissivities of the surfaces and on the temperatures in the enclosure. In the solved example, there was a considerable difference between the amounts of energy in each band.

6. ACKNOWLEDGEMENTS

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